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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	FLIGHT STABILITY AND CONTROL
COURSE CODE	:	BDL 30102
PROGRAMME CODE	:	BDC
EXAMINATION DATE	:	DECEMBER 2019/JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWERS FIVE (4) QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

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Q1 (a) Give the definition of static stability and list the necessary criteria for longitudinal balance and static stability.

(3 marks)

(b) Consider a model of a wing-body shape mounted in a low-speed wind tunnel. The flow conditions in the test section are standard sea level properties with a velocity of 100 m/s. The wing area and chord are 1.5 m^2 and 0.45 m, respectively. Using the wind tunnel force and moment measuring balance, the moment about the center of gravity when the lift is zero is found to be -12.4 Nm. When the model is pitched to another angle of attack, the lift and moment about the center of gravity are measured to be 3675 N and 20.67 Nm, respectively. Calculate the value of the moment coefficient about the aerodynamic center and the location of the aerodynamic center.

(6 marks)

(c) If a mass of lead is added to the rear of the model such that the center of gravity is shifted rearward by the length equal to 20% of the chord, calculate the moment about the center of gravity when the lift is 4000 N.

(3 marks)

(d) Assume that a horizontal tail with no elevator is added to the wing-body model in question **Q1(d)**. The distance from the aircraft's center of gravity to the tail's aerodynamic center is 1.0 m. The area of the tail is 0.4 m^2 and the tail setting angle is 2.0°. The lift slope of the tail is 0.12 per degree. From experimental measurement, $\varepsilon_0 = 0$ and $\partial\varepsilon/\partial\alpha = 0.42$. If the absolute angle of attack of the model is 5° and the lift at this angle of attack is 4134 N, calculate the moment about the center of gravity.

(7 marks)

(e) Consider the wing-body-tail model in question **Q1(d).** Does this model have longitudinal static stability and balance?

(6 marks)

Q2 (a) Describe the physical characteristics of Dutch Roll stability mode.

(3 marks)

(b) The Dutch Roll motion can be approximated using the following equation:

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_0} & -\left(1 - \frac{Y_r}{u_0}\right) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta r}}{u_0} \\ N_{\delta r} \end{bmatrix} \delta r$$

Assume the aircraft has the following stability derivative characteristics as follows:

$Y_\beta = -7.1$ ft/s^2	$Y_r = 2.1$ ft/s	$u_0 = 160$ ft/s
$N_\beta = 2.9$ s^{-2}	$N_r = -0.325$ s^{-1}	$N_{\delta r} = 0.615$ s^{-2}
	$Y_{\delta r} = -4.9$ ft/s^2	

(i) Determine the characteristic equation of the Dutch Roll mode.

(4 marks)

(ii) Determine the eigenvalues of the Dutch Roll mode.

(2 marks)

(iii) Determine the damping ratio, natural frequency, period, time to half amplitude and number of cycles to half amplitude for the Dutch Roll mode.

(5 marks)

(c) Compare your Dutch Roll mode natural frequency and damping ratio calculation for this particular aircraft to the handling quality criteria in **Table Q2.** Assume that the aircraft under consideration is a **Class IV aircraft** (High Maneuvering Aircraft) performing **CAT A mission** (Precision Tracking). Determine the minimum state feedback gain so that the damping ratio achieved **Level 1** handling qualities. Use feedback control design based on **rudder deflection**, δr, proportional only to the **yaw rate state**, Δr, i.e.:

$$\delta r = -K^T x = -\begin{bmatrix} 0 & K \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix}$$

(11 marks)

Q3 The longitudinal equations of motion for the McDonnell Douglas F-4C Phantom aircraft flying at Mach 1.1 is given as follows:

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.068 & -0.011 & 0 & -9.81 \\ 0.023 & -2.10 & 375 & 0 \\ 0.011 & -0.160 & -2.20 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} -0.41 & 1.00 \\ -77.0 & -0.99 \\ -61.0 & -0.11 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_E \\ \delta_T \end{bmatrix}$$

Note that two input variables are given in the model, elevator angle δ_E and thrust δ_T. The open loop characteristic equation was found to be:

$$\Delta(s) = (s^2 + 4.3s + 64.4)(s^2 + 0.07s + 0.003)$$

(a) Determine the corresponding damping ratio and natural frequency for each longitudinal stability mode.

(4 marks)

(b) Compare the obtained longitudinal stability characteristics by referring to the MIL-F-8785C flying qualities requirement (see **Table Q3(a)** and **Table Q3(b)**). Assume that the aircraft is a class IV aircraft in the most demanding category A flight phase. Does the aircraft require augmentation to improve the longitudinal stability?

(4 marks)

(c) List the design parameters (i.e. damping ratio and natural frequency) used if the pole placement method is to be used to achieve the required short period flying qualities requirement.

(4 marks)

(d) Design the stability augmentation system with longitudinal state feedback to the elevator using pole placement method. Assume that the original longitudinal equation of motion can be approximated using short period dynamics model and the feedback gain from the body vertical velocity, w is significantly small.

(13 marks)

Q4 The open loop pitch rate response to elevator transfer function for the Lockheed F-104 Starfighter is given by the following transfer function:

$$\frac{q(s)}{\delta_e(s)} = \frac{-4.66s(s+0.133)(s+0.269)}{(s^2+0.015s+0.021)(s^2+0.911s+4.884)}$$

(a) The root locus plot of the transfer function is given in **Figure Q4**. With the aid of the root locus plot, explain how can the root locus plot be used to evaluate the effect of feedback on the characteristics modes of motion?

(4 marks)

(b) Determine the damping ratio and undamped natural frequency for short period and phugoid mode.

(4 marks)

(c) Design a pitch rate feedback controller, K_q to bring the closed loop short period mode in agreement with minimum specification for damping ratio and natural frequency. Assume the following Level 1 flying qualities are used in the analysis:

Phugoid damping ratio $\zeta_p \geq 0.04$

Short period damping ratio $\zeta_s \geq 0.5$

Short period undamped natural frequency $0.8 \leq \omega_s \leq 3.0$ rad/s

(11 marks)

(d) Compare the augmented short period damping ratio and the natural frequency with those of the unaugmented aircraft. Examine the effect of pitch rate feedback that was used to improve the longitudinal flying qualities. Explain your answer based on your findings and the given root locus plot.

(6 marks)

Q5 (a) The longitudinal equation of motion and aerodynamic data for A-7A Corsair II aircraft were given in the following state space model. The flight condition corresponds to level cruising flight at an altitude of 15000 ft. at Mach 0.3:

$$\begin{bmatrix}\dot{u}\\ \dot{w}\\ \dot{q}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}0.0501 & 0.00464 & -72.9 & -31.34\\ -0.0857 & -0.545 & 309 & -7.4\\ -0.00185 & -0.00767 & -0.395 & 0.00132\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}u\\ w\\ q\\ \theta\end{bmatrix} + \begin{bmatrix}5.63\\ -23.8\\ -4.51576\\ 0\end{bmatrix}\eta$$

Establish the state space model of the aircraft based on the simplest form of short period mode approximation.

(6 marks)

(b) Determine whether the system is state controllable.

(4 marks)

(c) Design a pitch displacement autopilot system for the aircraft using Bass-Gura method (state feedback) so that the aircraft has the following short period characteristic:

$$\xi_{sp} = 0.6$$
$$\omega_{n,sp} = 1.5\ rad/s$$

(15 marks)

-END OF QUESTIONS-

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Table Q2 Minimum Dutch-Roll frequency and damping

Aircraft Class	Flight Phase	Minimum Values							
		Level 1			Level 2			Level 3	
		ζ	$\zeta\omega_n$	ω_n	ζ	$\zeta\omega_n$	ω_n	ζ	ω_n
I, IV	CAT A	0.19	0.35	1.0	0.02	0.05	0.5	0	0.4
II, III	CAT A	0.19	0.35	0.5	0.02	0.05	0.5	0	0.4
All	CAT B	0.08	0.15	0.5	0.02	0.05	0.5	0	0.4
I, IV	CAT C	0.08	0.15	1.0	0.02	0.05	0.5	0	0.4
II, III	CAT C	0.08	0.10	0.5	0.02	0.05	0.5	0	0.4

Table Q3(a) Short period mode damping ratio

	Level 1		*Level 2*		*Level 3*
Flight phase	ζ_s *min*	ζ_s *max*	ζ_s *min*	ζ_s *max*	ζ_s *min*
CAT A	0.35	1.30	0.25	2.00	0.10
CAT B	0.30	2.00	0.20	2.00	0.10
CAT C	0.50	1.30	0.35	2.00	0.25

Table Q3(b) Phugoid damping ratio

Level of flying qualities	*Minimum* ζ_p
1	0.04
2	0
3	Unstable, period $T_p > 55$ s

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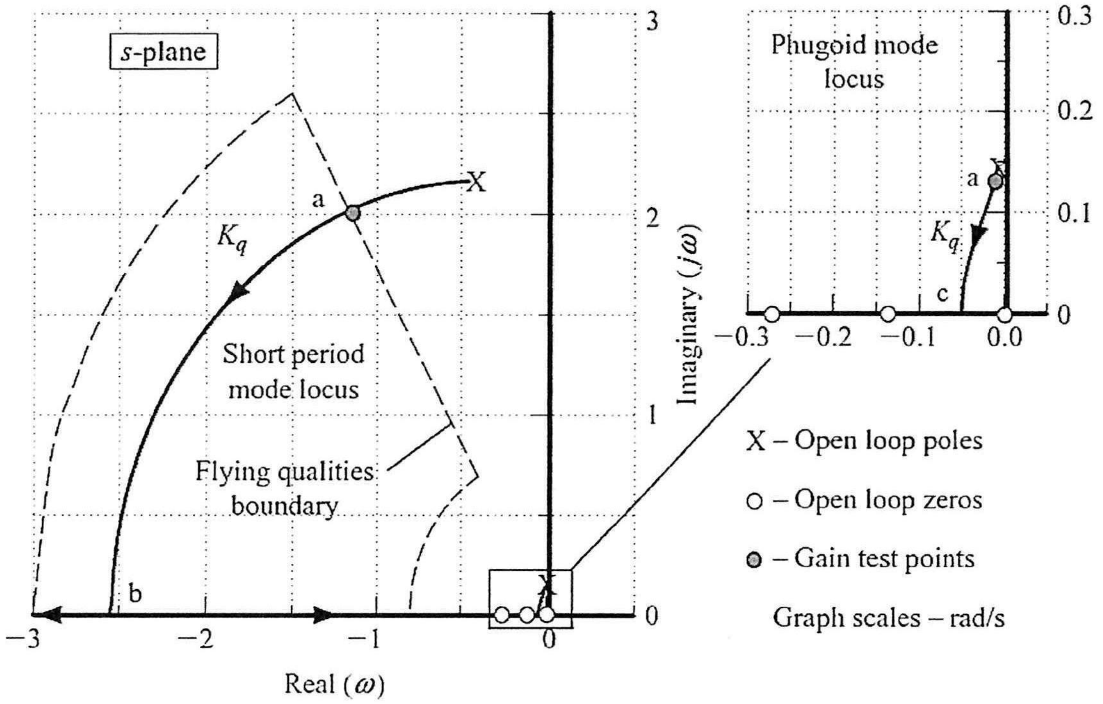

Figure Q4 Root locus plot showing pitch rate feedback to the elevator.

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A Key Equations

The relevant equations used in this examination are given as follows:

1. The determinant of a 3×3 matrix:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} \quad (1)$$

2. Partial fraction for $F(s)$ with real and distinct roots in denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2}{(s+p_2)} + \cdots + \frac{K_m}{(s+p_m)} \quad (2)$$

3. Partial fraction for $F(s)$ with complex or imaginary roots in denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2 s + K_3}{(s^2+as+b)} + \cdots \quad (3)$$

4. General first order transfer function:

$$G(s) = \frac{s}{s+a} \quad (4)$$

5. General second order transfer function:

$$G(s) = \frac{K}{s^2 + 2\xi\omega_n s + \omega_n} \quad (5)$$

6. The closed loop transfer function:

$$T(s) = \frac{G(s)}{1+G(s)H(s)} \quad (6)$$

where $G(s)$ is the transfer function of the open loop system and $H(s)$ is the transfer function in the feedback loop.

7. The final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \quad (7)$$

8. Time response:

$$T_r = \frac{2.2}{a} \quad (8)$$

$$T_s = \frac{4}{a} \quad (9)$$

$$\%OS = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\% \quad (10)$$

$$\xi = \frac{-\ln\left(\%\frac{OS}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\%\frac{OS}{100}\right)\right)^2}} \quad (11)$$

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$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\omega} \quad (12)$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{\eta} \quad (13)$$

$$P = \frac{2\pi}{\omega} \quad (14)$$

$$t_{1/2} = \frac{0.693}{|\eta|} \quad (15)$$

$$N_{1/2} = 0.110\frac{|\omega|}{|\eta|} \quad (16)$$

9. Estimation of parameter q (Paynter Numerical Method)

$$q = \max\left|A_{ij}\Delta t\right| \quad (17)$$

10. Estimation of integer value of p (Paynter Numerical Method)

$$\frac{1}{p!}(nq)^p e^{nq} \leq 0.001 \quad (18)$$

11. Numerical solution of state equation:

$$\mathbf{x}_{k+1} = \boldsymbol{M}\mathbf{x}_k + \boldsymbol{N}\boldsymbol{\eta}_k$$

with matrix $\boldsymbol{M}$ and $\boldsymbol{N}$ are given by the following matrix expansion:

$$\boldsymbol{M} = e^{A\Delta t} = \boldsymbol{I} + \boldsymbol{A}\Delta t + \frac{1}{2!}\boldsymbol{A}^2\Delta t^2 \ldots \quad (19)$$

$$\boldsymbol{N} = \Delta t\left(\boldsymbol{I} + \frac{1}{2!}\boldsymbol{A}\Delta t + \frac{1}{3!}\boldsymbol{A}^2\Delta t^2 + \cdots\right)\boldsymbol{B}$$

12. Characteristic equation of the closed loop system:

$$1 + KG(s)H(s) = 0 \quad (20)$$

13. Asymptotes: angle and real-axis intercept:

$$\sigma = \frac{[\sum Real\ parts\ of\ the\ poles - \sum Real\ parts\ of\ the\ zeros]}{n-m} \quad (21)$$

$$\phi_a = \frac{180°[2q+1]}{n-m} \quad (22)$$

14. Solution to find real axis break-in and breakaway points:

$$\frac{dK(\sigma)}{d\sigma} = 0 \quad (23)$$

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15. An alternative solution to find real axis break-in and breakaway points:

$$\sum \frac{1}{\sigma + z_i} = \sum \frac{1}{\sigma + p_i} \quad (24)$$

16. Angle of departure of the root locus from a pole of $G(s)H(s)$:

$$\theta = 180° + \sum (angles\ to\ zeros) - \sum (angles\ to\ poles) \quad (25)$$

17. The angle of arrival at a zero:

$$\theta = 180° - \sum (angles\ to\ zeros) + \sum (angles\ to\ poles) \quad (26)$$

18. The steady state error:

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) \quad (27)$$

where the error signal is given as:

$$E(s) = \frac{1}{1 + K(s)G(s)H(s)} \times U(s) \quad (28)$$

19. Linear system (SISO) with state feedback:

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}^T)\boldsymbol{x} + \boldsymbol{B}u$$

$$\text{or, } \dot{\boldsymbol{x}} = \boldsymbol{A}_{\boldsymbol{new}}\boldsymbol{x} + \boldsymbol{B}u \quad (29)$$

where $\boldsymbol{A}_{\boldsymbol{new}}$ is the augmented matrix and $u = \boldsymbol{K}^T\boldsymbol{x} + \delta_{ref}$

20. The characteristic equation for the standard form of the second-order differential equation:

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0$$

The roots of the characteristic equation are: (30)

$$\lambda_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{1 - \xi^2} \cdot i$$

$$\lambda_{1,2} = \sigma \pm \omega_d$$

21. Controllability matrix:

$$\boldsymbol{V} = [\boldsymbol{B} \quad \boldsymbol{A}\boldsymbol{B} \quad \boldsymbol{A}^2\boldsymbol{B} \quad \dots \quad \boldsymbol{A}^{n-1}\boldsymbol{B}] \quad (31)$$

22. Transformation matrix:

$$\boldsymbol{W} = \begin{bmatrix} 1 & a_1 & \cdots & a_{n-1} \\ 0 & 1 & \cdots & a_{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \quad (32)$$

23. Bass-Gura formula to determine feedback gains:

$$\boldsymbol{K} = [(\boldsymbol{V}\boldsymbol{W})^T]^{-1}[\bar{\boldsymbol{a}} - \boldsymbol{a}] \quad (33)$$

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24. The contribution of the wing-body to M_{cg}:

$$C_{M,cg_{wb}} = C_{M,ac_{wb}} + C_{L_{wb}}\left(h - h_{ac_{wb}}\right)$$
$$C_{M,cg_{wb}} = C_{M,ac_{wb}} + a_{wb}\alpha_{wb}\left(h - h_{ac_{wb}}\right) \quad (34)$$

25. The contribution of the wing-body-tail to M_{cg}:

$$C_{M,cg_{wb}} = C_{M,ac_{wb}} + a\alpha_a\left(h - h_{ac_{wb}} - V_H\frac{a_t}{a}\left[1 - \frac{\partial\varepsilon}{\partial\alpha}\right]\right) + V_H a_t(i_t + \varepsilon_0) \quad (35)$$

26. The equation for longitudinal static stability:

$$C_{M,0} = C_{M,ac_{wb}} + V_H a_t(i_t + \varepsilon_0)$$
$$\frac{\partial C_{M,cg}}{\partial\alpha_a} = a\left[h - h_{ac_{wb}} - V_H\frac{a_t}{a}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right] \quad (36)$$

27. Absolute angle of attack, α_a:

$$\alpha_a = \alpha + |\alpha_{L=0}| \quad (37)$$

where α is the geometric angle of attack.