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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAS 20603
PROGRAMME	:	2 DAE / 3 DAE
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **SECTION A** B) ANSWER **THREE (3)** QUESTIONS IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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SECTION A

Q1 Given the differential equation $y''(x) - 2y'(x) + y = \dfrac{e^x}{x^2+1}$.

(a) Find

(i) y_1 and y_2.

(6 marks)

(ii) the Wronskian, W.

(4 marks)

(iii) u_1 and u_2.

(6 marks)

(iv) the particular solution, $y_p(x)$.

(2 marks)

(b) Write the general solution, $y(x)$

(2 marks)

Q2 Given the ordinary differential equation $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 2y = 14e^x + \sin 3x$.

(a) Solve:

(i) the homogeneous equation $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 2y = 0$.

(5 marks)

(ii) the non homogeneous equation $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 2y = 14e^x$.

(6 marks)

(iii) the non homogeneous equation $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 2y = \sin 3x$.

(7 marks)

(b) Write the general solution of

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 2y = 14e^x + \sin 3x$$

(2 marks)

SECTION B

Q3 (a) Evaluate $\int \frac{10x^2 - 3x + 14}{(x-1)(x^2+2)}\, dx$ by using partial fraction method.

(10 marks)

(b) Find the approximate value for $\int_1^4 \frac{x}{\sqrt{x^2+2}}\, dx$ using $\frac{1}{3}$ Simpson's rule by taking step size, $h = 0.5$. Do the calculations in 3 decimal places.

(10 marks)

Q4 (a) Given $y = 3 - x^2$, $y = -x + 1$, $x = 0$ and $x = 2$.

(i) Sketch the graph.

(4 marks)

(ii) Evaluate the area of the region enclosed by $y = 3 - x^2$, $y = -x + 1$, $x = 0$ and $x = 2$.

(6 marks)

(b) Calculate the arc length of $x = \frac{2}{3}(y-1)^{\frac{3}{2}}$ between $1 \le y \le 4$.

(10 marks)

Q5 (a) Solve the initial value problem of the given differential equation.

$$xy' + 2y = \cos x \quad ; \quad y(\pi) = 0$$

(10 marks)

(b) Given differential equation of

$$2x \sin y + \left(x^2 \cos y - y^2\right)\frac{dy}{dx} = 0$$

(i) Determine that the given equation is an exact equation.

(2 marks)

(ii) Hence, solve the equation using the given initial value problem, $y(\pi) = 0$.

(8 marks)

Q6 (a) A bacteria culture starts with 500 bacteria and grows at a rate proportional to its original size. After 3 hours there are 8000 bacteria.

(i) If N_t is the number of culture at time t, N_0 the culture at $t = 0$, and k is the proportional constant, derive a mathematical equation showing the number of culture at any time, t.

(6 marks)

(ii) Evaluate the value of k from the given data.

(2 marks)

(iii) Calculate the number of bacteria after 24 hours.

(2 marks)

(b) In a murder investigation, a corpse was found by a detective at exactly 8 pm. Being alert, the detective also measured the body temperature and found it to be 70 °F. Two hours later, the detective measured the body temperature again and found it to be 60 °F. If the room temperature is 50 °F, the body temperature of the person before death was 98.6 °F, and assuming the rate of temperature change follows Newton's Law of Cooling, determine when did the murder occurred?

(10 marks)

Q7 (a) Evaluate

(i) $\int_0^1 (2+5x)e^{\frac{1}{3}x}\,dx$ using part by part technique only.

(7 marks)

(ii) $\int_1^\infty \frac{1}{4}e^{-2x}\,dx.$

(5 marks)

(b) Solve $y''-6y'+8y=0$ with initial value $y(0)=3$ and $y'(0)=1$.

(8 marks)

- END OF QUESTION -

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Formulae

Second Order Linear Differential Equation

Method of Undetermined Coefficients : $ay'' + by' + cy = f(x)$

$f(x)$	$y_k(x)$
$P_n(x) = A_n x^n + A_{n-1} x^{n-1} + \ldots + A_1 x + A_0$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r (Pe^{\alpha x})$
$C \cos \beta x$ atau $C \sin \beta x$	$x^r (p \operatorname{kos} \beta x + q \sin \beta x)$

Notes : r is the smallest non negative integers to ensure no alike terms between $y_k(x)$ and $y_h(x)$.

Variation of Parameter :

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1' \;;\quad u_1 = -\int \frac{y_2 f(x)}{W} dx \;;\quad u_2 = \int \frac{y_1 f(x)}{W} dx$$

The particular solution : $y_p(x) = u_1 y_1 + u_2 y_2$

The general solution : $y(x) = y_p(x) + y_h(x)$.

Trigonometric identity :

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

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Formulae

Differentiation :

$\frac{d}{dx}[c]=0$	$\frac{d}{dx}[\sin x]=\cos x$	$\frac{d}{dx}[\cot x]=-\csc^2 x$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\cos x]=-\sin x$	$\frac{d}{dx}[\csc x]=-\csc x\cot x$
$\frac{d}{dx}[c\,f(x)]=c\frac{d}{dx}[f(x)]$	$\frac{d}{dx}[\tan x]=\sec^2 x$	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$
$\frac{d}{dx}(uv)=vu'+uv'$	$\frac{d}{dx}[\sec x]=\sec x\tan x$	$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$

Integration :

$\int_a^b f(x)\,dx=F(b)-F(a)$	$\int\frac{dx}{x}=\ln\lvert x\rvert+C$
$\int x^n\,dx=\frac{x^{n+1}}{n+1}+C,\quad n\neq-1$	$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+C$
$\int\cos ax\,dx=\frac{1}{a}\sin ax+C$	$\int\sin ax\,dx=-\frac{1}{a}\cos ax+C$
$\int\sec^2 x\,dx=\tan x+C$	$\int\csc^2 x\,dx=-\cot x+C$
$\int\csc x\cot x\,dx=-\csc x+C$	$\int\sec x\tan x\,dx=\sec x+C$
$\int\csc x\,dx=-\ln\lvert\csc x+\cot x\rvert+C$	$\int\sec x\,dx=\ln\lvert\sec x+\tan x\rvert+C$
$\int\tan x\,dx=\ln\lvert\sec x\rvert+C$	$\int\frac{1}{x^2+1}dx=\tan^{-1}x+C$
$\int udv=uv-\int vdu+C$	

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Formulae

Area of region :

$$A = \int_a^b [f(x) - g(x)]\,dx \qquad \textbf{or} \qquad A = \int_c^d [w(y) - v(y)]\,dy$$

Volume cylindrical shells :

$$V = \int_a^b 2\pi x\, f(x)\,dx \qquad \textbf{or} \qquad V = \int_c^d 2\pi y\, f(y)\,dy$$

Arc length :

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx \qquad or \qquad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$

Simpson's rule :

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[(f_0 + f_n) + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i\right] \; ; \quad n = \frac{b-a}{h}$$

Trapezoidal rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[(f_0 + f_n) + 2\sum_{i=1}^{n-1} f_i\right] \; ; \quad n = \frac{b-a}{h}$$