UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BDA 34003
PROGRAMME	:	BDD
EXAMINATION DATE	:	DECEMBER 2018 / JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTIONS	:	(a) ANSWER **ALL** QUESTIONS IN **PART A** (b) ANSWER **TWO (2)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

PART A: ANSWER ALL QUESTIONS

Q1 a) State a suitable numerical method that can be used to determine

(i) The largest eigenvalue

(ii) The smallest eigenvalue

(2 marks)

b) Given

$$A = \begin{pmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{pmatrix}.$$

(i) Evaluate the largest eigenvalue using an appropriate numerical method. Use $x_0 = (1 \quad 1 \quad 1)^T$ and iterate until $|\lambda_{k+1} - \lambda_k| \leq 0.05$.

(10 marks)

(ii) Compare the obtained solution in **Q1(b)(i)** with reference to the solution given by the characteristic equation in terms of absolute error.

(8 marks)

Q2 State a suitable numerical method that can be used to determine The conservation of heat can be used to develop a heat balance for a long thin rod, as illustrated in **Figure Q2**. If the rod is not insulated along its length, and the system is at a steady state, the differential equation that results is

$$\frac{d^2T}{dx^2} + h'(T_a - T) = 0$$

Where h' is a heat transfer coefficient (cm^{-2}) that parameterizes the rate of heat dissipation to the surrounding air and T_a is the temperature of the surrounding air (°C). The conditions imposed to solve the differential equation can be fixed as $T(0) = T_1$ and $T(L) = T_2$. Consider a 10 cm rod with $T_a = 20$, $T_1 = 40$, $T_2 = 200$ and $h' = 0.01$.

(a) Transform the problem into a boundary value problem.

(3 marks)

(b) By dividing the length of rod into five equidistant nodes, deconstruct the differential equation using central finite difference approximation.

(7 marks)

(c) Determine the temperature at all nodes using finite difference approach.

(6 marks)

(d) Compare the accuracy of the obtained solution in terms of absolute error with reference to the analytical solution given by $T = 73.4523e^{0.1x} - 53.4523e^{-0.1x} + 20$.

(4 marks)

Q3 The heat transfer performance of a new conductor bar of length 20 cm is under inspection. The material is fully insulated such that the heat transfer is only one dimensional in axial direction (x-axis). The left end is maintained at temperature of 100°C, while the right end is maintained at temperature of 20°C, for $t > 0$. The distribution of the initial temperatures is shown in **Figure Q3.** The unsteady state heat conduction equation is given by

$$\frac{\partial T}{\partial t} - \kappa \frac{\partial^2 T}{\partial x^2} = 0$$

Where κ is a thermal diffusivity of material and x is the longitudinal coordinate of the bar. The thermal diffusivity of the material is given as $\kappa = 10$ cm²/s, and $\Delta t = 2$ second.

(a) By considering numerical differentiation $\frac{\partial^2 T}{\partial x^2} = \frac{T(x+1,t) - 2T(x,t) + T(x-1,t)}{\Delta x^2}$ and $\frac{\partial T}{\partial t} = \frac{T(x,t+1) - T(x,t)}{\Delta t}$, deduce that the temperature distribution along the bar at point $(x, t+1)$ in explicit finite-difference form is given by

$$T(x,t+1) = 0.8T(x-1,t) - 0.6T(x,t) + 0.8T(x+1,t)$$

(6 marks)

(b) Draw the finite difference grid to predict the temperature of all points up to 4 seconds. Label all unknown temperatures on the grid.

(6 marks)

(c) Determine all the unknown temperatures.

(4 marks)

(d) Analyze why the heat conduction in this question is more suitable to be solved using the implicit finite-difference approach.

(4 marks)

PART B: ANSWER TWO (2) QUESTIONS

Q4 a) The velocity of a car driven in a highway is give as $v(t) = t^3 - 20t^2 + 18$

(i) Use the secant method to locate the location of t where the car has zero velocity. Stop the iteration until $|x_{i+1} - x_i| \leq 0.05$. Start the interval $[2,5]$.

(5 marks)

(ii) Plot the convergence error graphically and discuss the trend.

(3 marks)

b) The upward velocity of Atlas V rocket model is represented as a function $v(t) = e^{0.091t}$ in the **Table Q4(b)**

Table Q4(b): The upward velocity of rocket at different time

t (sec)	0	20	40	70
v (ms^{-1})				

(i) Calculate the upward velocity of the rocket and complete the **Table Q4(b)**.

(2 marks)

(ii) Then, identify $v(60)$ by using Newton divided difference method.

(5 marks)

(iii) If (50, 94.6324) is added into the data above, determine $v(60)$ by using Newton divided difference method.

(5 marks)

Q5 a) Formula One is about to commence. Ferrari team develops a simulation on the track The velocity equation can be expressed as;

$$v(t) = 25t^3 + 15t^2 + 10t$$

Where the unit of velocity, v is measured in m/s and t in second.
By using $h = 10^{-5}$, determine the acceleration of the Ferrari car at $t = 3$ seconds using;

(i) 3-point central difference

(5 marks)

(ii) 5-point difference

(5 marks)

b) The time required to fulfill 50% lubricant to be pumped into an orifice is given by;

$$T = -\int_{1.22\times10^{-6}}^{0.61\times10^{-6}} \left(\frac{6.73x + 4.3025\times10^{-7}}{2.316\times10^{-11}x} \right) dx$$

(i) Approximate the time to pump 50% of lubricant by using 2-point Gauss Quadrature.

(8 marks)

(ii) Considering the relative error, analyze the obtained result in **Q5(b)(i)** if the exact value is 1% less from the result in **Q5(b)(i)**

(2 marks)

Q6 a) Use the Gauss-Seidel Method to obtain the solution of the following system:

$$\begin{aligned}
0.4x_1 - 4x_2 + 1.5x_3 &= 4.56 \\
7x_1 + 0.2x_2 - 1.9x_3 &= 1.73 \\
2.5x_1 - 1.6x_2 + 5x_3 &= 8.45
\end{aligned}$$

Stop the iteration until max $\{|x^{k+1} - x_i^k|\} < 0.003$.

(10 marks)

b) For a simple RL circuit, Kirchhoff's voltage law requires that

$$L\frac{di}{dt} + Ri = 0$$

Where i = current, L = inductance, and R = resistance. Given that $L = 1$, $R = 2$ and $i(0) = 0.6$.

(i) Solve for i using Midpoint method over the interval from $t = 0$ to $t = 1$. Use $h = 0.25$.

(5 marks)

(ii) Solve the differential equation analytically. Subsequently, find the difference between the value obtained by Midpoint method and actual value at $t = 1$.

(5 marks)

-END OF QUESTIONS-

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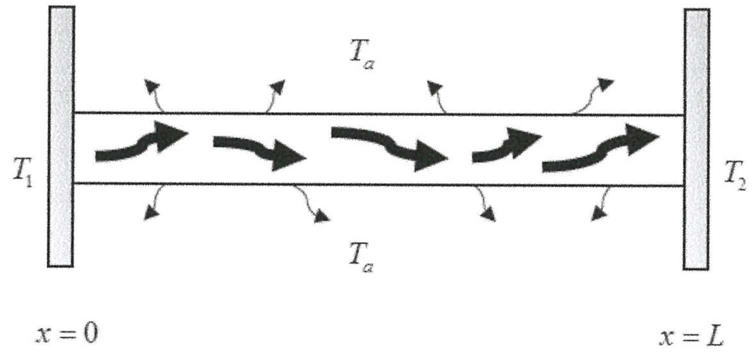

Figure Q2: Heat balance for a long thin rod

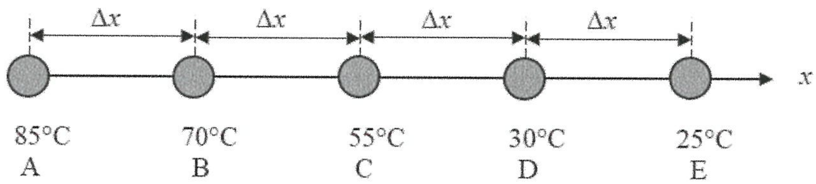

Figure Q3: Distribution of the initial temperatures

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FORMULA

Newton Raphson

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Secant

$$x_{i+1} = \frac{x_{i-1}y(x_i) - x_i y(x_{i-1})}{y(x_i) - y(x_{i-1})}$$

Doolittle

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

Crout

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

Thomas

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} = \begin{pmatrix} \alpha_1 & 0 & 0 & 0 \\ l_{21} & \alpha_1 & 0 & 0 \\ l_{31} & l_{32} & \alpha_1 & 0 \\ l_{41} & l_{42} & l_{43} & \alpha_1 \end{pmatrix} \begin{pmatrix} 1 & \beta_1 & 0 & 0 \\ 0 & 1 & \beta_2 & 0 \\ 0 & 0 & 1 & \beta_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Gauss-Seidel

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, \forall i = 1,2,\ldots,n$$

Lagrange

$$L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

$$f(x) = P_n(x) = \sum_{i=0}^{n} L_i(x) f_i$$

Newton Divided

$$f(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

Spline

$$h_k = (x_{k+1} - x_k)$$

$$d_k = \frac{f_{k+1} - f_k}{h_k}$$

$$b_k = 6(d_{k+1} - d_k)$$

$$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k$$

$$S_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i$$

$$a_i = \frac{m_{i+1} - m_i}{6h_i}$$

$$b_i = \frac{m_i}{2}$$

$$c_i = \left(\frac{f_{i+1} - f_i}{h_i}\right) - \left(\frac{2h_i m_i + h_i m_{i+1}}{6}\right)$$

$$d_i = f_i$$

2 Point Forward

$$y'(x) = \frac{y(x+h) - y(x)}{h}$$

2 Point Backward

$$y'(x) = \frac{y(x) - y(x-h)}{h}$$

3 Point Central

$$y'(x) = \frac{y(x+h) - y(x-h)}{2h}$$

$$y''(x) = \frac{y(x+h) - 2y(x) + y(x-h)}{h^2}$$

3 Point Forward

$$y'(x) = \frac{-y(x+2h) + 4y(x+h) - 3y(x)}{2h}$$

3 Point Backward

$$y'(x) = \frac{3y(x) - 4y(x-h) + y(x-2h)}{2h}$$

5 Point

$$y'(x) = \frac{-y(x+2h) + 8y(x+h) - 8y(x-h) + y(x-2h)}{12h}$$

$$y''(x) = \frac{-y(x+2h) + 16y(x+h) - 30y(x) + 16y(x-h) - y(x-2h)}{12h^2}$$

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FORMULA

Trapezoidal

$$\int_{x_0}^{x_n} y(x)dx = \frac{h}{2}(y_0 + y_n + 2\sum_{i=1}^{n-1} y_i)$$

Simpson

$$\int_{x_0}^{x_n} y(x)dx = \frac{h}{3}(y_0 + y_n + 4\sum_{i=1,3,5..}^{n-1} y_i + 2\sum_{i=2,4,6..}^{n-2} y_i)$$

$$\int_{x_0}^{x_n} y(x)dx = \frac{3}{8}h[y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5 + \ldots) + 2(y_3 + y_6)]$$

Gauss Quadrature

$$x_\xi = \frac{1}{2}[(1-\xi)x_0 + (1+\xi)x_n]$$

$$I = \left(\frac{x_n - x_0}{2}\right) I_\xi$$

$$I_\xi = R_1\phi(\xi_1) + R_2\phi(\xi_2) + \ldots + R_n\phi(\xi_n)$$

n	$\pm\xi_j$	R_j
1	0.0	2.0
2	0.5773502692	1.0
3	0.7745966692 0.0	0.555555556 0.888888889
4	0.8611363116 0.3399810436	0.3478548451 0.6521451549
5	0.9061798459 0.5384693101 0.0	0.2369268851 0.4786286705 0.5688888889

Characteristic Equation

$$|[A] - \lambda[I]| = 0 \qquad \{V\}^{k+1} = \frac{[A]\{V\}^k}{\lambda_{k+1}}$$

Euler Method

$$y(x_{i+1}) = y(x_i) + hy'(x_i)$$

Midpoint

$$f(x_i, y_i) = y'(x_i)$$
$$y_{i+1} = y_i + k_2$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$

Heun's

$$f(x_i, y_i) = y'(x_i)$$
$$y_{i+1} = y_i + \frac{1}{4}k_1 + \frac{3}{4}k_2$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf\left(x_i + \frac{2h}{3}, y_i + \frac{2k_1}{3}\right)$$