

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

# FINAL EXAMINATION SEMESTER I SESSION 2014/2015

**COURSE NAME** 

: BASIC CALCULUS

COURSE CODE

BBR33903

PROGRAMME CODE

BBR

EXAMINATION DATE :

DECEMBER 2014 / JANUARY 2015

**DURATION** 

3 HOURS

**INSTRUCTION** 

ANSWER FIVE QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF SIX (6) PAGES

Q1 (a) Sketch  $y = -(x-5)^2 - 4$ . State clearly the x- intercept and y- intercept.

(8 marks)

(b) Sketch  $y = \frac{1}{(x+2)^2} - 2$ .

(6 marks)

(c) Sketch  $y = e^x$ ,  $y = e^{2x}$  and  $y = 2e^x$  at the same graph. State clearly the y-intercept.

(6 marks)

- Q2 (a) Find the one side limits for
  - (i)  $\lim_{x \to 2^+} \frac{1}{x 2}$
  - (ii)  $\lim_{x \to 2^{-}} \frac{1}{x 2}$

Hence, or use other method, determine  $\lim_{x\to 2} \frac{1}{x-2}$ .

(7 marks)

(b) Evaluate  $\lim_{x\to 6} \frac{x^2 - 2x - 24}{-6 + x}$  without using L' Hôpital's rule.

(6 marks)

(c) Find  $\lim_{x \to -4} f(x)$  and  $\lim_{x \to 4} f(x)$  where

$$f(x) = \begin{cases} x^2 + x + 4 & \text{if } x < 4 \\ 6x & \text{if } x > 4 \end{cases}$$

(7 marks)

Q3 (a) Find  $\frac{dy}{dx}$  for the  $y = (2 - x^2)(x^3 + x - 6)$ .

(4 marks)

(b) Find  $\frac{dy}{dx}$  for the following functions

(ii) 
$$y = \frac{2x}{\sqrt{x} + 6}$$

(iii) 
$$y = \frac{\ln x}{e^x}$$

(8 marks)

- (c) Using the chain rule, differentiate
  - $(i) y = (2 \ln x)^6$

(ii) 
$$y = \frac{1}{\sqrt{4 - 6x}}$$

(8 marks)

- Q4 (a) Sand is poured onto a growing conical pile of sand, at the constant rate of  $6\text{m}^3\text{s}^{-1}$ . Suppose the side of the cone makes a 60 degree angle to cone's base. Given  $V = \frac{1}{3}\pi r^2 h$ .
  - (i) How fast is the height of the sand pile increasing when the volume of the cone is 180m<sup>3</sup>?

(9 marks)

(ii) If the volume of the cone does not change, find the rate of change of its height, h with respect to its radius, r.

(4 marks)

(b) Given  $f(x) = x^3 - 3x + 1$ . Find all critical points of f(x). Then use the Second Derivative Test to determine the properties of all the local extreme points.

(7 marks)

**Q5** (a) Evaluate  $\int_{-10}^{-5} 6 \, dx$ .

(4 marks)

(b) Evaluate  $\int \cos^2 3x \sin 3x \, dx$  by using substitution method.

(7 marks)

(c) (i) Change  $\frac{1}{x^2 + x - 6}$  into partial fraction.

(5 marks)

(ii) Thus, use the answer from Q5(c)(i) to evaluate  $\int \frac{1}{x^2 + x - 6} dx$ .

(4 marks)

Q6 (a) Find the area of f(x) = x(x-3) for  $0 \le x \le 5$ .

(5 marks)

(b) Find the area of the region enclosed by  $x = y^2$  and y = x - 2, by integration with respect to y.

(7 marks)

(c) Find the volume of the solid generated when the region enclosed between  $y = \sqrt{x}$ , x = 1 and x = 4 is revolved about the x-axis.

(8 marks)

- END OF QUESTION -

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### FORMULAS

#### Differentiation:

$$\frac{d}{dx}[c] = 0$$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[c f(x)] = c \frac{d}{dx}[f(x)]$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[f(x)g(x)] = f(x) \cdot \frac{d}{dx}[g(x)] + g(x) \cdot \frac{d}{dx}[f(x)]$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot \frac{d}{dx}[f(x)] - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$\frac{d}{dx}\left[\cos^{-1}x\right] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \left[ \tan^{-1} x \right] = \frac{1}{1 + x^2}$$
$$\frac{d}{dx} \left[ \sec^{-1} x \right] = \frac{1}{|x| \sqrt{x^2 - 1}}$$

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#### Integration:

$$\int c f(x) dx = c F(x) + C$$

$$\int \tan x \, dx = \ln |\sec x| + C$$

$$\int [f(x) \pm g(x)] dx = F(x) \pm G(x) + C \qquad \int \sec^2 x \, dx = \tan x + C$$

$$\sec^2 x \, dx = \tan x + C$$

$$\int x^r dx = \frac{x^{r+1}}{r+1} + C, (r \neq -1)$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sec x \, \tan x \, dx = \sec x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int \csc x \, dx = -\ln\left|\csc x + \cot x\right| + C$$

$$\int \sec x \, dx = \ln\left|\sec x + \tan x\right| + C$$

#### Area of region:

$$A = \int_{a}^{b} [f(x) - g(x)] dx \qquad \text{or} \qquad A = \int_{a}^{d} [w(y) - v(y)] dy$$

$$A = \int_{0}^{d} \left[ w(y) - v(y) \right] dy$$

## Volume cylindrical shells:

$$V = \int_{a}^{b} 2\pi x f(x) dx$$

$$V = \int_{0}^{d} 2\pi y \, f(y) \, dy$$