CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	DAS 10303
PROGRAM ME	:	1 DAE / 3 DAL
EXAMINATION DATE	:	MARCH 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**.

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

CONFIDENTIAL

PART A

Q1 (a) Find the Laplace transforms of the following functions.

(i) $f(t) = 3t - 5$

(ii) $f(t) = 2t + 5e^{-8t}$

(iii) $f(t) = 3\sin 2t$

(iv) $f(t) = 7\cos t + 8\sin 2t$

(v) $f(t) = \cosh 2t - 6\sinh 3t$

(10 marks)

(b) By using First Shift Theorem, Multiply with tn and/or Linearity, find the Laplace transform of each function below.

(i) $f(t) = e^{2t}\cos 3t$

(ii) $f(t) = t\sinh 2t$

(iii) $f(t) = t^2 e^{-3t} - 5t\sin 2t$

(iv) $f(t) = (2t-1)^3$

(10 marks)

Q2 (a) Find the inverse Laplace transform for each function below.

(i) $F(s) = \dfrac{4}{s^6}$

(ii) $F(s) = \dfrac{7s}{s^2+9}$

(iii) $F(s) = \dfrac{1}{(s+1)(s-2)}$

(iv) $F(s) = \dfrac{1}{s^2-2s+1}$

(10 marks)

(b) Use the Laplace and inverse Laplace transform to solve the following Initial Value Problems (IVP).

(i) $y'+y=3t, \qquad y(0)=1$

(ii) $y''+4y'+4y=e^{t}, \qquad y(0)=1, \quad y'(0)=1$

(10 marks)

PART B

Q3 (a) Find the domain and range of the following functions.

(i) $f(x)=x^5+\sqrt{x}$

(ii) $g(x)=\dfrac{1}{x-2}$

(iii) $h(x)=\sqrt{x^2-2x-3}$

(9 marks)

(b) Let $f(x)=x^3$, $g(x)=\sin x$, and $h(x)=\sqrt{x}$.

(i) Find $h \circ f\,(x)$ and $f \circ g \circ h\,(x)$.

(ii) Evaluate $h \circ g\left(\dfrac{\pi}{2}\right)$.

(8 marks)

(c) Find the inverse function of $f(x)=3\sqrt{x}-7$.

(3 marks)

Q4 (a) Calculate the following limits.

(i) $\lim_{x\to 3} \frac{1}{2}x - 7$

(ii) $\lim_{x\to 4} \frac{3x^2 - 4x}{5x^3 - 36}$

(iii) $\lim_{x\to 1} \frac{\sqrt{x} - 1}{x - 1}$

(iv) $\lim_{x\to 2} \frac{x^2 - 6x + 8}{x^2 - 4}$

(10 marks)

(b) Calculate the following limits.

(i) $\lim_{x\to 3^-} \frac{2 - 5x}{x - 3}$

(ii) $\lim_{x\to -4^+} \frac{-x^3 + 5x^2 - 6x}{-x^3 - 4x^2}$

(5 marks)

(c) Given $f(x) = \begin{cases} x^2 + 1 & , \quad x \leq 0 \\ 3x + 5 & , \quad x > 0 \end{cases}$.

(i) Sketch the graph of $f(x)$.
(ii) Check whether $f(x)$ is continuous at $x = 0$.

(5 marks)

Q5 (a) By using the formula $\dfrac{dy}{dx} = f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$,

prove that

$$\frac{d}{dx}\left(16x^2 + \frac{3}{4}x\right) = 32x + \frac{3}{4}.$$

(4 marks)

(b) Given $y = \sqrt{x}$.

(i) Compute $\dfrac{dy}{dx}$.

(ii) Find an equation of the line tangent to the graph $y = \sqrt{x}$ at (4, 2).

(5 marks)

(c) Evaluate the following derivatives.

(i) $\dfrac{d}{dx}\left(-\dfrac{7x^{11}}{8}\right)$

(ii) $\dfrac{d}{dt}\left(\dfrac{7}{8}\sqrt{t}\right)$

(iii) $\dfrac{d}{ds}\left(\dfrac{s^2+1}{s^2-4}\right)$

(iv) $\dfrac{d}{dx}\left(\dfrac{4x\left(2x^3 - 3x^{-1}\right)}{x^2+1}\right)$

(11 marks)

Q6 (a) A 5 feet-tall woman walks at 9 ft/s toward a street light that is 20 ft above the ground.

(i) What is the rate of change of the length of her shadow when she is 15 ft from the street light.

(ii) At what rate is the tip of her shadow moving?

(6 marks)

(b) Given a function, $f(x) = 2x^3 + 3x^2 + 1$. By finding $f'(x)$ and $f''(x)$,

(i) find the local minimum, local maximum and inflection point.

(iii) Sketch the graph of $f(x)$.

(8 marks)

(c) By using L'hopital's Rule, find

(i) $\lim_{x \to 0} \frac{\cos x - 1}{x^2}$

(ii) $\lim_{x \to 1} \frac{x^3 + x^2 - 2x}{x^2 - 1}$

(iii) $\lim_{x \to \infty} \frac{4x^3 - 6x^2 + 1}{2x^3 - 10x + 3}$

(6 marks)

Q7 (a) Find the Laplace transform for the functions below.

(i) $f(t) = \frac{3}{4}$

(ii) $f(t) = 30t^7$

(iii) $f(t) = 3t - 5$

(iv) $f(t) = 3t(t - 5)$

(v) $f(t) = (t - 2)H(t - 2)$

(10 marks)

(b) Find the Laplace transform for $g(t) = \begin{Bmatrix} 2 & 0 < t < 3 \\ 3 & 3 \le t \le 6 \\ 4 & t > 6 \end{Bmatrix}$.

(4 marks)

(c) Find the inverse Laplace transform for the functions below.

(i) $F(s) = \frac{3}{s} + \frac{5}{s-4}$

(ii) $F(s) = \frac{8}{(s-4)^3}$

(iii) $F(s) = \frac{s-3}{s^2+16}$

(6 marks)

- END OF QUESTION -

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2012/2013 PROGRAMME: 1 DAE, 3 DAL
COURSE : ENGINEERING MATHEMATICS I COURSE CODE: DAS 10303

FORMULAE

Table 1 : Laplace transform.

$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}\,dt=F(s)$	
$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n,\ n=1,2,..$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
The First Shift Theorem	
$e^{at}f(t)$	$F(s-a)$
Multiply with t^n	
$t^n f(t),\ n=1,2,..$	$(-1)^n\frac{d^nF(s)}{ds^n}$
The Unit Step Function	
$H(t-0)$	$\frac{1}{s}$
$H(t-a)$	$\frac{e^{-as}}{s}$
The Second Shift Theorem	
$f(t-a)\,H(t-a)$	$e^{-as}F(s)$

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2012/2013
COURSE : ENGINEERING MATHEMATICS I

PROGRAMME: 1 DAE, 3 DAL
COURSE CODE: DAS 10303

Heaviside Function
$g(t) = g_1 + [g_2 - g_1]H(t-a) + [g_3 - g_2]H(t-b)$

Initial Value Problem (IVP)
$\mathcal{L}\{y(t)\} = Y(s)$ $\mathcal{L}\{y'(t)\} = sY(s) - y(0)$ $\mathcal{L}\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$

Table 2: Differentiation
$\frac{d}{dx}x^n = nx^{n-1}$
$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$
$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\left(\frac{dt}{dx}\right)$
$\frac{d}{dx}\ln u = \frac{1}{u}\cdot\frac{du}{dx}$
$\frac{d}{dx}e^u = e^u\cdot\frac{du}{dx}$
$\frac{d}{dx}\sin u = \cos u\cdot\frac{du}{dx}$
$\frac{d}{dx}\tan u = \sec^2 u\cdot\frac{du}{dx}$
$\frac{d}{dx}\cot u = -\csc^2 u\cdot\frac{du}{dx}$
$\frac{d}{dx}\sec u = \sec u\tan u\cdot\frac{du}{dx}$
$\frac{d}{dx}\csc u = -\csc u\cot u\cdot\frac{du}{dx}$
$\frac{d}{dx}\cos u = -\sin u\cdot\frac{du}{dx}$