UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BDA 14103
PROGRAMME	:	BDD
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	**(PART A)** ANSWER **TWO (2)** QUESTIONS OUT OF TWO. **(PART B)** ANSWER **THREE (3)** QUESTIONS OUT OF FOUR .

THIS EXAMINATION PAPER CONTAINS **EIGHT (8)** PRINTED PAGES

PART A

Q1 (a) By using the method of **Variation Parameter**, solve the general solution for:

$$y'' + y = \tan(x)$$

(*Note*: $\tan(x) = \sin(x)/\cos(x)$ and $sin^2(x) + cos^2(x) = 1$)

(10 marks)

(b) By using the method of **Undetermined Coefficients**, obtain the general solution for:

$$y'' - 2y' - 3y = 1 - x^2$$

(10 marks)

Q2 (a) Express $\dfrac{3}{s(s^2+4)}$ in partial fraction.

(8 marks)

(b) Find the Laplace transform of $f(t) = \cos(2t)$.

(2 marks)

(c) By using the obtained result in Q2(b), show that

$$\mathcal{L}^{-1}\left\{\frac{3}{s(s^2+4)}e^{-4s}\right\} = \frac{3}{4}H(t-4) - \frac{3}{4}H(t-4)\cos(2(t-4)).$$

(3 marks)

(d) By using the obtained results in Q2 (a)-(c), solve the following initial value problem.

$$y'' + 4y = f(t), \quad y(0) = 1, \quad y'(0) = 0, \text{ with}$$

$$f(t) = \begin{cases} 0, & \text{for} \quad 0 \le t < 4 \\ 3, & \text{for} \quad t \ge 4 \end{cases}.$$

(7 marks)

PART B

Q3 (a) Solve the differential equation:

$$ax\frac{dy}{dx} - ay = \frac{ax}{x+1}$$

(6 marks)

(b) By using the method of Laplace transform, solve the initial value problem of:

$$2\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 4y = 2e^{-t}$$

with the initial condition $y(0) = y'(0) = 0$.

(14 marks)

Q4 (a) Solve the differential equation:

$$\frac{dy}{dx} = xe^{x^2 - \ln(y^2)}$$

(6 marks)

(b) Solve $\frac{dy}{dx} + y\tan x = \sec x$ if $y = 5$ when $x = 0$.

(8 marks)

(c) Find the general solution for the following equation:

$$y'' - 8y' + 16y = 0,$$

With the initial condition $y(0) = 2, y'(0) = 2$

(6 marks)

Q5 A 3 cm length silver bar with a constant cross section area 1 cm^2 (density 10 g/cm^3, thermal conductivity 1.5 cal/(cm sec°C), specific heat 0.075 cal/(g °C), is perfectly insulated laterally, with ends kept at temperature 0°C and initial uniform temperature $f(x) = 25$ °C.

The heat equation is:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$$

(a) Show that $c^2 = 2$.

(2 marks)

(b) By using the method of separation of variable, and applying the boundary condition, prove that

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{3} e^{-\frac{2n^2\pi^2 t}{9}}$$

where b_n is an arbitrary constant.

(12 marks)

(c) By applying the initial condition, find the value of b_n.

(6 marks)

Q6 A periodic function is defined as:

$$f(x)=\begin{cases}-1 & , \quad -\pi<x<0 \\ 0 & , \quad x=0 \\ 1 & , \quad 0<x<\pi\end{cases}$$

$$f(x)=f(x+2\pi)$$

(a) Determine whether the given function is even, odd or neither.

(2 marks)

(b) Prove that the corresponding Fourier series to this periodic function is given by:

$$\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin[(2n-1)x]}{2n-1}, \qquad -\pi<x<\pi$$

(12 marks)

(c) By choosing an appropriate value for x, show that

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$$

(6 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER / SESSION	: SEM II /20162017	PROGRAMME	: BDD
COURSE	: ENGINEERING MATHEMATICS II	COURSE CODE	: BDA14103

FORMULAS

First Order Differential Equation

Type of ODEs	General solution
Linear ODEs: $y' + P(x)y = Q(x)$	$y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx} Q(x)dx + C\right\}$
Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$	$F(x,y) = \int f(x,y)dx$ $F(x,y) - \int\left\{\frac{\partial F}{\partial y} - g(x,y)\right\}dy = C$

Characteristic Equation and General Solution for Second Order Differential Equation

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1e^{m_1x} + c_2e^{m_2x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1e^{mx} + c_2xe^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

Method of Undetermined Coefficient

$g(x)$	y_p
Polynomial: $P_n(x) = a_nx^n + \ldots + a_1x + a_0$	$x^r(A_nx^n + \ldots + A_1x + A_0)$
Exponential: e^{ax}	$x^r(Ae^{ax})$
Sine or Cosine: $\cos \beta x$ or $\sin \beta x$	$x^r(A\cos \beta x + B\sin \beta x)$

Note: r is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

Method of Variation of Parameters

The particular solution for $y''+by'+cy=g(x)$ (b and c constants) is given by $y(x)=u_1y_1+u_2y_2$, where

$$u_1=-\int\frac{y_2g(x)}{W}dx,$$

$$u_2=\int\frac{y_1g(x)}{W}dx,$$

Where

$$W=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

Laplace Transform

$$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}dt=F(s)$$

$f(t)$	$F(s)$
a	$\frac{a}{s}$
$t^n, n=1,2,3,...$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t), n=1,2,3,...$	$(-1)^n\frac{d^nF(s)}{ds^n}$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$\dot{y}(t)$	$sY(s)-y(0)$
$\ddot{y}(t)$	$s^2Y(s)-sy(0)-y'(0)$

Fourier Series

Fourier series expansion of periodic function with period 2 π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Half Range Series

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$