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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2018/2019

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BDA 34003/ BWM 30603
PROGRAMME	:	BDD
EXAMINATION DATE	:	JUNE / JULY 2019
DURATION	:	3 HOURS
INSTRUCTIONS	:	(a) ANSWER **ALL** QUESTIONS IN **PART A** (b) ANSWER **TWO (2)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

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PART A: ANSWER ALL QUESTIONS

Q1 a) State a suitable numerical method that can be used to determine
(i) The largest eigenvalue
(ii) The smallest eigenvalue

(2 marks)

b) Given

$$A = \begin{pmatrix} 2 & 2 & 10 \\ 8 & 3 & 4 \\ 10 & 4 & 5 \end{pmatrix}$$

(i) Evaluate the largest eigenvalue and its corresponding eigenvector using an appropriate numerical method. Use $x_0 = (1 \quad 0 \quad 1)^T$ and iterate until $|\lambda_{k+1} - \lambda_k| \le 0.01$.

(10 marks)

(ii) Compare the obtained solution in **Q1(b)(i)** with reference to the solution given by the characteristic equation in terms of absolute error.

(8 marks)

Q2 The temperature distribution in a tapered conical cooling fin as illustrated in **Figure Q2** is described by the following differential equation, which has been nondimensionalized

$$\frac{d^2u}{dx^2} + \left(\frac{2}{x}\right)\left(p\frac{du}{dx}\right) = 0$$

where u = temperature $(0 \le u \le 1)$, x = axial distance $(0 \le x \le 1)$, and p is a non-dimensional parameter that describes the heat transfer and geometry. The differential equation has the boundary condition $u(x=0)=0$ and $u(x=1)=1$.

(a) By dividing the axial distance into six equidistant nodes, construct the differential equation using central finite difference approximation. Use $p = 10$.

(8 marks)

(b) Determine the temperature at all nodes using finite difference approach. You have to use the Thomas Algorithm to solve the obtained system of linear equations.

(12 marks)

Q3 A straight metal bar of length 2.5 cm is illustrated in **Figure Q3**. The temperatures on its ends are maintained for 2 seconds. The initial temperatures of the bar are shown in the figure. The temperature at point A is maintained at 100°C while point F is maintained at 5°C. This bar is fully insulated on its surface so the heat transfer occurs only in its longitudinal axis. The unsteady state heating equation follows a heat equation, given as:

$$\frac{\partial T}{\partial t} - \mathrm{K}\frac{\partial^2 T}{\partial x^2} = 0$$

where K is thermal diffusivity of material and x is the longitudinal coordinate of the bar. The thermal diffusivity of the material is given as 0.03125 cm^2 /s.

(a) By using Implicit Crank Nicolson method, deduce that the temperature distribution along the metal bar for every 1 second is given as

$$-T(x-1,t+1)+18T(x,t+1)-T(x+1,t+1)=T(x-1,t)+14T(x,t)+T(x+1,t)$$

(9 marks)

(b) Draw clearly the finite difference grid to predict the temperature of all points up to 2 seconds. Label all unknown temperatures in the grid.

(6 marks)

(c) Construct the simultaneous equations based on Implicit Crank Nicolson method to determine the temperature of points A, B, C, D, E and F when time is 1 second. You do not need to determine the unknown temperature of each point.

(5 marks)

PART B: ANSWER TWO (2) QUESTIONS

Q4 The differential equation for steady state condition of heat conduction through a wall with considering internal heat generation is given by

$$\frac{d^2T}{dx^2}+\frac{G}{k}=0$$

where T = temperature (°C), x = position (cm), G = internal heat source (W/cm³), and k = thermal conductivity (W/cm/°C). An experimental work was done and the temperatures of the wall at specific positions were measured and tabulated in Table Q4(a).

Table Q4(a): Wall temperature at different positions

x (cm)	-3.00	-2.25	-1.50	-0.75	0.75	1.50	2.25	3.00
T (°C)	40.00	42.81	44.82	46.03	46.03	44.82	42.81	40.00

(a) Conclude that by using data from $x=-1.5$ to $x=1.5$, the Newton's divided difference interpolation polynomial is given by:

$$T(x)=-0.7170x^2+46.4334$$

Subsequently, predict the temperature at $x=0$.

(10 marks)

(b) By using Secant method, determine at which position the temperature of the wall will become 46°C. Start with the interval [-1.5 0.75]. Iterate the calculation until $|x_{i+1}-x_i|<0.05$.

(10 marks)

Q5 (a) The distance x, measured in metres, of a downhill skier from a fixed point is measured at intervals of 0.25 sec. The data gathered is shown in Table Q5.

Table Q5: The distance of a downhill skier from a fixed point at different times

t	x
0.00	0.00
0.25	4.30
0.50	10.20
0.75	17.20
1.00	26.20
1.25	33.10

Utilize numerical differentiation using 2-Point Forward Difference to approximate the skier's velocity and acceleration at each value of time.

(10 marks)

(b) A new prototype rocket engine test show that the distance covered by a rocket from t = 8 sec to t = 30 sec is given by

$$x = \int_{8}^{30} \left(2000 \ln\left(\frac{140000}{140000 - 2100t} \right) - 9.8t \right) dt$$

Comparing two and three points Gauss Quadrature, investigate which method gives the highest accuracy (in terms of relative error) in predicting the distance covered by a rocket from t = 8 sec to t = 30 sec.

(10 marks)

Q6 (a) Given the following system of linear equations:

System 1: $$\begin{pmatrix} 2 & 2 & 3 & 0 \\ 9 & 3 & 0 & 1 \\ 1 & 8 & 0 & 9 \\ 0 & 3 & 1 & 8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -5 \\ 8 \end{pmatrix}$$

System 2: $$\begin{pmatrix} 2 & -3 & 0 \\ 1 & 3 & -7 \\ 0 & 2 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 8 \end{pmatrix}$$

System 3: $$\begin{pmatrix} -2 & 3 & 1 \\ 4 & 3 & -17 \\ 0 & 8 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 19 \\ 7 \\ 0 \end{pmatrix}$$

(i) Classify the system of linear equations into two different categories.

(2 marks)

(ii) Determine which system of linear equations can be solved by LU Decomposition: Thomas Method (Variant 1) and solve it subsequently.

(8 marks)

(b) Given the initial value problem as follows:

$$\frac{dy}{dx} = \frac{y^2}{x+2} \quad \text{at } x = 0(0.2)0.6$$

Solve the initial value problem with initial condition $y(0) = 1$ using

(i) Euler's method

(5 marks)

(ii) Modified Euler's method

(5 marks)

-END OF QUESTIONS-

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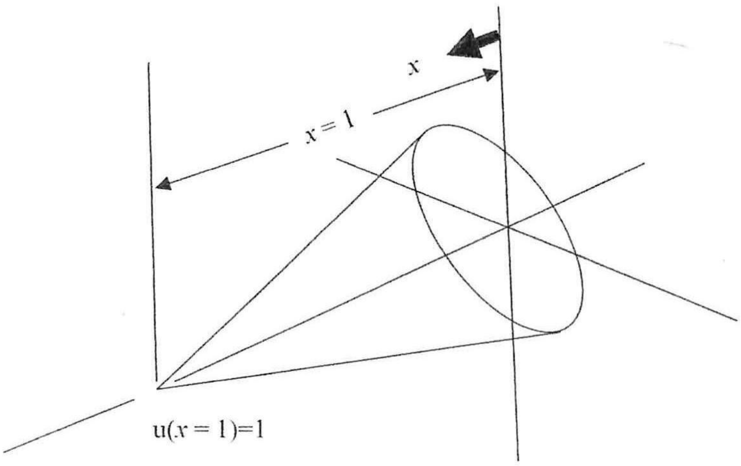

Figure Q2

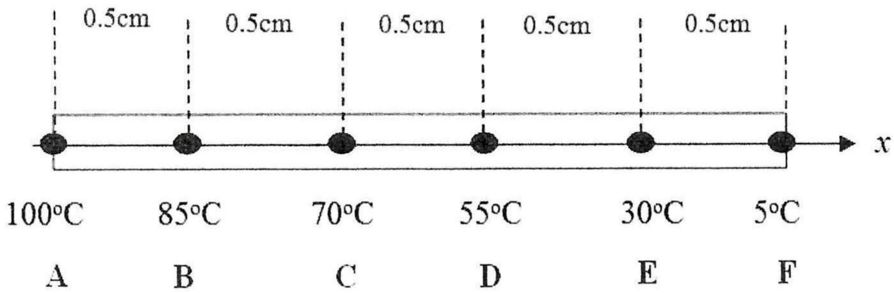

Figure Q3

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FORMULA

Secant Method: $x_{i+1} = \dfrac{x_{i-1}y(x_i) - x_i y(x_{i-1})}{y(x_i) - y(x_{i-1})}$

Thomas Method (Variant 1):

$$\begin{bmatrix} d_1 & e_1 & 0 & \cdots & 0 \\ c_2 & d_2 & e_2 & \cdots & 0 \\ 0 & c_3 & \ddots & & 0 \\ \vdots & \vdots & & d_{n-1} & e_{n-1} \\ 0 & 0 & \cdots & c_n & d_n \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 & 0 & \cdots & 0 \\ c_2 & \alpha_2 & 0 & \cdots & 0 \\ 0 & c_3 & \ddots & & 0 \\ \vdots & \vdots & & \alpha_{n-1} & 0 \\ 0 & 0 & \cdots & c_n & \alpha_n \end{bmatrix} \begin{bmatrix} 1 & \beta_1 & 0 & \cdots & 0 \\ 0 & 1 & \beta_2 & \cdots & 0 \\ 0 & 0 & \ddots & & 0 \\ \vdots & \vdots & & 1 & \beta_{n-1} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

Thomas Algorithm:

i	1	2	…	n
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

Newton's Divided Difference Interpolating Polynomial:

$y(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2)$

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FORMULA

2-Point Forward Difference: $y'(x) = \dfrac{y(x+h) - y(x)}{h}$

Gauss Quadrature:

$$x_\xi = \frac{1}{2}[(1-\xi)x_0 + (1+\xi)x_n]$$

$$I = \left(\frac{x_n - x_0}{2}\right) I_\xi$$

$$I_\xi = R_1\phi(\xi_1) + R_2\phi(\xi_2) + \ldots + R_n\phi(\xi_n)$$

n	$\pm\xi_j$	R_j
1	0.0	2.0
2	0.5773502692	1.0
3	0.7745966692 0.0	0.555555556 0.888888889

Power Method: $\{V\}^{k+1} = \dfrac{[A]\{V\}^k}{\lambda_{k+1}}$

Inverse Power Method: $\{V\}^{k+1} = \dfrac{[A]^{-1}\{V\}^k}{\lambda_{k+1}}$

Characteristic Equation: det(A-λI)=0

Euler's Method: $y(x_{i+1}) = y(x_i) + hy'(x_i)$

Modified Euler's Method:

$$f(x_i, y_i) = y'(x_i)$$

$$y_{i+1} = y_i + \frac{1}{2}k_1 + \frac{1}{2}k_2$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf(x_i + h, y_i + k_1)$$

Numerical Differentiation: $y_i' = \dfrac{y_{i+1} - y_{i-1}}{2h}$ $\quad y_i'' = \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$