CONFIDENTIAL

Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS I
COURSE CODE	:	BDX 10102
PROGRAMME CODE	:	BDX
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	ANSWER **FOUR (4)** QUESTIONS **ONLY**

THIS QUESTION PAPER CONSISTS OF SIX (6) PAGES

CONFIDENTIAL

Q1 (a) Solve the $\lim\limits_{x\to\infty}$ for the function of $f(x)=\dfrac{2-x}{\sqrt{7+6x^2}}$

(5 marks)

(b) Evaluate the value of c if $f(x)$ is continuous at every x,

$$f=\begin{cases}x^2-2 & x<2\\ 4cx & x\geq 2\end{cases}$$

(5 marks)

(c) Solve the derivative of $y^2+\sin(3x)=12-y^4$.

(5 marks)

(d) Prove $\displaystyle\int_0^1\frac{36-8x-2x^2}{(4-3x)(1+x)^2}dx=3+\frac{14}{3}\ln 2$

(10 marks)

Q2 (a) Find the real and imaginary parts of the complex number $z+\dfrac{1}{z}$ for $z=\dfrac{3+2i}{2-i}$.

(7 marks)

(b) Find the Taylor series for $\dfrac{1}{x}$ about $x=2$.

(6 marks)

(c) Newton's Law of Cooling states that the rate of changes of the temperature, T, of a body is proportional to the difference between T and the temperature of the surrounding medium, T_s, multiplies to thermal conductivity k.

(i) Interpret this cooling law in the form of first order ordinary differential equation, and subsequently find its general solution.

(6 marks)

(ii) If a thermometer with a reading of 10°C, is brought into a room whose temperature is 23°C, and the reading of the thermometer is 18°C after two minutes later, how long will it take until the reading is 23°C?

(6 marks)

Q3 (a) Find the Laplace transform of $f(t) = \cos 2t$.

(2 marks)

(b) Evaluate the inverse Laplace transform for $F(s) = \dfrac{1}{s^2(s+4)}$

(8 marks)

(c) Use root test to determine whether the following series converge or diverge.

$$\sum_{k=2}^{\infty}\left(\frac{4k-5}{2k+1}\right)^k$$

(8 marks)

(d) A person is standing 350 meters away from a model rocket that is fired straight up into the air at a rate of 15 m/s. At what rate is the distance between the person and the rocket increasing 20 seconds after lift-off?

(7 marks)

Q4 (a) By using method of variation parameter, obtain the general solution for:

$$y'' + y = 2\sin x$$

(*Note*: $\sin 2x = 2\cos x \sin x \qquad \cos 2x = 1 - 2\sin^2 x$)

(12 marks)

(b) A model for forced spring mass system is given by:

$$y'' + 8y' + 8y = 4\cos t$$

(i) State the value of the mass, spring constant and damping constant for this system.

(ii) If another 1 kg mass is added to this system, what will happen to this differential equation?

(iii) Find the steady state solution for case with 2kg mass.

(iv) Find the particular solution for the answer in Q4(b)(iii) that satisfies $y(0) = 1$ and $y'(0) = 2$.

(13 marks)

Q5 (a) Solve the differential equation:

$$2y'' - 2y' + 5y = 0$$

(6 marks)

(b) Determine the Laplace transform of the following functions:

$$f(t) = 4t^2 - 5\sin 3t + 6$$

(5 marks)

(c) By using the Laplace transform, solve the initial value problem of the given ordinary differential equation:

$$y'' - 4y' + 3y = 2, \qquad y(0) = 2, \qquad y'(0) = 6$$

(14 marks)

-END OF QUESTION-

FINAL EXAMINATION

SEMESTER / SESSION	: SEM I /20192020	PROGRAMME	: 1 BDX
COURSE	:ENGINEERING TECHNOLOGY MATHEMATICS I	COURSE CODE	: BDX 10102

FORMULAS

Ratio Test

Let $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to\infty} \frac{u_{k+1}}{u_k}$

Root Test

Let $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to\infty} \sqrt[k]{u_k}$

Characteristic Equation and General Solution for Second Order Differential Equation

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1 e^{mx} + c_2 x e^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

Method of Undetermined Coefficient

$g(x)$	y_p
Polynomial: $P_n(x) = a_n x^n + \ldots + a_1 x + a_0$	$x^r(A_n x^n + \ldots + A_1 x + A_0)$
Exponential: e^{ax}	$x^r(Ae^{ax})$
Sine or Cosine: $\cos\beta x$ or $\sin\beta x$	$x^r(A\cos\beta x + B\sin\beta x)$

Note: r is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

Method of Variation of Parameters

The particular solution for $y'' + by' + cy = g(x)$ (b and c constants) is given by $y(x) = u_1 y_1 + u_2 y_2$, where

$$u_1 = -\int \frac{y_2 g(x)}{W} dx,$$

$$u_2 = \int \frac{y_1 g(x)}{W} dx,$$

where

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

FINAL EXAMINATION

SEMESTER / SESSION	: SEM I /20192020	PROGRAMME	: 1 BDX
COURSE	:ENGINEERING TECHNOLOGY MATHEMATICS I	COURSE CODE	: BDX10102

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$
a	$\frac{a}{s}$
$t^n, n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t), n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^nF(s)}{ds^n}$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$\dot{y}(t)$	$sY(s)-y(0)$
$\ddot{y}(t)$	$s^2Y(s)-sy(0)-y'(0)$