UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	ELECTROMECHANICAL AND CONTROL SYSTEM
COURSE CODE	:	BDU 20302
PROGRAMME	:	2 BDC/2 BDM
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FOUR (4)** QUESTIONS **ONLY**

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THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

Q1 (a) List the characteristics of the longitudinal short period and phugoid stability modes.

(5 marks)

(b) Consider an aircraft model in a wind tunnel setup where the aircraft is constrained at its center of gravity. The aircraft is free to perform a pitching motion about its center of gravity. The governing equation of this simple motion is obtained from the Newton's second law and is given as:

$$\Delta\ddot{\alpha} - (M_q + M_{\dot{\alpha}})\Delta\dot{\alpha} - M_\alpha \Delta\alpha = M_{\delta e}\Delta\delta_e$$

where $\Delta\alpha$ is the change in angle of attack (the change in angle of attack and pitch angles are identical), $\Delta\delta e$ is the change in elevator angle. M_q and M_α are longitudinal derivatives due to pitching velocity and angle of attack. Find the transfer function relating the change in angle of attack, $\Delta\alpha(s)$ and the change in elevator angle $\Delta\delta e(s)$. Use the Laplace transform theorem in **Figure Q1.**

(2 marks)

(c) Determine the solution, $\alpha(t)$ for the governing equation in (b) if a step input for elevator is applied to the dynamic system using the following assumptions:

$$M_q = -2.05\ s^{-1}$$
$$M_\alpha = -8.80\ s^{-2}$$
$$M_{\dot{\alpha}} = -0.95\ s^{-2}$$
$$M_{\delta e} = -5.5\ s^{-2}$$

Use partial fraction and inverse Laplace theorem to obtain the output response of the system, $\alpha(t)$ with the initial conditions of $\alpha(0) = 0$ and $\frac{d\alpha(0)}{dt} = 0$.

(14 marks)

(d) Analyze the effect of parameter $(M_q + M_{\dot{\alpha}})$ and M_a on the stability of short period mode.

(4 marks)

Q2 A simplified pitch control system is shown in **Figure Q2** with transfer functions for each individual component in the control system are given as:

$$K(s) = K_P + \frac{K_I}{s} + K_D s$$

$$G_1(s) = \frac{10}{s + 10}$$

$$G_2(s) = \frac{3}{s^2 + 3s + 4}$$

(a) Find the damped frequency, ω_d and gain, K, values at the imaginary axis crossing using the characteristic equation of the closed loop system.

(10 marks)

(b) Design the automatic controllers (i.e. P, PD and PID control) for the dynamic system under consideration using the ***Ziegler and Nichols*** tuning method.

(8 marks)

(c) Compare the steady-state error performance of the compensated systems (i.e. P, PD and PID control). Describe any problems with your design.

(7 marks)

Q3 (a) Describe the physical characteristics of Dutch Roll stability mode.

(3 marks)

(b) The Dutch Roll motion can be approximated using the following equation:

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_0} & -\left(1 - \frac{Y_r}{u_0}\right) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta r}}{u_0} \\ N_{\delta r} \end{bmatrix} \delta r$$

Assume the aircraft has the following stability derivative characteristics as follows:

$Y_\beta = -7.1$ ft/s^2 $\quad$ $Y_r = 2.1$ ft/s

$N_\beta = 2.9$ s^{-2} $\quad$ $N_r = -0.325$ s^{-1}

$Y_{\delta r} = -4.9$ ft/s^2 $\quad$ $N_{\delta r} = 0.615$ s^{-2}

$u_0 = 160$ ft/s

(i) Determine the characteristic equation of the Dutch Roll mode.

(4 marks)

(ii) Determine the eigenvalues of the Dutch Roll mode.

(2 marks)

(iii) Determine the damping ratio, natural frequency, period, time to half amplitude and number of cycles to half amplitude for the Dutch Roll mode.

(5 marks)

(c) Compare your Dutch Roll mode natural frequency and damping ratio calculation for this particular aircraft to the handling quality criteria in **Table Q3.** Assume that the aircraft under consideration is a **Class IV aircraft** (High Maneuvering Aircraft) performing **CAT A mission** (Precision Tracking). Determine the minimum state feedback gain so that the damping ratio achieved **Level 1** handling qualities. Use feedback control design based on **rudder deflection**, δr, proportional only to the **yaw rate state**, Δr, i.e.:

$$\delta r = -K^T x = -[0 \quad K] \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix}$$

(11 marks)

Q4 The open loop pitch rate response to elevator transfer function for the Lockheed F-104 Starfighter is given by the following transfer function:

$$\frac{q(s)}{\delta_e(s)} = \frac{-4.66s(s+0.133)(s+0.269)}{(s^2+0.015s+0.021)(s^2+0.911s+4.884)}$$

(a) Explain how the root locus plot can be used to evaluate the effect of feedback on the characteristics modes of motion?

(4 marks)

(b) Determine the damping ratio and undamped natural frequency for short period and phugoid mode.

(4 marks)

(c) Design a pitch rate feedback controller, K_q to bring the closed loop short period mode in agreement with minimum specification for damping ratio and natural frequency. The root locus plot of the transfer function is given in **Figure Q4**. Assume the following **Level 1** flying qualities are used in the analysis:

Phugoid damping ratio $\zeta_p \geq 0.04$
Short period damping ratio $\zeta_s \geq 0.5$
Short period undamped natural frequency $0.8 \leq \omega_s \leq 3.0$ rad/s

(11 marks)

(d) Compare the augmented short period damping ratio and natural frequency with those of the unaugmented aircraft. How does pitch rate feedback to elevator input improve the longitudinal flying qualities? Explain your answer based from your findings and the given root locus plot.

(6 marks)

Q5 A unity feedback system with forward transfer function:

$$G(s) = \frac{K}{s(s+4)(s+6)}$$

(a) Determine the settling time of the uncompensated system and the compensator gain when the system operate at 16% overshoot.

(5 marks)

(b) Determine the steady state error value for a unit step input.

(2 marks)

(c) Design an **ideal derivative (PD) compensator** to yield a 16% overshoot with a threefold reduction in settling time. Summaries the performance characteristics (i.e. dominant poles, compensator gain, damping ratio, natural frequency, settling time, peak time etc.) for the uncompensated and compensated system in a table.

(11 marks)

(d) Analyze the effect of the location of the compensator zero on the system settling time if the location of the compensator zero is placed further left than the location obtained in **Q5(c).** Assume that the compensated system operates at 16% overshoot. Provide a root locus sketch to support your claim.

(7 marks)

-END OF QUESTION-

FINAL EXAMINATION

SEMESTER/SESSION	:	*SEM 2 / 2016/2017*	*PROGRAMME*	:	*2 BDC/2 BDM*
COURSE	:	*ELECTROMECHANICAL AND CONTROL SYSTEM*	*COURSE CODE*	:	*BDU20302*

Laplace transforms – Table

$f(t)=L^{-1}\{F(s)\}$	$F(s)$	$f(t)=L^{-1}\{F(s)\}$	$F(s)$
$a \quad t\ge 0$	$\frac{a}{s} \quad s>0$	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$at \quad t\ge 0$	$\frac{a}{s^2}$	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t+\theta)$	$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t+\theta)$	$\frac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$
$\frac{1}{2}t^2e^{-at}$	$\frac{1}{(s+a)^3}$	$t\sin\omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s>a$	$\sinh\omega t$	$\frac{\omega}{s^2-\omega^2} \quad s>\lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh\omega t$	$\frac{s}{s^2-\omega^2} \quad s>\lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at}-e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$\frac{1}{a^2}[1-e^{-at}(1+at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n=1,2,3..$	$e^{at}\sin\omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
t^ne^{at}	$\frac{n!}{(s-a)^{n+1}} \quad s>a$	$e^{at}\cos\omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}} \quad s>a$	$1-e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at-1+e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s>0$	$f(t-t_1)$	$e^{-t_1 s}F(s)$
$g(t)\cdot p(t)$	$G(s)\cdot P(s)$	$f_1(t)\pm f_2(t)$	$F_1(s)\pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s}+\frac{f^{-1}(0)}{s}$	$\delta(t)$ *unit impulse*	$1 \quad$ *all s*
$\frac{df}{dt}$	$sF(s)-f(0)$	$\frac{d^2f}{df^2}$	$s^2F(s)-sf(0)-f'(0)$
$\frac{d^nf}{dt^n}$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)-s^{n-3}f''(0)-\cdots-f^{n-1}(0)$		

Figure Q1 The Laplace transform theorems.

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FINAL EXAMINATION

SEMESTER/SESSION	:	*SEM 2 / 2016/2017*	*PROGRAMME*	:	*2 BDC/2 BDM*
COURSE	:	*ELECTROMECHANICAL AND CONTROL SYSTEM*	*COURSE CODE*	:	*BDU20302*

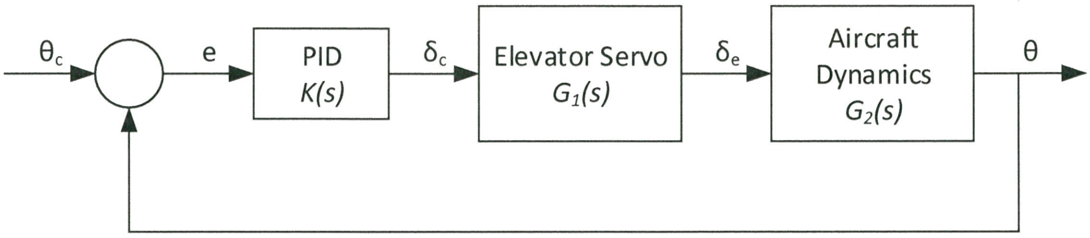

Figure Q5 Simplified block diagram for pitch angle control system.

Table Q3 Minimum Dutch-Roll frequency and damping

Aircraft Class	**Flight Phase**	**Minimum Values**							
		Level 1			**Level 2**			**Level 3**	
		ζ	$\zeta\omega_n$	ω_n	ζ	$\zeta\omega_n$	ω_n	ζ	ω_n
I, IV	CAT A	0.19	0.35	1.0	0.02	0.05	0.5	0	0.4
II, III	CAT A	0.19	0.35	0.5	0.02	0.05	0.5	0	0.4
All	CAT B	0.08	0.15	0.5	0.02	0.05	0.5	0	0.4
I, IV	CAT C	0.08	0.15	1.0	0.02	0.05	0.5	0	0.4
II, III	CAT C	0.08	0.10	0.5	0.02	0.05	0.5	0	0.4

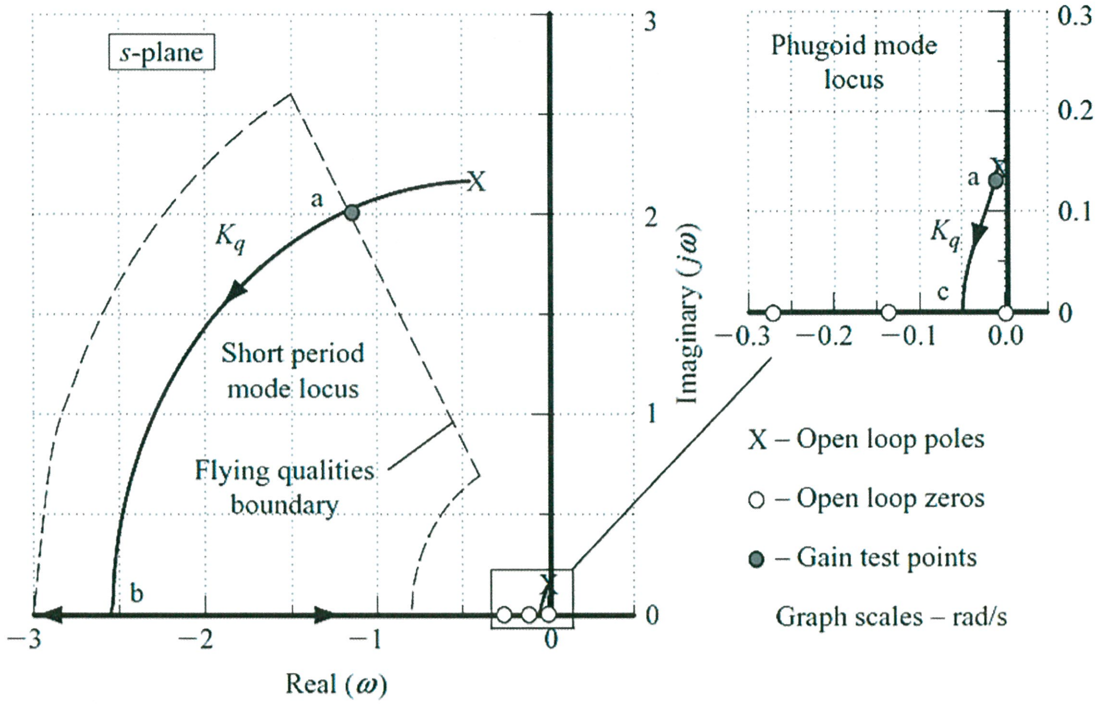

Figure Q4 Root locus plot showing pitch rate feedback to elevator.

FINAL EXAMINATION

SEMESTER/SESSION	: SEM 2 / 2016/2017	PROGRAMME	: 2 BDC/BDM
COURSE	: ELECTROMECHANICAL AND CONTROL SYSTEM	COURSE CODE	: BDU20302

A Key Equations

The relevant equations used in this examination are given as follows:

1. Determinant of 3 × 3 matrix:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} \quad (1)$$

2. Partial fraction for $F(s)$ with real and distinct roots in denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2}{(s+p_2)} + \cdots + \frac{K_m}{(s+p_m)} \quad (2)$$

3. Partial fraction for $F(s)$ with complex or imaginary roots in denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2 s + K_3}{(s^2+as+b)} + \cdots \quad (3)$$

4. General first order transfer function:

$$G(s) = \frac{a}{s+a} \quad (4)$$

5. General second order transfer function:

$$G(s) = \frac{K}{s^2 + 2\xi\omega_n s + \omega_n} \quad (5)$$

6. The closed loop transfer function:

$$T(s) = \frac{G(s)}{1+G(s)H(s)} \quad (6)$$

where $G(s)$ is the transfer function of the open loop system and $H(s)$ is the transfer function in the feedback loop.

7. The final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \quad (7)$$

8. Time response:

Rise time (first order system): $$T_r = \frac{2.2}{a} \quad (8)$$

Settling time (first order system): $$T_s = \frac{4}{a} \quad (9)$$

Percentage overshoot (second order system): $$\%OS = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\% \quad (10)$$

Damping ratio (second order system): $$\xi = \frac{-\ln\left(\%\frac{OS}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\%\frac{OS}{100}\right)\right)^2}} \quad (11)$$

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM 2 / 2016/2017	PROGRAMME	:	2 BDC/BDM
COURSE	:	ELECTROMECHANICAL AND CONTROL SYSTEM	COURSE CODE	:	BDU20302

Peak time (second order system): $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = \dfrac{\pi}{\omega_d}$ (12)

Settling time (second order system): $T_s = \dfrac{4}{\xi\omega_n} = \dfrac{4}{\sigma}$ (13)

Period (second order system): $P = \dfrac{2\pi}{\omega_d}$ (14)

Time to half amplitude (second order system): $t_{1/2} = \dfrac{0.693}{|\eta|}$ (15)

Number of cycles for halving the amplitude (second order system): $N_{1/2} = 0.110\dfrac{|\omega_d|}{|\sigma|}$ (16)

9. Estimation of parameter q (Paynter Numerical Method)

$$q = \max|A_{ij}\Delta t| \quad (17)$$

10. Estimation of integer value of p (Paynter Numerical Method)

$$\frac{1}{p!}(nq)^p e^{nq} \leq 0.001 \quad (18)$$

11. Numerical solution of state equation (Paynter Numerical Method):

$$\mathbf{x}_{k+1} = \boldsymbol{M}\mathbf{x}_k + \boldsymbol{N}\boldsymbol{\eta}_k$$

with matrix $\boldsymbol{M}$ and $\boldsymbol{N}$ are given by the following matrix expansion:

$$\boldsymbol{M} = e^{\boldsymbol{A}\Delta t} = \boldsymbol{I} + \boldsymbol{A}\Delta t + \frac{1}{2!}\boldsymbol{A}^2\Delta t^2 \ldots \quad (19)$$

$$\boldsymbol{N} = \Delta t\left(\boldsymbol{I} + \frac{1}{2!}\boldsymbol{A}\Delta t + \frac{1}{3!}\boldsymbol{A}^2\Delta t^2 + \cdots\right)\boldsymbol{B}$$

12. Finding eigenvalues from state space model:

$$|\lambda\boldsymbol{I} - \boldsymbol{A}| = 0 \quad (20)$$

13. Characteristic equation of the closed loop system:

$$1 + KG(s)H(s) = 0 \quad (21)$$

14. Asymptotes: angle and real-axis intercept:

$$\sigma = \frac{[\sum \text{Real parts of the poles} - \sum \text{Real parts of the zeros}]}{n-m} \quad (22)$$

$$\phi_a = \frac{180°[2q+1]}{n-m} \quad (23)$$

15. Solution to find real axis break-in and breakaway points:

$$\frac{dK(\sigma)}{d\sigma} = 0 \quad (24)$$

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM 2 / 2016/2017	PROGRAMME	:	2 BDC/BDM
COURSE	:	ELECTROMECHANICAL AND CONTROL SYSTEM	COURSE CODE	:	BDU20302

16. Alternative solution to find real axis break-in and breakaway points:

$$\sum \frac{1}{\sigma + z_i} = \sum \frac{1}{\sigma + p_i} \quad (25)$$

17. Angle of departure of the root locus from a pole of $G(s)H(s)$:

$$\theta = 180° + \sum(angles\ to\ zeros) - \sum(angles\ to\ poles) \quad (26)$$

18. Angle of arrival at a zeros:

$$\theta = 180° - \sum(angles\ to\ zeros) + \sum(angles\ to\ poles) \quad (27)$$

19. The steady state error:

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) \quad (28)$$

where the error signal is given as:

$$E(s) = \frac{1}{1 + K(s)G(s)H(s)} \times U(s) \quad (29)$$

20. Linear system (SISO) with state feedback:

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} - \boldsymbol{BK^T})\boldsymbol{x} + \boldsymbol{B}u$$

$$\text{or, } \dot{\boldsymbol{x}} = \boldsymbol{A_{new}}\boldsymbol{x} + \boldsymbol{B}u \quad (30)$$

where $\boldsymbol{A_{new}}$ is the augmented matrix and $u = \boldsymbol{K^T x} + \delta_{ref}$

21. The characteristic equation for the standard form of the second-order differential equation:

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0$$

The roots of the characteristic equation are: (31)

$$\lambda_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{1-\xi^2} \cdot i$$

$$\lambda_{1,2} = \sigma \pm \omega_d$$

22. The controller gains calculation using Ziegler-Nichols method: (32)

Table 1 The Ziegler-Nichols tuning method.

Control Type	K_p	K_I	K_D
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2\,K_p/T_u$	-
PD	$0.8K_u$	-	$K_PT_u/8$
Classic PID	$0.6K_u$	$2\,K_p/T_u$	$K_PT_u/8$
Pessen Integral Rule	$0.7K_u$	$2.5\,K_p/T_u$	$3K_PT_u/20$
Some Overshoot	$0.33K_u$	$2\,K_p/T_u$	$K_PT_u/3$
No Overshoot	$0.2K_u$	$2\,K_p/T_u$	$K_PT_u/3$