UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAS 20403
PROGRAMME	:	2 DAA
EXAMINATION DATE	:	MARCH 2013
DURATION	:	3 HOURS
INSTRUCTIONS	:	ANSWER **FIVE (5)** QUESTIONS ONLY.

THIS QUESTION PAPER CONSISTS OF SIX (6) PAGES

Q1 (a) For each of the following, find $F(s)$ or $f(t)$ as indicated.

(i) $\mathscr{L}\left[2\sin t + \cos 2t\right]$

(ii) $\mathscr{L}\left[te^{2t}\right]$

(iii) $\mathscr{L}^{-1}\left[\dfrac{s+4}{s^2+4s+8}\right]$

(14 marks)

(b) Sketch the graph of the function $f(t) = \begin{cases} 1 & 0 \le t < 2 \\ 2 & 2 \le t < 4 \\ -1 & t \ge 4 \end{cases}$

Describe these functions in terms of unit step functions.

(6 marks)

Q2 (a) (i) By using the convolution theorem, find

$$f(t) = \mathscr{L}^{-1}\left[\frac{3}{s^2-1}\right]$$

(ii) Evaluate $f(0)$ and $f(1)$.

(12 marks)

(b) By using the partial fraction method, find $\mathscr{L}^{-1}\left[\dfrac{2}{s(s^2+4)}\right]$

(8 marks)

Q3 (a) Given $y = A\cos 2x + B\sin 2x - \frac{1}{5}\sin 3x$.

(i) Show that y is a general solution of the equation $y'' + 4y = \sin 3x$.

(ii) Find the particular solution if $y = 1$ and $y' = -\frac{1}{5}$ when $x = 0$.

(10 marks)

(b) Determine whether the differential equation $\dfrac{dy}{dx} = \dfrac{xy + y^2}{x^2}$ is homogeneous.

Thus, solve the equation.
(Hint: $y = xv$)

(10 marks)

Q4 (a) Consider an animal population $P(t)$ that is modelled by the equation

$$\frac{dP}{dt} = 0.004P(P-150).$$

Find $P(t)$, if $P(0) = 200$.

(10 marks)

(b) A body at a temperature of 50° F is placed outdoors where the temperature is 100° F. If after 5 minutes the temperature of the body is 60° F, find

(i) how long it will take the body to reach a temperature of 75° F.
(ii) the temperature of the body after 20 minutes.

The Newton's Law of Cooling equation for this problem is

$$\frac{dT}{dt} + kT = 100k..$$

where t is the variable for temperature in $^\circ$F, t is the variable for time in minutes and k is a constant of proportionality.

(10 marks)

Q5 (a) Find an integating factor for $y' - 2xy = x$.

(8 marks)

(b) Given $y'' - y' - 2y = e^{3x}$. By using the method of variation of parameters, answer the following questions.

(i) Find the characteristic equation, y_h of the given equation if $y'' - y' - 2y = 0$.

(ii) By using $y_p = v_1 e^{-x} + v_2 e^{2x}$ and $\phi(x) = e^{3x}$, solve the simultaneous equation

$$v_1'(e^{-x}) + v_2'(e^{2x}) = 0$$
$$v_1'(-e^{-x}) + v_2'(2e^{2x}) = e^{3x}.$$

to determine v_1 and v_2. Substitute this result into y_p.

(iii) Finally, write the general solution as $y = y_h + y_p$.

(12 marks)

Q6 (a) Given the initial value problem as follows;

$$x'' + 4x' + 4x = 1, \quad x(0) = x'(0) = 0.$$

(i) Show that the laplace transform for the given problem is

$$X(s) = \frac{1}{s(s+2)^2}.$$

(ii) Then, use partial fractions to prove that

$$X(s) = \frac{(\frac{1}{4})}{s} + \frac{(-\frac{1}{4})}{(s+2)} + \frac{(-\frac{1}{2})}{(s+2)^2}.$$

(iii) Finally, solve the given problem.

(10 marks)

(b) A tank which initially holds 100 litres of a solution containing 90% of water and 10% of salt is drawn off at the rate of 5 litres/min. At the same time the tank is refilled at the rate of 4 litres/min with solution that contains 50% of water and 50% of salt. Assuming that the mixture is kept uniform. How much saltis present
(i) at any time $t > 0$?
(ii) at the end of 10 min.

(10 marks)

Q7 (a) Solve the exact equation below:

$$(y \sin x + xy \cos x)\, dx + (x \sin x + 5)\, dy = 0.$$

(11 marks)

(b) Given the R-L circuit model as

$$L\frac{dI(t)}{dt} + RI(t) = E(t),$$

where the circuit has inductance $L = 0.5$ henry, resistance $R = 10$ ohms, electromotive force $E(t) = 3e^{2t}$ and $I(t)$ is the current flowing in the circuit. The initial current is 6 Amperes.

(i) Determine the initial condition for the circuit.
(ii) Find the current flowing in the circuit for all time

(9 marks)

- END OF QUESTION -

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2012/2013 PROGRAMME : 2 DAA
COURSE : ENGINEERING MATHEMATICS II COURSE CODE : DAS 20403

FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay'' + by' + cy = f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2012/2013 PROGRAMME : 2 DAA
COURSE : ENGINEERING MATHEMATICS II COURSE CODE : DAS 20403

The method of variation of parameters

If the solution of the homogeneous equation $ay'' + by' + cy = 0$ is $y_h = Ay_1 + By_2$, then the particular solution for $ay'' + by' + cy = f(x)$ is

$$y = uy_1 + vy_2,$$

where $u = -\int \frac{y_2 f(x)}{aW} dx + A$, $v = \int \frac{y_1 f(x)}{aW} dx + B$ and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$.

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s) - y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n}{ds^n} F(s)$		