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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BDA 24003
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	DECEMBER 2018/ JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

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Q1 (a) Given the vector-valued function $\boldsymbol{r}\ (t) = (3 + 2\ cos\ \omega t)\ \mathbf{i} + (2 + 2\ sin\ \omega t)\ \mathbf{j}$, for $0 \le t \le 2\pi$.

(i) unit tangent vector $\boldsymbol{T}\ (t)$, principle unit normal vector N (t), curvature κ, and radius of curvature ρ.

(ii) Sketch the graph of $\boldsymbol{r}\ (t)$ and $\boldsymbol{T}\ (\frac{\pi}{4})$ in one coordinate system

(10 marks)

(b) A particle moves along a path which its position vector $\mathbf{r}(t)$ is given as a function of time t by $\mathbf{r}(t) = \cos 2t\ \mathbf{i} + \sin 2t\ \mathbf{j} + 4t^2\ \mathbf{k}$

(i) Determine the instantaneous velocity, speed and acceleration of the particle at time t.

(ii) Determine the time at which position vector is perpendicular to the acceleration vector.

(10 marks)

Q2 (a) The scalar function given by $f(x, y) = x \cos y$. Find

(i) the gradient of $f(x, y)$, and

(ii) the directional derivative of $f(x, y)$ in the direction of unit vector $\boldsymbol{v} = 2\mathbf{i} + \mathbf{j}$

(5 marks)

(b) Evaluate $\int_C (2y - 3z)dx + (2x + z)dy + (y - 3x)dz$ where C is line segment from the point (0, 0, 0) to (0, 1, 1).

(7 marks)

(c) Let C be the boundary of first quadrant region bounded by the line $y = x$ and curve $y = x^2$ oriented counterclockwise. Evaluate $\int_C xy\ dx + y^2 dy$ using Green's Theorem.

(8 marks)

Q3 (a) Suppose that $w = x^2 + y^2 - z^2$ and $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\cos\theta$, $z = \rho\cos\phi$. Find the rate of change of w with respect to θ and ϕ. (5 marks)

(b) Analyze all relative maxima, relative minima and saddle points, if any for $f(x,y) = x^2 + xy + y^2 - 3x$. (5 marks)

(c) Given $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{9-y^2-x^2}} \sqrt{x^2+y^2+z^2}\, dzdydx$

i. Sketch the solid represented by the triple integral and its projection on *xy*-plane.

ii. Transform the integral to spherical coordinates. Then calculate the triple integral and show the answer in form of surd.

(10 marks)

Q4 (a) Given function $f(y,z) = y^2 + 4z^2$.

(i) Sketch the contour map of z = 1, 6 and 10,

(ii) Sketch the graph.

(10 marks)

(b) Find the coordinate $\bar{x}$ and $\bar{z}$ for the centroid of a solid enclosed by surface $z = x^2$ and plane $y = 0$, $y = 1$ and $z = 1$. Assume that the solid has constant density 1.

(10 marks)

Q5 (a) Evaluate $\iint_R x + y + 2 \;\; dxdy$, where ***R*** is the region inside the unit square in which $x + y \geq 0.5$. (5 marks)

(b) Sketch the region ***R*** in the *xy*-plane bounded by the curves $y^2 = 4x$ and $y = 2x$, and find its area. (5 marks)

(c) Given the triple integral $\iiint_R \sqrt{x^2+y^2} \quad dV$, where *R* is the region lying above the *xy*-plane, and below cone $z = 3 - \sqrt{x^2 - y^2}$.

(i) Sketch the 3D-graph of the integral

(ii) Evaluate the integral

(10 marks)

Q6 (a) Given the force field $\mathbf{F}(x,y,z) = (5/3)y^3\,\mathbf{i} + 5xy^2\,\mathbf{j} + 2\,\mathbf{k}$

(i) Prove that F is conservative

(ii) By using formula $\nabla\emptyset = \mathbf{F}$, find a scalar potential $\emptyset$ for **F**.

(iii) Hence, compute the amount of work done against the force field F in moving an object from the point (1, 1, 1) to (2, 3, 4).

(10 marks)

(b) Let $\mathbf{F}(x,y,z) = z\,\mathbf{i} + y\,\mathbf{j} - x\,\mathbf{k}$ and oriented outward. Suppose that σ is the surface of the plane $x + y + z = 2$ contained within the triangle, with the vertices (2, 0, 0), (0, 2, 0) and (0, 0, 2) as shown in **Figure 6(b)**. Evaluate the surface integral.

(10 marks)

-END OF QUESTIONS –

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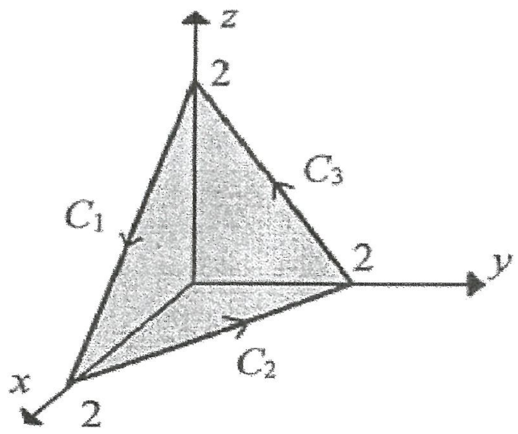

Figure 6(b)

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FORMULA

Total Differential

For function $z = f(x, y)$, the total differential of z, dz is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Relative Change

For function $z = f(x, y)$, the relative change in z is given by:

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

Implicit Differentiation

Suppose that z is given implicitly as a function $z = f(x, y)$ by an equation of the form $F(x, y, z) = 0$, where $F(x, y, f(x, y)) = 0$ for all (x, y) in the domain of f, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
$f(x, y)$ has a local maximum value at (a, b)

b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
$f(x, y)$ has a local minimum value at (a, b)

c. If $D < 0$
$f(x, y)$ has a saddle point at (a, b)

d. If $D = 0$
The test is inconclusive

Surface Area

$$\textit{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$$

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Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$$\iint_R f(x,y)\,dA = \iint_R f(r,\theta)\,r\,dr\,d\theta$$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$$

Spherical Coordinates:

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(\rho,\phi,\theta)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

In 2-D: Lamina

Given that $\delta(x,y)$ is a density of lamina

Mass, $m = \iint_R \delta(x,y)\,dA$, where

Moment of Mass

a. About x-axis, $M_x = \iint_R y\delta(x,y)\,dA$

b. About y-axis, $M_y = \iint_R x\delta(x,y)\,dA$

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Centre of Mass

Non-Homogeneous Lamina:

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m} \right)$$

Centroid

Homogeneous Lamina:

$$\bar{x} = \frac{1}{Area\,of \quad R} \iint_R x\,dA \text{ and } \bar{y} = \frac{1}{Area\,of \quad R} \iint_R y\,dA$$

Moment Inertia:

a. $I_y = \iint_R x^2 \delta(x,y)\,dA$

b. $I_x = \iint_R y^2 \delta(x,y)\,dA$

c. $I_o = \iint_R (x^2 + y^2) \delta(x,y)\,dA$

In 3-D: Solid

Given that $\delta(x,y,z)$ is a density of solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$

If $\delta(x,y,z) = c$, where c is a constant, $m = \iiint_G dA$ is volume.

Moment of Mass

a. About *yz*-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

b. About *xz*-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

c. About *xy*-plane, $M_{xy} = \iiint_G z\delta(x,y,z)\,dV$

Centre of Gravity

$$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m} \right)$$

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Moment Inertia

a. About x-axis, $I_x = \iiint\limits_G (y^2 + z^2)\delta(x, y, z)dV$

b. About y-axis, $I_y = \iiint\limits_G (x^2 + z^2)\delta(x, y, z)dV$

c. About z-axis, $I_z = \iiint\limits_G (x^2 + y^2)\delta(x, y, z)dV$

Directional Derivative

$D_u f(x, y) = (f_x\mathbf{i} + f_y\mathbf{j}) \cdot \mathbf{u}$

Del Operator

$\nabla = \dfrac{\partial}{\partial x}\mathbf{i} + \dfrac{\partial}{\partial y}\mathbf{j} + \dfrac{\partial}{\partial z}\mathbf{k}$

Gradient of $\phi = \nabla\phi$

Let $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla.\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

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Let C is smooth curve defined by $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t)=\dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t)=\dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The **Binormal Vector,** $\mathbf{B}(t)=\mathbf{T}(t)\times\mathbf{N}(t)$

Curvature

$$\kappa=\frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

Radius of Curvature

$$\rho=\frac{1}{\kappa}$$

Green's Theorem

$$\oint_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right) dA$$

Gauss's Theorem

$$\iint_S \mathbf{F}\cdot\mathbf{n}dS = \iiint_G \nabla\cdot\mathbf{F}dV$$

Stoke's Theorem

$$\oint_C \mathbf{F}\cdot dr = \iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}dS$$

Arc Length

If $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}, t\in[a,b]$, hence, the **arc length**,

$$s=\int_a^b \|\mathbf{r}'(t)\|\, dt=\int_a^b \sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\, dt$$