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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER 1
SESSION 2019/2020

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BDA 14103
PROGRAMME	:	BDD
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	PART A: ANSWER **ALL** QUESTIONS. PART B: ANSWER **THREE (3)** QUESTIONS ONLY OUT OF FOUR.

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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PART A

Q1 Let $f(x)$ be a function of period 2π such that

$$f(x)=\begin{cases}-5, & -\pi\le x<0\\ 5, & 0\le x<\pi\end{cases}$$

$$f(x)=f(x+2\pi)$$

(a) Sketch the graph of $f(x)$ in the interval $-2\pi < x < 2\pi$

(2 marks)

(b) Prove that the Fourier series for $f(x)$ is:

$$f(x)=\frac{20}{\pi}\left[\sin x+\frac{1}{3}\sin 3x+\frac{1}{5}\sin 5x+\cdots\right]$$

(12 marks)

(c) By giving an appropriate value for x, demonstrate that

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$$

(6 marks)

Q2 A rod of length 2 m is perfectly insulated laterally, with ends kept at temperature 0°C and its temperature at x is initially $f(x)=100\sin(\pi x/2)$ °C. The heat equation is,

$$\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$$

(a) By using the method of separation of variable, and applying the boundary condition, prove that

$$u(x,t)=\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2}e^{-\frac{n^2\pi^2 t}{2}}$$

where b_n is an arbitrary constant.

(14 marks)

(b) By applying the initial condition, find the value of b_n.

(6 marks)

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PART B

Q3 (a) Sketch the graph and express the following function in term of unit step functions.

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ (t-1), & 1 \le t < 2 \\ 1, & t \ge 2 \end{cases}$$

(5 marks)

(b) By using the obtained relation in **Q3** (a), solve the Laplace transforms of the function by using a second shift property.

(5 marks)

(c) Solve the inverse Laplace transforms of the following expressions.

$$\frac{s+1}{(s+3)^2+16}$$

(10 marks)

Q4 (a) For the differential equation given below:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 5y = 3e^{2x}$$

Determine:

(i) the general solution; and
(ii) the particular solution to satisfy the conditions: $y(0) = 2$ and $y'(0) = 2$.

(8 marks)

(b) An object at temperature θ (in °C) is placed in a room that is at temperature θ_{room} = 18 °C. The object takes 5 minutes to cool down from 70 °C to 57 °C. This problem can be modelled through the following differential equation:

$$\frac{d\theta}{dt} = -k(\theta - \theta_{room})$$

Determine;

(i) the general solution; and
(ii) time taken for the object to cool to $\theta = 40$ °C.

(12 marks)

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Q5 (a) Solve the following differential equation:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$$

(5 marks)

(b) Using the solution in **Q5 (a)** find the general solution to a force oscillation system represented by the following differential equation:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 2\sin(2x)$$

(10 marks)

(c) Obtain the particular solution for **Q5 (b)** to satisfy the following set of conditions:

$$y(0) = 1/2, \qquad y'(0) = 0$$

(5 marks)

Q6 (a) Determine whether the following differential equation are exact or inexact.

(i) $x' = (1 - 2xt)/(t^2 + 3x^2)$

(ii) $xy' + 2y = x^2 - x$

(6 marks)

(b) If $L\{f(t)\} = F(s)$, find the Laplace transform for the following function:

$$f(t) = \frac{1 - \cos 2t}{t}$$

(6 marks)

(c) Determine whether the following periodic functions are EVEN or ODD

(i) $f(x) = x^2 + 1 \qquad [-2\pi \le x \le 2\pi]$

(ii) $g(x) = x^3 - 3x \qquad [-2\pi \le x \le 2\pi]$

(iii) $h(x) = x^4 - 4x^2 \qquad [-2\pi \le x \le 2\pi]$

(iv) $f(x)g(x) \qquad [-2\pi \le x \le 2\pi]$

(8 marks)

- END OF QUESTION -

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FORMULAS

First Order Differential Equation

Type of ODEs	General solution
Linear ODEs: $y' + P(x)y = Q(x)$	$y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx}Q(x)dx + C\right\}$
Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$	$F(x,y) = \int f(x,y)dx$ $F(x,y) - \int \left\{\frac{\partial F}{\partial y} - g(x,y)\right\} dy = C$
Inexact ODEs: $M(x,y)dx + N(x,y)dy = 0$ $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Integrating factor; $i(x) = e^{\int f(x)dx}$ where $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $i(y) = e^{\int g(y)dy}$ where $g(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$	$\int iM(x,y)dx - \int \left\{ \frac{\partial\left(\int iM(x,y)dx\right)}{\partial y} - iN(x,y) \right\} dy = C$

Characteristic Equation and General Solution for Second Order Differential Equation

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1 e^{mx} + c_2 x e^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

Method of Undetermined Coefficient

$g(x)$	y_p
Polynomial: $P_n(x) = a_n x^n + \ldots + a_1 x + a_0$	$x^r(A_n x^n + \ldots + A_1 x + A_0)$
Exponential: e^{ax}	$x^r(Ae^{ax})$
Sine or Cosine: $\cos\beta x$ or $\sin\beta x$	$x^r(A\cos\beta x + B\sin\beta x)$

Note: r is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

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Method of Variation of Parameters

The particular solution for $y'' + ay' + by = r(x)$, is given by $y(x) = u_1y_1 + u_2y_2$, where;

$$u_1 = -\int \frac{y_2 r(x)}{W} dx \quad \text{and} \quad u_2 = \int \frac{y_1 r(x)}{W} dx \qquad W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix} = y_1y'_2 - y_2y'_1$$

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$
a	$\frac{a}{s}$
$t^n, n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t), n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^nF(s)}{ds^n}$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$y'(t)$	$sY(s) - y(0)$
$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$

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Fourier Series

Fourier series expansion of periodic function with period 2 π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Half Range Series

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

Trigonometric Series

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\{\cos(\alpha-\beta) - \cos(\alpha+\beta)\}$$