Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BDA 34003
PROGRAMME	:	BDD
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTIONS	:	(a) ANSWER **ALL** QUESTIONS IN **PART A** (b) ANSWER **TWO (2)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

PART A: ANSWER ALL QUESTIONS

Q1 (a) **Figure Q1** shows a vibrating system with 2 masses and 2 springs. The masses are constrained to move only in the vertical direction. Prove that the Characteristic Equation for the system is as follow: $|[A\} - \lambda[I]| = 0$

(10 marks)

(b) Given

$$A = \begin{bmatrix} 7 & 6 & 3 \\ 5 & 4 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

Evaluate the largest eigenvalue and its corresponding eigenvector using Power Method. Use $V^0 = \{0\ 1\ 1\}\ T$ and iterate until $|\lambda_{k+1} - \lambda_k| \leq 0.0005$.

(10 marks)

Q2 Given the initial value problem as follows

$$f(x) = \frac{dy}{dx} = \frac{5x^2 - y}{e^{x+y}}$$

with initial condition y(0) =1, $0 \leq x \leq 2.0$ and h = 0.4

Solve the initial value problem using

(i) Euler's method (4 marks)

(ii) Modified Euler's method (8 marks)

(iii) Midpoint / Improved Euler's method (8 marks)

Q3 The heat transfer performance of a new conductor bar of length 20 cm is under inspection. The material is fully insulated such that the heat transfer is only one dimensional in axial direction (x-axis). The left end is maintained at temperature of 100°C, while the right end is maintained at temperature of 20°C, for $t > 0$. The distribution of the initial temperatures is shown in **Figure Q3.** The unsteady state heat conduction equation is given by

$$\frac{\partial T}{\partial t} - \kappa \frac{\partial^2 T}{\partial x^2} = 0$$

Where κ is a thermal diffusivity of material and x is the longitudinal coordinate of the bar. The thermal diffusivity of the material is given as $\kappa = 10$ cm²/s, and $\Delta t = 2$ second.

(a) By considering numerical differentiation $\frac{\partial^2 T}{\partial x^2} = \frac{T(x+1,t) - 2T(x,t) + T(x-1,t)}{\Delta x^2}$ and $\frac{\partial T}{\partial t} = \frac{T(x,t+1) - T(x,t)}{\Delta t}$, deduce that the temperature distribution along the bar at point $(x, t+1)$ in explicit finite-difference form is given by

$$T(x,t+1) = 0.8T(x-1,t) - 0.6T(x,t) + 0.8T(x+1,t)$$

(6 marks)

(b) Draw the finite difference grid to predict the temperature of all points up to 4 seconds. Label all unknown temperatures on the grid.

(6 marks)

(c) Determine all the unknown temperatures.

(4 marks)

(d) Analyze why the heat conduction in this question is more suitable to be solved using the implicit finite-difference approach.

(4 marks)

PART B: ANSWER TWO (2) QUESTIONS

Q4 (a) Given the data as in **Table Q4(a)**

Table Q4(a)

x	1	3	5	6	7
f(x)	2.9777	16.5237	67.5817	114.9927	181.1597

Conclude that by using data from $x=3$ to $x=5$, the Newton's divided difference interpolation polynomial is given by:

$$f(x)=0.521x^3+2.4567$$

Subsequently, predict the $f(4)$.

(10 marks)

(b) Given the graph of $y_1=5x^3$ and $y_2=0.5\sin x+0.2x$ on the same plane as in **Figure Q4(b)**. Estimate the point of intersection for y_1 and y_2 using Secant method. Start with the interval [0.2, 0.5]. Iterate the calculation until $|x_{i+1}-x_i|<0.0005$. Correct to 6 decimal places.

(10 marks)

Q5 (a) The movement of an object is represented by

$$s(t)=8e^{-0.5t}$$

Utilize numerical differentiation using 3 point central formula with $h=1$ and $h=0.5$ to approximate the object's velocity and acceleration at 15 second. State which h gives better approximation. Correct to 4 decimal places.

(10 marks)

(b) Approximate the value of

$$\int_1^5 \frac{x}{\sqrt{x+2}}\,dx$$

Using two and three points Gauss Quadrature.

(10 marks)

Q6 (a) Given the following system of linear equations:

$$\text{System 1: } \begin{pmatrix} 2 & 2 & 3 & 0 \\ 9 & 3 & 0 & 1 \\ 1 & 8 & 0 & 9 \\ 0 & 3 & 1 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -5 \\ 8 \end{pmatrix}$$

$$\text{System 2: } \begin{pmatrix} 1 & 5 & 0 & 0 \\ 8 & 2 & 6 & 0 \\ 0 & 9 & 3 & 7 \\ 0 & 0 & 10 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

$$\text{System 3: } \begin{pmatrix} -2 & 3 & 1 \\ 4 & 3 & -17 \\ 0 & 8 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 19 \\ 7 \\ 0 \end{pmatrix}$$

(i) Classify the system of linear equations into tridiagonal and non-tridiagonal categories.

(2 marks)

(ii) Determine which system of linear equations can be solved by LU Decomposition: Thomas Method (Variant 1) and solve it subsequently.

(8 marks)

(b) Given the following data which is generated by the function $y = \frac{1}{3x^2}$:

Table Q6(b)

x	1	2	3	4	5
$y = f(x)$	0.33333	0.08333	0.03704	0.02083	0.01333

Comparing quadratic (x within [2,4]) and cubic (x within [2,5]) Lagrange interpolations method, investigate which method gives the highest accuracy (in terms of absolute error) in predicting the value of $f(3.5)$.

(10 marks)

-END OF QUESTIONS-

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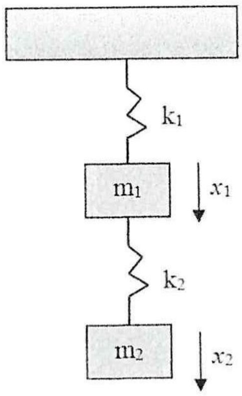

Figure Q1

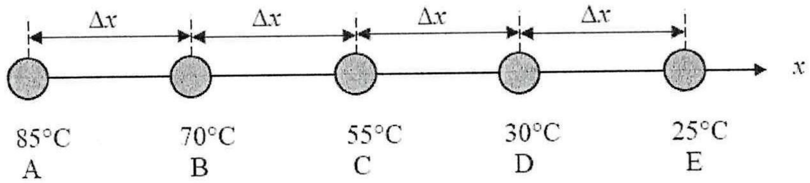

Figure Q3: Distribution of the initial temperatures

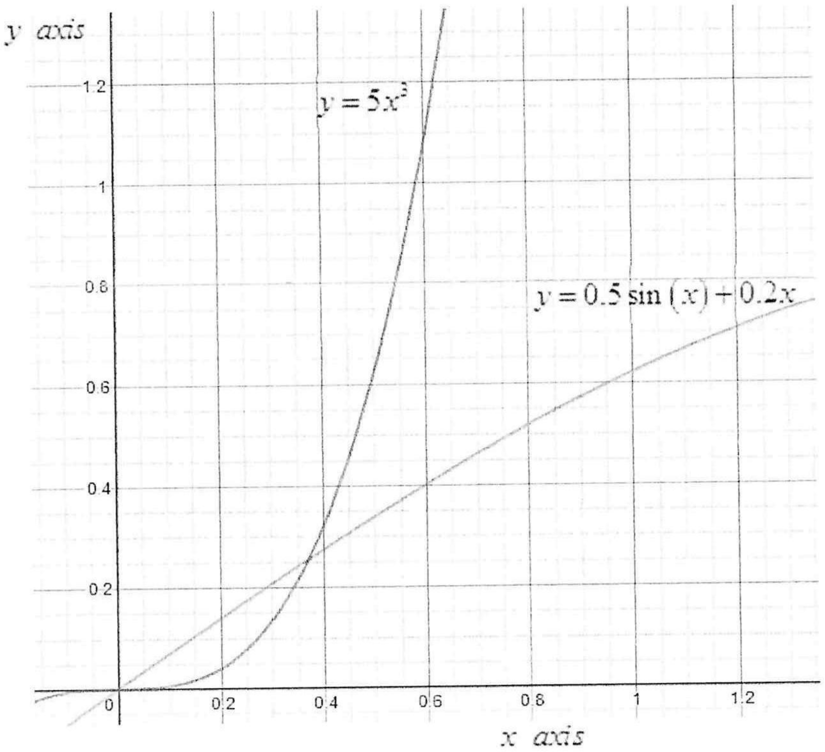

Figure Q4(a)

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FORMULA

Secant Method: $x_{i+1} = \dfrac{x_{i-1}y(x_i) - x_i y(x_{i-1})}{y(x_i) - y(x_{i-1})}$

Thomas Method (Variant 1):

$$\begin{bmatrix} d_1 & e_1 & 0 & \cdots & 0 \\ c_2 & d_2 & e_2 & \cdots & 0 \\ 0 & c_3 & \ddots & & 0 \\ \vdots & \vdots & & d_{n-1} & e_{n-1} \\ 0 & 0 & \ldots & c_n & d_n \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 & 0 & \cdots & 0 \\ c_2 & \alpha_2 & 0 & \cdots & 0 \\ 0 & c_3 & \ddots & & 0 \\ \vdots & \vdots & & \alpha_{n-1} & 0 \\ 0 & 0 & \ldots & c_n & \alpha_n \end{bmatrix} \begin{bmatrix} 1 & \beta_1 & 0 & \cdots & 0 \\ 0 & 1 & \beta_2 & \cdots & 0 \\ 0 & 0 & \ddots & & 0 \\ \vdots & \vdots & & 1 & \beta_{n-1} \\ 0 & 0 & \ldots & 0 & 1 \end{bmatrix}$$

Thomas Algorithm:

i	1	2	$\cdots$	n
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_i$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

Lagrange interpolation:

$$f(x) = P_n(x) = \sum_{i=0}^{n} L_i(x) f_i \quad where \quad L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

Newton's Divided Difference interpolating polynomial:

$$f(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2)$$

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FORMULA

Gauss Quadrature:

$$x_\xi = \frac{1}{2}\left[(1-\xi)x_0 + (1+\xi)x_n\right]$$

$$I = \left(\frac{x_n - x_0}{2}\right) I_\xi$$

$$I_\xi = R_1\phi(\xi_1) + R_2\phi(\xi_2) + K + R_n\phi(\xi_n)$$

n	$\pm\xi_j$	R_j
1	0.0	2.0
2	0.5773502692	1.0
3	0.7745966692	0.555555556
	0.0	0.888888889

3 Point Central Difference:

$$y'(x) = \frac{y(x+h) - y(x-h)}{2h}$$

$$y''(x) = \frac{y(x+h) - 2y(x) + y(x-h)}{h^2}$$

Power Method:

$$\{V\}^{k+1} = \frac{[A]\{V\}^k}{\lambda_{k+1}}$$

Inverse Power Method:

$$\{V\}^{k+1} = \frac{[A]^{-1}\{V\}^k}{\lambda_{k+1}}$$

Characteristic Equation:

$$\det(A - \lambda I) = 0$$

Euler 's Method:

$$f(x_i, y_i) = y'(x_i)\ ;\ y(x_{i+1}) = y(x_i) + hy'(x_i)$$

Modified Euler's Method:

$$f(x_i, y_i) = y'(x_i)$$

$$y_{i+1} = y_i + \frac{1}{2}k_1 + \frac{1}{2}k_2$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf(x_i + h, y_i + k_1)$$

Midpoint/ Improved Euler Method:

$$f(x_i, y_i) = y'(x_i)$$

$$y_{i+1} = y_i + k_2$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf(x_i + 0.5h, y_i + 0.5k_1)$$