Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAS 20403
PROGRAMME	:	2 DAA / DAM 3 DAA / DAM
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **PART A** ONLY. B) ANSWER **THREE (3)** QUESTIONS IN **PART B** ONLY.

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

PART A

Q1 (a) Find the inverse Laplace of following transform

(i) $\dfrac{4s+7}{s^2+9}$

(ii) $\dfrac{3}{(2s-6)}$

(iii) $\dfrac{2s+5}{(s+3)^2}$

(12 marks)

(b) (i) Express $\dfrac{2s^2-3s+2}{(s+2)(s^2+4)}$ as partial fraction.

(ii) Find the inverse Laplace of the answer from (b) (i).

(8 marks)

Q2 Solve the differential equation below by using Laplace transform.

(a) $y''-6y'+8y=0$; $y(0)=0,\ \ y'(0)=-3.$

(10 marks)

(b) $y''-7y'+12y=2$; $y=1,\ y'=5,$ when $t=0$.

(10 marks)

PART B

Q3 (a) Given

$$\left(2y+x^2+1\right)+\left(2xy-9x^2\right)\frac{dx}{dy}=0$$

(i) Show that the equation is an exact ordinary differential equation.
(ii) Find the general solution of the equation.

(11 marks)

(b) Find the solution of the given IVP differential equation.

$$x\frac{dy}{dx}+2y=x^2-x+1 \quad ; \quad y(1)=\frac{1}{2}$$

(9 marks)

Q4 (a) Just before midday the body of an apparent homicide victim is found in a room that is kept at a constant temperature of 21°C. At 12 noon the temperature of the body is 27°C and at 1 P.M. it is 24°C. Assuming that the temperature of the body at the time of death was 37 °C and the decreasing in temperature follows the Newton's Law of Cooling, determine the time of death?

(11 marks)

(b) A culture initially has N_0 number of bacteria. At time t = 1 hour the number of bacteria is measured to be $\frac{3}{2}N_o$. If the rate of growth is proportional to the number of bacteria present at any time, determine the time necessary for the number of bacteria to triple.

(9 marks)

Q5 (a) By using undetermined coefficient method, solve the equation

$$y''+4y'+5y=13e^{3x},$$

given that when $x=0$, $y=\frac{5}{2}$ and $y'=\frac{1}{2}$.

(10 marks)

(b) By using variations of parameter method, solve the equation

$$y''-4y=10e^{3x}.$$

(10 marks)

Q6 (a) Find the Laplace transforms of the following functions.

(i) $\mathcal{L}\left\{\frac{t}{2a}\sin at - 4e^{2t+3}\right\}$

(ii) $\mathcal{L}\left\{e^{2t}(t-2)(t-3)\right\}$

(iii) $\mathcal{L}\left\{\cos(\alpha t+\beta)\right\}$

Hint: $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

(15 marks)

(b) Show that $\mathcal{L}\left\{3e^{-t/2}\sin^2 t\right\} = \frac{48}{(2s+1)(4s^2+4s+17)}$

Hints : $\sin^2 t = \frac{1}{2}(1-\cos 2t)$

(5 marks)

Q7 (a) Solve the second order homogeneous differential equation.

$$y''+2y'+y=0 \quad ; \quad y(0)=1 \quad \text{and} \quad y(1)=3$$

(7 marks)

(b) Find the Inverse Laplace Transform

(i) $\mathcal{L}^{-1}\left\{\frac{7}{5s-1}-\frac{1}{s^3}+\frac{12}{s^4}\right\}$

(ii) $\mathcal{L}^{-1}\left\{\frac{5s}{2s^2+8}-\frac{2}{s^2-3}\right\}$

(6 marks)

(c) Use Laplace Transform to solve the differential equation $y'-4y=8$ given $y(0)=2$.

(7 marks)

END OF QUESTIONS

FINAL EXAMINATION

SEMESTER / SESSION: I / 2013/2014 PROGRAMME : 2 DAA / DAM
: 3 DAA / DAM

COURSE : ENGINEERING MATHEMATICS II COURSE CODE : DAS 20403

Formula

Table 1 : Laplace transform.

$L\{f(t)\}=\int_0^\infty f(t)e^{-st}\,dt = F(s)$

$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n,\ n = 1, 2, ..$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t),\ n = 1, 2, ..$	$(-1)^n \frac{d^n F(s)}{ds^n}$

Table 3 : Indefinite differentiation

$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$
$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\left(\frac{dt}{dx}\right)$
$\frac{d}{dx}\ln u = \frac{1}{u}\cdot\frac{du}{dx}$
$\frac{d}{dx}e^u = e^u\cdot\frac{du}{dx}$
$\frac{d}{dx}\sin u = \cos u\cdot\frac{du}{dx}$
$\frac{d}{dx}\tan u = \sec^2 u\cdot\frac{du}{dx}$
$\frac{d}{dx}\cot u = -\csc^2 u\cdot\frac{du}{dx}$
$\frac{d}{dx}\sec u = \sec u\tan u\cdot\frac{du}{dx}$
$\frac{d}{dx}\csc u = -\csc u\cot u\cdot\frac{du}{dx}$
$\frac{d}{dx}\cos u = -\sin u\cdot\frac{du}{dx}$

Table 2 : Initial and Boundary Value Problem

If $L\{y(t)\} = Y(s)$ then

$L\{y'(t)\} = sY(s) - y(0)$

$L\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$

FINAL EXAMINATION

SEMESTER / SESSION: I / 2013/2014 PROGRAMME : 2 DAA / DAM
: 3 DAA / DAM

COURSE : ENGINEERING MATHEMATICS II COURSE CODE : DAS 20403

Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$; Characteristic equation : $am^2 + bm + c = 0$		
Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 5 : Solution of particular solution $ay'' + by' + cy = f(x)$

$f(x)$	$y_k(x)$
$P_n(x) = A_n x^n + A_{n-1} x^{n-1} + \ldots + A_1 x + A_0$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r (Pe^{\alpha x})$
$C \cos\beta x$ atau $C \sin \beta x$	$x^r (p \text{ kos } \beta x + q \sin \beta x)$
$P_n(x)e^{\alpha x}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, e^{\alpha x}$
$P_n(x)\begin{cases} kos\beta x & atau \\ \sin\beta x \end{cases}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, kos\, \beta x$ + $x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0) \sin \beta x$
$Ce^{\alpha x}\begin{cases} kos\beta x & atau \\ \sin\beta x \end{cases}$	$x^r e^{\alpha x} (p\, kos\, \beta x + q \sin \beta x)$
$P_n(x)e^{\alpha x}\begin{cases} kos\beta x & atau \\ \sin\beta x \end{cases}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, e^{\alpha x}\, kos\, \beta x$ + $x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, e^{\alpha x} \sin \beta x$

Notes : r is the smallest non negative integers to ensure no alike terms between $y_k(x)$ and $y_h(x)$.

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: 3 DAA / DAM

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COURSE CODE : DAS 20403

Table 6 : Variation of parameters method.

Homogeneous solution, $y_h(x) = Ay_1 + By_2$

Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$

$u_1 = -\int \frac{y_2 f(x)}{aW} dx$ $\qquad$ $u_2 = \int \frac{y_1 f(x)}{aW} dx$

Particular solution, $y_p = u_1y_1 + u_2y_2$

Final solution, $y(x) = y_h(x) + y_p(x)$