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FINAL EXAMINATION
SEMESTER 1
SESSION 2018/2019

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BDA 14103
PROGRAMME	:	BDD
EXAMINATION DATE	:	DECEMBER 2018 / JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	PART A: ANSWER **ALL** QUESTIONS. PART B: ANSWER **THREE (3)** QUESTIONS ONLY OUT OF FOUR.

THIS QUESTION PAPER CONSISTS OF **EIGHT (9)** PAGES

PART A

Q1 (a) The heat flux through the faces at the ends of bar is found to be proportional to $u_n = \partial u/\partial n$ at the ends. If the bar is perfectly insulated, also at the ends $x = 0$ and $x = L$ are adiabatic conditions,

$$u_x(0,t) = 0 \qquad u_x(L,t) = 0$$

prove that the solution of the heat transfer problem above (adiabatic conditions at both ends) gives as,

$$u(x,t) = A_o + \sum_{n=1}^{\infty} A_n \cos\frac{n\pi x}{L} e^{-\left(\frac{\alpha n\pi}{L}\right)^2 t}$$

where A_o and A_n are an arbitrary constant.

The heat equation is:

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

(17 marks)

(b) If $L = \pi$ and $\alpha = 1$ for the solution of heat transfer problem in **Q1** (a), find the temperature in the bar with the initial temperature, $f(x) = k =$ constant.

(3 marks)

Q2 A half-range expansions given as the following function:

$$f(x) = \begin{cases} \frac{2k}{L}x & for \quad 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & for \quad \frac{L}{2} < x < L \end{cases}$$

(a) Sketch a graph of $f(x)$ in the interval $0 < x < L$

(2 marks)

(b) Solve the given half-range expansion if the function $f(x)$ is extended to the interval $-L < x < L$ as an ***even function*** and sketch the periodic extension for the series.

(12 marks)

(c) Solve the given half-range expansion if the function $f(x)$ is extended to the interval $-L < x < L$ as an ***odd function*** and sketch the periodic extension for the Fourier series.

(6 marks)

PART B

Q3 (a) Express the graph shown in **Figure Q4 (a)** in terms of unit step function, and then find the Laplace transform.

(8 marks)

(b) Find the inverse Laplace transform of the following function using convolution theorem.

$$\frac{s}{(s^2+1)^2}$$

(12 marks)

Q4 (a) Obtain the general solution for the following differential equation:

$$(6x^2 - 10xy + 3y^2)dx + (-5x^2 + 6xy - 3y^2)dy = 0$$

(8 marks)

(b) The rate of cooling a body is given by the equation,

$$\frac{dT}{dt} = -k(T - 10)$$

where T is the temperature in degree Celsius, k is a constant and t is the time in minutes. When $t = 0$, $T = 90$ °C and when $t = 5$, $T = 60$ °C.

Show that when $t = 10$,

$$T(t) = 80e^{\ln\frac{(5/8)}{5}} + 10$$

(12 marks)

Q5 (a) Find the particular solution of the following differential equation that satisfies the given condition.

$$y'' + 4y' + 20y = 0; \qquad y(0) = 9, y'(0) = 10$$

(10 marks)

(b) By using a variation of parameter method, obtain the general solution for the following differential equation.

$$y'' - 5y' + 4y = e^{3x}$$

(10 marks)

Q6 (a) Determine whether the following differential equation are homogeneous or not.

$$\frac{dy}{dx} = \frac{x^2 + y^2}{(x - y)(x + y)}$$

(4 marks)

(b) If $L\{f(t)\} = F(s)$ and a is a constant, prove the First Shift Theorem that

$$\mathcal{L}\{e^{at} f(t)\} = F(s - a)$$

(6 marks)

(c) By using the Convolution Theorem, determine the inverse Laplace transforms of the following function.

$$\frac{2}{s^2(s - 2)}$$

(10 marks)

- **END OF QUESTION** -

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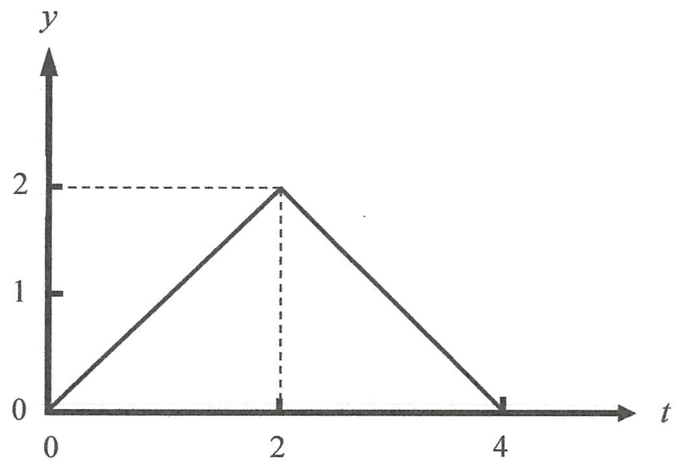

Figure Q4(a)

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FORMULAS

First Order Differential Equation

Type of ODEs	General solution
Linear ODEs: $y' + P(x)y = Q(x)$	$y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx}Q(x)dx + C\right\}$
Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$	$F(x,y) = \int f(x,y)dx$ $F(x,y) - \int\left\{\frac{\partial F}{\partial y} - g(x,y)\right\}dy = C$
Inexact ODEs: $M(x,y)dx + N(x,y)dy = 0$ $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Integrating factor; $i(x) = e^{\int f(x)dx}$ where $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $i(y) = e^{\int g(y)dy}$ where $g(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$	$\int iM(x,y)dx - \int\left\{\frac{\partial\left(\int iM(x,y)dx\right)}{\partial y} - iN(x,y)\right\}dy = C$

Characteristic Equation and General Solution for Second Order Differential Equation

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1 e^{mx} + c_2 x e^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

Method of Undetermined Coefficient

$g(x)$	y_p
Polynomial: $P_n(x) = a_n x^n + \ldots + a_1 x + a_0$	$x^r(A_n x^n + \ldots + A_1 x + A_0)$
Exponential: e^{ax}	$x^r(Ae^{ax})$
Sine or Cosine: $\cos\beta x$ or $\sin\beta x$	$x^r(A\cos\beta x + B\sin\beta x)$

Note: r is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

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Method of Variation of Parameters

The particular solution for $y'' + ay' + by = r(x)$, is given by $y(x) = u_1y_1 + u_2y_2$, where;

$$u_1 = -\int \frac{y_2 r(x)}{W} dx \quad \text{and} \quad u_2 = \int \frac{y_1 r(x)}{W} dx \qquad W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix} = y_1y'_2 - y_2y'_1$$

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$
a	$\frac{a}{s}$
$t^n, n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2 + a^2}$
$\cos at$	$\frac{s}{s^2 + a^2}$
$\sinh at$	$\frac{a}{s^2 - a^2}$
$\cosh at$	$\frac{s}{s^2 - a^2}$
$e^{at} f(t)$	$F(s-a)$
$t^n f(t), n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$y'(t)$	$sY(s) - y(0)$
$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$

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Fourier Series
Fourier series expansion of periodic function with period 2π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nxdx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nxdx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Half Range Series

$$a_0 = \frac{2}{L}\int_0^L f(x)dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$