UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BDA 34003
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	PART A: ANSWER **FOUR** (4) QUESTIONS PART B: ANSWER **ONE** QUESTION ONLY

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

PART A: ANSWER ALL QUESTIONS

Q1 The heat transfer performance of a new conductor material is under inspection. The material is fully insulated such that the heat transfer is only one dimensional in axial direction (x-axis). The initial temperature of the material is at room temperature of 25°C. At one end (point A) is heated while another end (point E) is attached to a cooler system, as shown in **Figure Q1**.

The unsteady state heating equation follows a heat equation

$$\frac{\partial T}{\partial t} - K\frac{\partial^2 T}{\partial x^2} = 0,$$

where K is thermal diffusivity of the material and x is the longitudinal coordinate of the bar. The thermal diffusivity of the steel is given as $K = 13$ mm²/s, $\Delta x = h = 10mm$ and $\Delta t = k = 4s$.

(a) Draw finite difference grid to predict the temperature of all points up to 8 seconds and label all unknown temperatures in the grid.

(4 marks)

(b) From the grid illustration in **Q1(a)**, propose simultaneous equations based on Implicit Crank Nicolson method to determine the temperature of point A, B, C, D, E and F.

(8 marks)

(c) Determine the unknown temperatures of each point, at t = 4 second.

(8 marks)

Q2 (a) Given the nonlinear function

$$f(x) = -2 + 6x - 4x^2 + 0.5x^3$$

(i) Perform two iterations of Newton Raphson method to locate the root for $f(x)$ with different initial guess of

- $x_0 = 4$
- $x_0 = 4.43$

(5 marks)

(ii) From the obtained answers, how is the initial guess related to the significant difference in the answer?

(3 marks)

(b) A constant temperature, pressure-volume thermodynamic process has the following data:

Table 1

Pressure (kPa)	420	368	333	316	326	312	242	207
Volume (m^3)	0.5	2	3	4	6	8	10	11

Knowing that

$$Work = \int p\,dV$$

where p is pressure and V is volume.

(i) Analyze the work done using rectangular rule.

(4 marks)

(ii) If the recorded data is unequally spaced, propose a suitable combination of the rectangular rule, Simpon's 1/3 rule and Simpson's 3/8 rule that can be used to analyze the work done.

(4 marks)

(iii) If the volume $V = 1.88$ m^3, estimate the pressure at using three points Lagrange interpolation polynomial.

(4 marks)

Q3. Analyse the following system of linear equations to find the unknown variables using Thomas Algorithm.

$$\begin{aligned} x_1 + 2x_2 \quad\quad &= 9 \\ 6x_1 + 6x_2 - 8x_3 &= 1 \\ -3x_2 + x_3 &= 0 \end{aligned}$$

(20 marks)

Q4. Determine the largest eigenvalue and its corresponding eigenvector for the system matrix A, by using power method.

$$A = \begin{pmatrix} 2 & 2 & 10 \\ 8 & 3 & 4 \\ 10 & 4 & 5 \end{pmatrix}$$

Use the initial value $(1 \quad 0 \quad 1)^T$ and calculate until the convergence less than 0.005.

(20 marks)

PART B: ANSWER ONE QUESTION ONLY

Q5. The period of vibration, T of a spring-supported mass of m is recorded from an experimental series and shown in Table Q5. The experiment conducted to determine the stiffness of the spring.

Table Q5

m (kg)	0.5	1.0	1.5	2.0	2.5	3.0	3.5
T(s)	0.125	0.17	0.2	0.235	0.24	0.27	0.28

(a) Predict the value for $T(2.25)$ using Newton's Divided Difference.

(7 marks)

(b) Consider an ordinary differential equation

$$\frac{dy}{dx} - \frac{xy}{1+x^2} = 0$$

with initial condition $y(2) = 5$, $2 \le x \le 3$ and $h = 0.25$

(i) Solve the initial value problem using Euler method.

(10 marks)

(ii) Calculate the absolute error for each approximation if the exact solution is given by

$$y(x) = \sqrt{5(1+x^2)}$$

(3 marks)

Q6. Table Q6 gives corresponding values of two quantities x and y.

Table Q6

x	6	6.5	7	7.5	8	8.5	9
y	4.10	4.25	4.38	4.54	4.66	4.80	4.96

(a) Predict the value for $y(7.25)$ using Lagrange interpolating polynomial of fourth order.

(7 marks)

(b) Consider an initial value problem (IVP)

$$x\frac{dy}{dx} - y - \frac{x}{x+1} = 0$$

with initial condition $y(1) = 0$, $1 \le x \le 1.4$ and h = 0.1

(i) Solve the initial value problem using fourth order Runge-Kutta method.

(10 marks)

(ii) Calculate the absolute error for each approximation if the exact solution is given by

$$y(x) = x \ln\left(\frac{2x}{x+1}\right)$$

(3 marks)

END OF QUESTIONS

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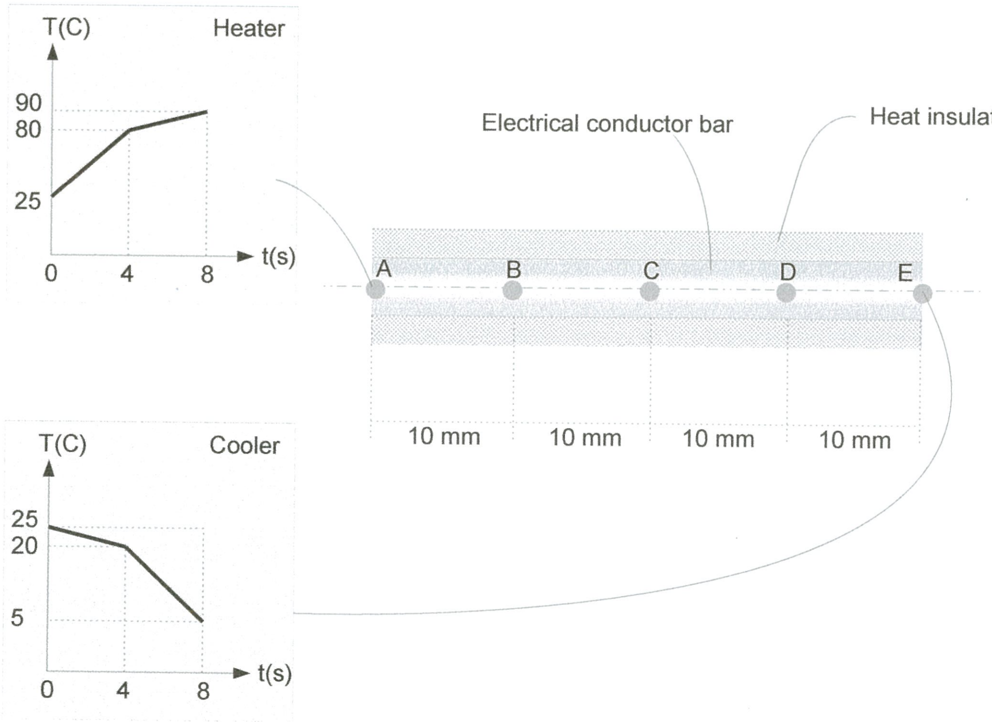

FIGURE Q1

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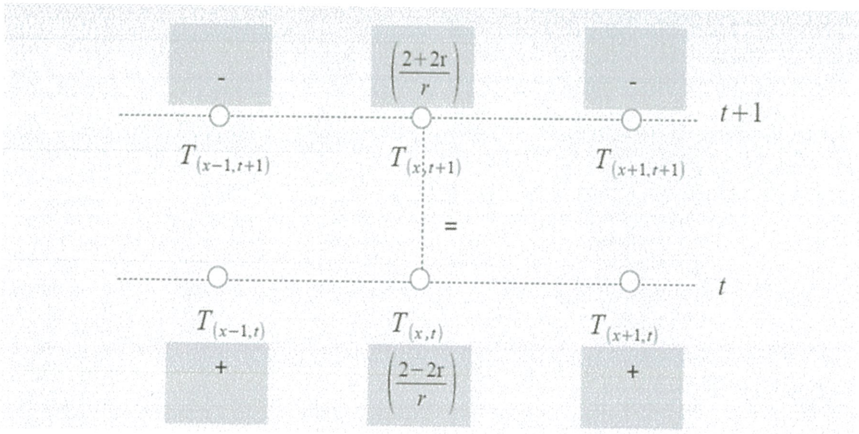
Useful Formulas

Implicit Crank Nicolson Method



Thomas Algorithm:

i	**1**	**2**	**…**	***n***
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

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Useful Formulas

Fourth-order Runge-Kutta method:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where $k_1 = hf(x_i, y_i)$, $k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$, $k_4 = hf(x_i + h, y_i + k_3)$