Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	FINITE ELEMENT METHOD
COURSE CODE	:	BDA 31003 / BDA 40303
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWERS **FOUR (4)** QUESTIONS **ONLY**

THIS QUESTION PAPER CONSISTS OF **TWELVE (12)** PAGES

Q1 A two-dimensional plate is isolated in upper and lower edges, so no convection heat transfer is allowed. This plate has been modelled as 2 quadrilateral elements as illustrated in **Figure Q1**. The left edge is maintained at 50°C and heat flux, $q = 100W/m^2$°C is applied on this edge. The right edge is exposed to ambient temperature of 25°C with heat transfer convection coefficient, $h = 30W/m^2$°C. By following the node and element definitions as seen in **Figure Q1**,

(a) Calculate the conductance matrix and the thermal load vector of every element. Do you have any thermal constraints in this problem?

(8 marks)

(b) Considering your answer in (a), formulate the global conductance matrix and the global thermal load that will be used to solve the temperature vector. You can use either elimination method or penalty method.

(10 marks)

(c) Evaluate how to determine the temperature distribution vector of the fin (no calculation required!)

(7 marks)

Q2 Oil with dynamic viscosity of $\mu = 0.5Ns/m^2$ and density of $\rho = 800kg/m^3$ flows through the piping network as shown in **Figure Q2**. The pressure at the inlet point (node 1) is 46,400 Pa and the pressure at outlet point (node 6) is -4,000 Pa. An additional pressure at point (node 3) is 34,974 Pa. Given the dimensions of the piping system and the laminar flow throughout the system;

(a) Measure the pressure distribution in the system.

(17 marks)

(b) Determine the flow rate in each branch.

(6 marks)

(c) Evaluate either the pipes are in series or in parallel.

(2 marks)

Q3 **Figure Q3** shows a small rectangular element of a thin plate. The dimensions and the boundary conditions are such that its stress distribution cannot accurately be approximated by one-dimensional function. The stress varies in both the x and y directions.

(a) Formulate the stress distribution function for two-dimensional rectangular element as shown below.

$$\sigma^{(e)} = [S_i \quad S_j \quad S_m \quad S_n]\begin{Bmatrix} \sigma_i \\ \sigma_j \\ \sigma_m \\ \sigma_n \end{Bmatrix}$$

where

$$S_i = \left(1 - \frac{x}{l}\right)\left(1 - \frac{y}{w}\right),\ S_j = \frac{x}{l}\left(1 - \frac{y}{w}\right),\ S_m = \frac{xy}{lw},\ S_n = \frac{y}{w}\left(1 - \frac{x}{l}\right)$$

(15 marks)

(b) If $\sigma_i = 2500N/m^2$, $\sigma_j = 2000N/m^2$, $\sigma_m = 1500N/m^2$ and $\sigma_n = 3000N/m^2$, determine the value of stress at $x = \frac{1}{4}l$ and $y = \frac{1}{2}w$ of this element.

(6 marks)

(c) Specify how many merits on the usage of 3D element over 2D element in finite element modelling.

(4 marks)

Q4 Aluminum fin shown in **Figure Q4** is used to remove heat from a surface whose temperature is 150°C. The temperature of the surrounding air is 20°C. The natural heat transfer coefficient associated with the surrounding air is $30W/m^2$°C. The thermal conductivity of aluminum is $168W/m^2$°C.

(a) Construct the simplest finite element model to estimate the temperature at point *P*. Point *P* is located at the midpoint of the line separating rectangular and triangular shape sections. Label all the nodes, elements, constraints and loads clearly.

(5 marks)

(b) Estimate the temperature at point *P*.

(20 marks)

Q5 One of CST (constant strain triangular) elements in a thick structure is shown in **Figure Q5**. Since the structure is thick in *z* direction, it does not have any deformation in *z* direction. The material has a modulus elasticity $E = 200GPa$ and the Poison's ratio, ν = 0.25.

(a) Based on the given information, decide the problem case (plane stress or plane strain) and analyze the strain displacement matrix [B] of the element.

(5 marks)

(b) Calculate the elasticity matrix and the area of the element.

(5 marks)

(c) Determine the stiffness matrix of the element, use one unit thickness.

(5 marks)

(d) If the nodal displacements are known as $u_{11} = 1mm$, $u_{12} = 1mm$, $u_{13} = 0mm$, $v_{11} = 2mm$, $v_{12} = 1.5mm$, $v_{13} = 0mm$, evaluate the elemental stresses.

(10 marks)

-END OF QUESTIONS -

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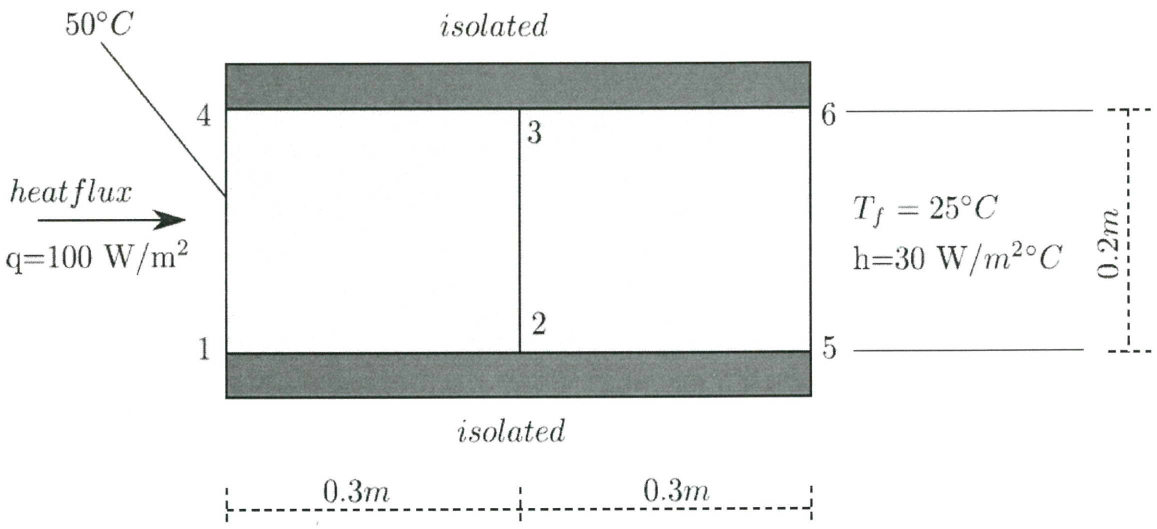

Figure Q1

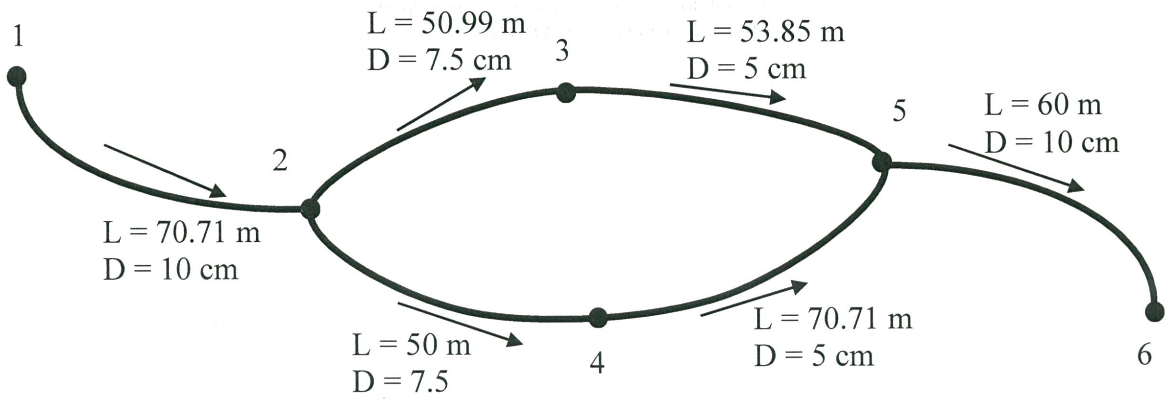

Figure Q2

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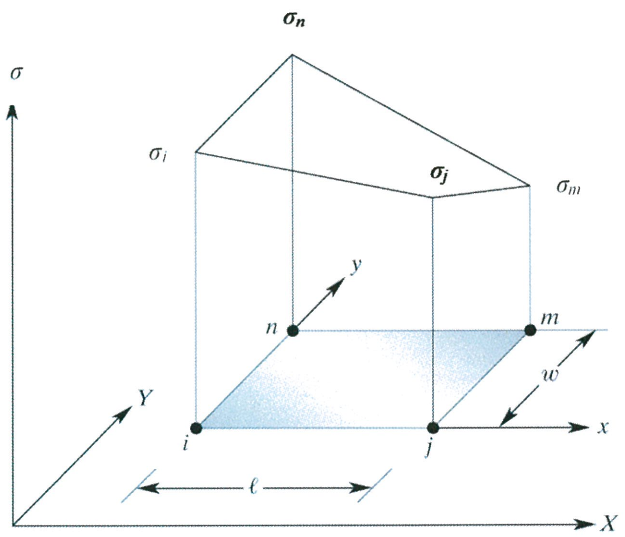

Figure Q3

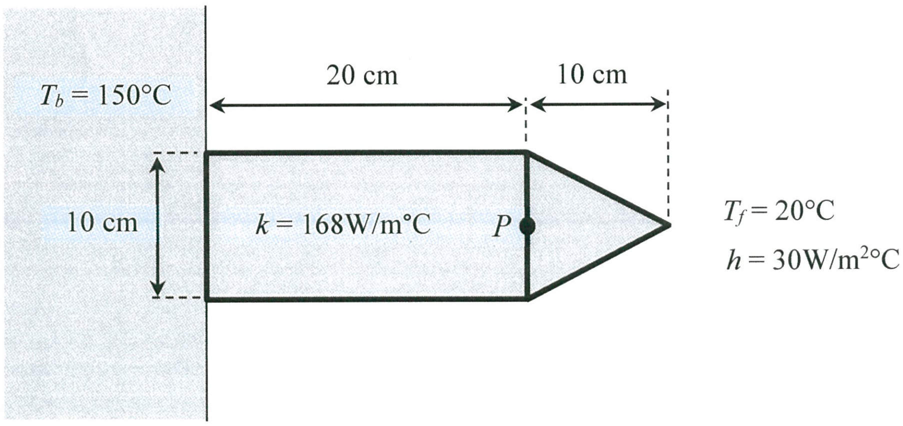

Figure Q4

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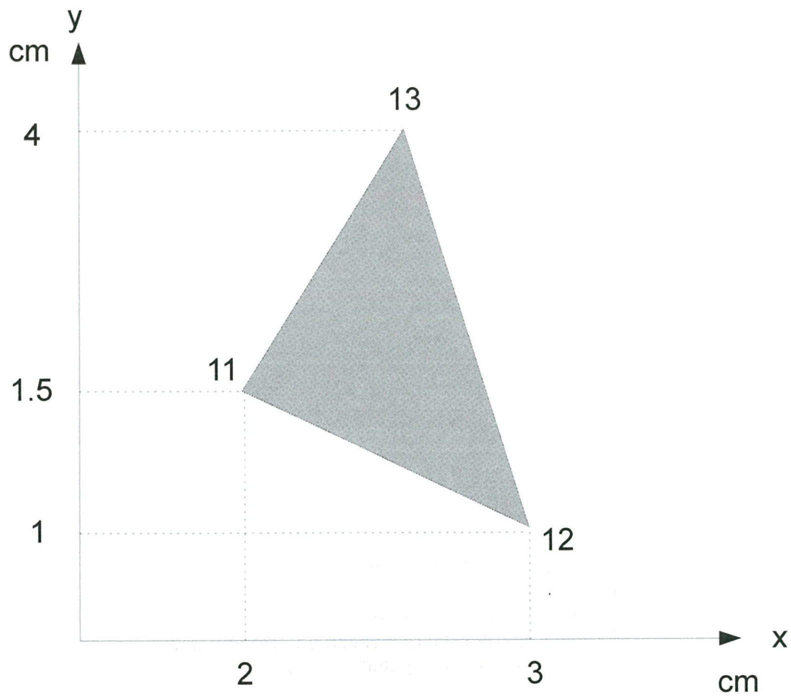

Figure Q5

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USEFUL EQUATIONS

Conductance matrix

$$[k^e] = \begin{bmatrix} h_{fi} & 0 \\ 0 & 0 \end{bmatrix} + \frac{kA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{h_f pL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & h_{fj}A \end{bmatrix}$$

Thermal load vector:

(i) Due to convection

$$\{f^e\} = \begin{Bmatrix} h_i A T_{fi} \\ 0 \end{Bmatrix} + \frac{h_p L T_f}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + \begin{Bmatrix} 0 \\ h_j A T_{fj} \end{Bmatrix}$$

(ii) Due to heat source

$$\{f^e\} = \frac{QAL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

(iii) Due to heat flux

$$\{f^e\} = q_i A\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + \frac{qpL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + q_j A\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

Flow-resistance matrix

$$[R] = \begin{bmatrix} \frac{\pi D^4}{128L\mu} & -\frac{\pi D^4}{128L\mu} \\ -\frac{\pi D^4}{128L\mu} & \frac{\pi D^4}{128L\mu} \end{bmatrix}$$

Flowrate, $Q = C(P_i + P_{i+1})$

Plane stress:

$$[E] = \frac{E}{(1-\gamma^2)}\begin{bmatrix} 1 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & \frac{1-\gamma}{2} \end{bmatrix}$$

Plain strain:

$$[E] = \frac{E}{(1+\gamma)(1-2\gamma)}\begin{bmatrix} 1-\gamma & \gamma & 0 \\ \gamma & 1-\gamma & 0 \\ 0 & 0 & \frac{1-2\gamma}{2} \end{bmatrix}$$

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Strain displacement matrix:

$$[B] = \frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$x_{ij} = x_i - x_j \qquad y_{ij} = y_i - y_j$$

$$A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

Stiffness matrix for CST element:

$$[K^e] = tA^e[B^e]^T[E^e][B^e]$$

Jacobian matrix, $[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \xi} \\ \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \xi} \end{bmatrix}$

Strain displacement matrix [B] for four-node quadrilateral element:

$$[B] = \begin{bmatrix} D_{G11} & 0 & D_{G12} & 0 & D_{G13} & 0 & D_{G14} & 0 \\ 0 & D_{G21} & 0 & D_{G22} & 0 & D_{G23} & 0 & D_{G24} \\ D_{G21} & D_{G11} & D_{G22} & D_{G12} & D_{G23} & D_{G13} & D_{G24} & D_{G14} \end{bmatrix}$$

Four-node quadrilateral element:

Stiffness matrix, $[k^e] = t\int_A [B(\xi,\eta)^e]^T[E^e]\,[B((\xi,\eta)^e]|J(\xi,\eta)|d\xi d\eta$

Bilinear rectangular element:

$$[K^e] = \frac{k_x w}{6l}\begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{k_y l}{6w}\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

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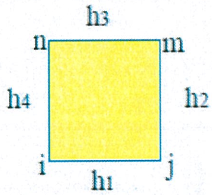
Conductance matrix, [K] due to convection:

$$[K^e]=\frac{h_3 L_{mn}}{6}\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&2&1\\0&0&1&2\end{bmatrix}$$

$$[K^e]=\frac{h_4 L_{ni}}{6}\begin{bmatrix}2&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&2\end{bmatrix}$$

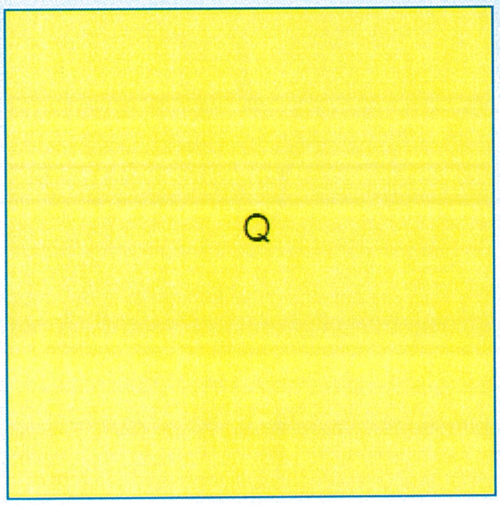


$$[K^e]=\frac{h_2 L_{jm}}{6}\begin{bmatrix}0&0&0&0\\0&2&1&0\\0&1&2&0\\0&0&0&0\end{bmatrix}$$

$$[K^e]=\frac{h_1 L_{ij}}{6}\begin{bmatrix}2&1&0&0\\1&2&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

Thermal load due to heat source:

$$\{F^e\}=\frac{QA}{4}\begin{Bmatrix}1\\1\\1\\1\end{Bmatrix}$$



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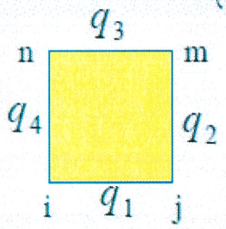
Thermal load due to heat flux

$$\{F^e\} = \frac{q_3 l_{mn}}{2}\begin{Bmatrix}0\\0\\1\\1\end{Bmatrix}$$

$$\{F^e\} = \frac{q_4 l_{ni}}{2}\begin{Bmatrix}1\\0\\0\\1\end{Bmatrix}$$



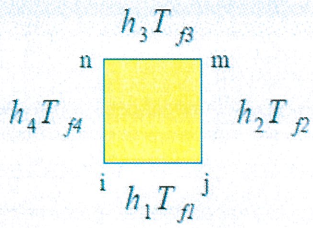
$$\{F^e\} = \frac{q_2 l_{jm}}{2}\begin{Bmatrix}0\\1\\1\\0\end{Bmatrix}$$

$$\{F^e\} = \frac{q_1 l_{ij}}{2}\begin{Bmatrix}1\\1\\0\\0\end{Bmatrix}$$

Thermal load due to convection:

$$\{F^e\} = \frac{h_3 T_{f3} L_{mn}}{2}\begin{Bmatrix}0\\0\\1\\1\end{Bmatrix}$$

$$\{F^e\} = \frac{h_4 T_{f4} L_{ni}}{2}\begin{Bmatrix}1\\0\\0\\1\end{Bmatrix}$$



$$\{F^e\} = \frac{h_2 T_{f2} L_{jm}}{2}\begin{Bmatrix}0\\1\\1\\0\end{Bmatrix}$$

$$\{F^e\} = \frac{h_1 T_{f1} L_{ij}}{2}\begin{Bmatrix}1\\1\\0\\0\end{Bmatrix}$$

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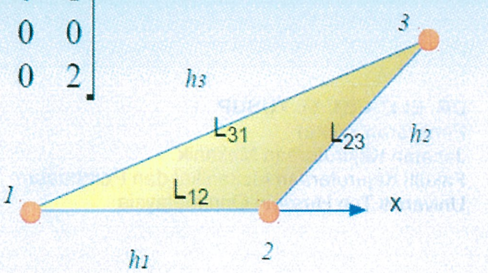
Linear triangular element:

$$[K^e]=\frac{k_X}{4A}\begin{bmatrix} y_{23}^2 & y_{31}y_{23} & y_{12}y_{23} \\ y_{23}y_{31} & y_{31}^2 & y_{12}y_{31} \\ y_{23}y_{12} & y_{31}y_{12} & y_{12}^2 \end{bmatrix}+\frac{k_Y}{4A}\begin{bmatrix} x_{32}^2 & x_{13}x_{32} & x_{21}x_{32} \\ x_{32}x_{13} & x_{13}^2 & x_{21}x_{13} \\ x_{32}x_{21} & x_{13}x_{21} & x_{21}^2 \end{bmatrix}$$

Conductance matrix due to convection:

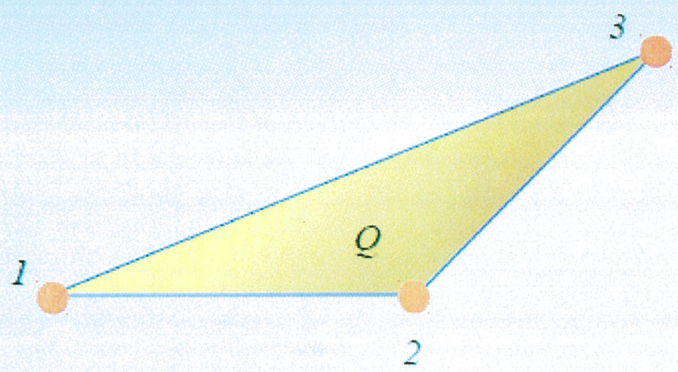
$$[K^e]=\frac{h_3L_{31}}{6}\begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$



$$[K^e]=\frac{h_2L_{23}}{6}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$[K^e]=\frac{h_1L_{12}}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thermal load due to heat source:



$$\{F^e\}=\frac{QA}{3}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

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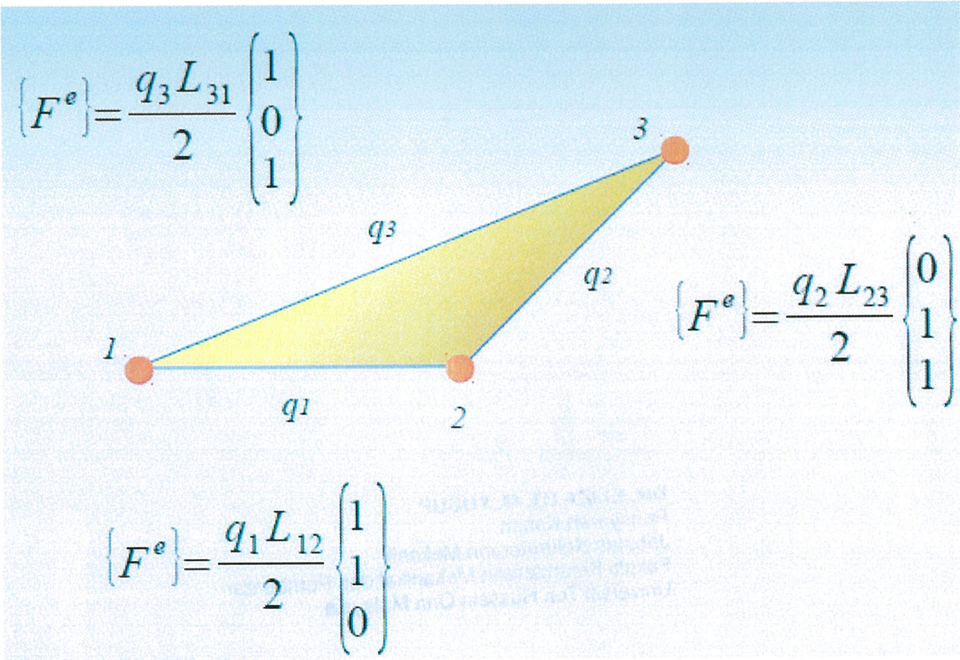
Thermal load due to heat flux:



Thermal load due to convection: