UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BDA 24003
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) Sketch the domain of the following function function :

$$f(x,y) = \ln((x^2 + y^2 - 4)(9 - x^2 - y^2)$$

(6 marks)

(b) Let $z = f(x,y)$ and $f(x,y) = x^2 + 9z^2$. By letting $z = 1$, 4 and 9, sketch the level curves of f. Hence sketch the 3D graph of f.

(7 marks)

(c) Given $x^2 - 3yz^2 = 2 - xyz$, evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ by implicit differentiation.

(7 marks)

Q2 (a) Let $z = \frac{x}{y}$, $x = 2\cos u$ and $y = 3\sin v$. Use the chain rule to find $\frac{\delta z}{\delta u}$

(4 marks)

(b) Evaluate all the extreme points (if exist) for $f(x,y) = e^y \cos x$

(8 marks)

(c) The dimension of a closed rectangular box is 3 m, 4 m and 5 m respectively with the possible error $\frac{100}{192}$ cm. Use partial derivatives to estimate the maximum possible error in calculating volume of the box.

(8 marks)

Q3 (a) Use double integrals to calculate the volume of the tetrahedron $3x + 2y + 4z = 12$ in the first octant.

(5 marks)

(b) Analyze all relative maxima, relative minima and saddle points, if any for $f(x,y) = x^3 - 3xy + y^3$.

(8 marks)

(c) Transform the integral $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-y^2-x^2}} z^2 \sqrt{x^2 + y^2 + z^2}\, dzdydx$ to spherical coordinates. Then calculate the triple integral.

(7 marks)

Q4 (a) Prove that the curvature of a circle of radius r is $\frac{1}{r}$

(10 marks)

(b) Convert to spherical coordinates and evaluate

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} (x^2+y^2+z^2)dzdydx$$

(10 marks)

Q5 (a) Evaluate $\int_C 3x - yds$, where C is the line segment from P (3,5) to Q (1,2).

(10 marks)

(b) Verify the Green's Theorem for the line integral $\int_C (x^2+y^2)dx - x\,dy$, where C consists of the portion of arc of circle $x^2 + y^2 = 1$ counterclockwise from (1,0) to (0,1), straight line segment from (0,1) to (0,0) and (0,0) to (1,0).

(10 marks)

Q6 (a) Use Gauss Theorem to evaluate $\iint_\sigma \boldsymbol{F}.\boldsymbol{n}dS$, where $\boldsymbol{F}(x,y,z) = x^2\,\mathbf{i} + y^2\,\mathbf{j} + z^2\,\mathbf{k}$, $\boldsymbol{n}$ is oriented outward and σ is the surface enclosed by the cylinder $x^2 + y^2 = 4$, planes $z = 0$ and $z = 5$.

(10 marks)

(b) Evaluate the surface integral $\iint_\sigma x^2 + y^2 dS$ where σ is the portion of the cone $z = \sqrt{3(x^2+y^2)}$ for $0 \le z \le 3$.

(10 marks)

-END OF QUESTION –

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2019/2020
COURSE NAME: ENGINEERING MATHEMATICS III
PROGRAMME CODE : BDD
COURSE CODE : BDA 24003

FORMULA

Total Differential

For function $z = f(x,y)$, the total differential of z, dz is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Relative Change

For function $z = f(x,y)$, the relative change in z is given by:

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

Implicit Differentiation

Suppose that z is given implicitly as a function $z = f(x,y)$ by an equation of the form $F(x,y,z) = 0$, where $F(x,y,f(x,y)) = 0$ for all (x,y) in the domain of f, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
   $f(x,y)$ has a local maximum value at (a,b)

b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
   $f(x,y)$ has a local minimum value at (a,b)

c. If $D < 0$
   $f(x,y)$ has a saddle point at (a,b)

d. If $D = 0$
   The test is inconclusive

Surface Area

$$\text{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\,dA$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2019/2020
COURSE NAME: ENGINEERING MATHEMATICS III
PROGRAMME CODE : BDD
COURSE CODE : BDA 24003

Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$$\iint_R f(x,y)dA = \iint_R f(r,\theta)r\,dr\,d\theta$$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)r\,dz\,dr\,d\theta$$

Spherical Coordinates:

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

In 2-D: Lamina

Given that $\delta(x,y)$ is a density of lamina

Mass, $m = \iint_R \delta(x,y)dA$, where

Moment of Mass

a. About x-axis, $M_x = \iint_R y\delta(x,y)dA$

b. About y-axis, $M_y = \iint_R x\delta(x,y)dA$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2019/2020
COURSE NAME: ENGINEERING MATHEMATICS III
PROGRAMME CODE : BDD
COURSE CODE : BDA 24003

Centre of Mass

Non-Homogeneous Lamina:

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m} \right)$$

Centroid

Homogeneous Lamina:

$$\bar{x} = \frac{1}{Area\ of\quad R} \iint_R x\,dA \text{ and } \bar{y} = \frac{1}{Area\ of\quad R} \iint_R y\,dA$$

Moment Inertia:

a. $I_y = \iint_R x^2 \delta(x, y)\,dA$

b. $I_x = \iint_R y^2 \delta(x, y)\,dA$

c. $I_o = \iint_R (x^2 + y^2) \delta(x, y)\,dA$

In 3-D: Solid

Given that $\delta(x, y, z)$ is a density of solid

Mass, $m = \iiint_G \delta(x, y, z)\,dV$

If $\delta(x, y, z) = c$, where c is a constant, $m = \iiint_G dA$ is volume.

Moment of Mass

a. About yz-plane, $M_{yz} = \iiint_G x\delta(x, y, z)\,dV$

b. About xz-plane, $M_{xz} = \iiint_G y\delta(x, y, z)\,dV$

c. About xy-plane, $M_{xy} = \iiint_G z\delta(x, y, z)\,dV$

Centre of Gravity

$$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m} \right)$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2019/2020
COURSE NAME: ENGINEERING MATHEMATICS III

PROGRAMME CODE : BDD
COURSE CODE : BDA 24003

Moment Inertia

a. About x-axis, $I_x = \iiint_G (y^2 + z^2)\delta(x, y, z)dV$

b. About y-axis, $I_y = \iiint_G (x^2 + z^2)\delta(x, y, z)dV$

c. About z-axis, $I_z = \iiint_G (x^2 + y^2)\delta(x, y, z)dV$

Directional Derivative

$D_u f(x, y) = (f_x \mathbf{i} + f_y \mathbf{j}) \cdot \mathbf{u}$

Del Operator

$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$

Gradient of $\phi = \nabla \phi$

Let $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla.\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2019/2020
COURSE NAME: ENGINEERING MATHEMATICS III
PROGRAMME CODE : BDD
COURSE CODE : BDA 24003

Let C is smooth curve defined by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The **Binormal Vector,** $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

Curvature

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

Radius of Curvature

$$\rho = \frac{1}{\kappa}$$

Green's Theorem

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

Gauss's Theorem

$$\iint_S \mathbf{F} \cdot \mathbf{n}\,dS = \iiint_G \nabla \cdot \mathbf{F}\,dV$$

Stoke's Theorem

$$\oint_C \mathbf{F} \cdot dr = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\,dS$$

Arc Length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}, t \in [a,b]$, hence, the **arc length**,

$$s = \int_a^b \|\mathbf{r}'(t)\|\,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$$