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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

TERBUKA

COURSE NAME	:	FINITE ELEMENT METHOD
COURSE CODE	:	BDA 31003/BDA 40303
PROGRAMME	:	BDD
EXAMINATION DATE	:	DECEMBER 2016/ JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTIONS	:	ANSWER **FOUR (4)** QUESTIONS **ONLY** FROM OVERALL FIVE (5) QUESTIONS

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

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Q1 A tinted glass with a thickness of 8 mm has a thermal conductivity of k = 0.7 W/m °C. The outside surface is exposed to free air (with convection coefficient h = 115 W/m^2 °C) with temperature of 4 °C. The inner surface is exposed to 27 °C with convection coefficient of moving the internal air is 45 W/ m^2 °C. The area (A) is one unit area.

(a) Determine the temperature distribution of the glass using 2 equally length element.

(20 marks)

(b) Discuss the effect to the temperature distribution if a heat source Q is introduced to the glass element. Explain with appropriate equation for the thermal load.

(5 marks)

Q2 Oil with dynamic viscosity of μ = 0.3 Ns/m^2 and density of ρ = 900 kg/m^3 flows through the piping network shown in **Figure Q2**. The 2-4-5 branch was added in parallel to the 2-3-5 branch to allow for the flexibility of performing maintenance on one branch while the oil flows through the other branch. The dimensions of the piping system are shown in **Figure Q2**.

(a) Determine the pressure distribution in the system if both branches are on line. The flow rate at node 1 is 5 X 10^{-4} m^3/s. The pressure at node 1 and 3 are 39,182 Pa (g) and 29,366 Pa (g), respectively, and the pressure at node 6 is -3,665 Pa (g). For the given conditions, the flow is laminar throughout the system.

(17 marks)

(b) How does the flow divide in each branch?

(4 marks)

(c) How to verify your finding based on the values of flow rate for each piping line.

(4 marks)

Q3 **Figure Q3** shows a small rectangular element of a thin plate. The dimensions and the boundary conditions are such that its stress distribution cannot accurately be approximated by one-dimensional function. The stress varies in both the *x* and *y* directions.

(a) Show that the stress distribution over the two-dimensional rectangular elements can be expressed as

$$\sigma^{(e)} = \begin{bmatrix} S_i & S_j & S_m & S_n \end{bmatrix} \begin{Bmatrix} \sigma_i \\ \sigma_j \\ \sigma_m \\ \sigma_n \end{Bmatrix}$$

where

$$S_i = \left(1-\frac{x}{l}\right)\left(1-\frac{y}{w}\right), \quad S_j = \frac{x}{l}\left(1-\frac{y}{w}\right), \quad S_m = \frac{xy}{lw}, \quad S_n = \frac{y}{w}\left(1-\frac{x}{l}\right)$$

(15 marks)

(b) If σ_i = 2500 N/m^2, σ_j = 2000 N/m^2, σ_m = 1500 N/m^2, and σ_n = 3000 N/m^2, determine the value of stress at $x = \frac{1}{4}l$ and $y = \frac{1}{2}w$ of this element.

(6 marks)

(c) Suggest **TWO (2)** alternative approaches that could be taken to obtain more accurate stress value. Explain your suggestions.

(4 marks)

Q4 A long bar of rectangular cross section, having thermal conductivity of 1.5 W/m°C, is subjected to the boundary conditions shown in **Figure Q4**. Two opposite sides are maintained at a uniform temperature of 180°C; one side is insulated, and the remaining side is subjected to a convection process with $T_\infty = 25\,^{\circ}C$ and $h = 50 W/m^2$. Evaluate the temperature distribution in the bar by using rectangular element with the size of 0.4 x 0.15 m for all elements.

(25 Marks)

Q5 A thin plate with thickness 0.013 m was loaded as shown in **Figure Q5**. The material's modulus of elasticity, E is 200 GPa and Poisson ratio, υ of 0.3.

(a) Determine whether this is a plane stress or plane strain problem and generate the stiffness matrix, [K] from elasticity matrix, [E] and the strain displacement matrix, [B].

(10 marks)

(b) Construct the global stiffness matrix and the global force vector before considering any constraints and present it in the form of [K] {x} = [f}.

(10 marks)

(c) Solve for the displacement vector {x}

(5 marks)

-END OF QUESTIONS-

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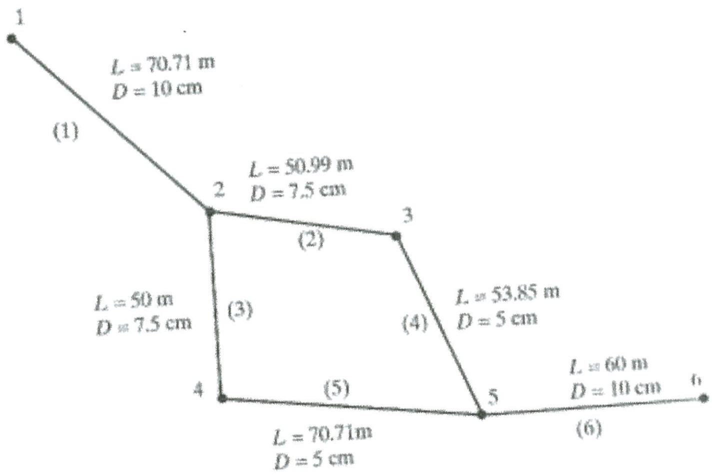

Figure Q2

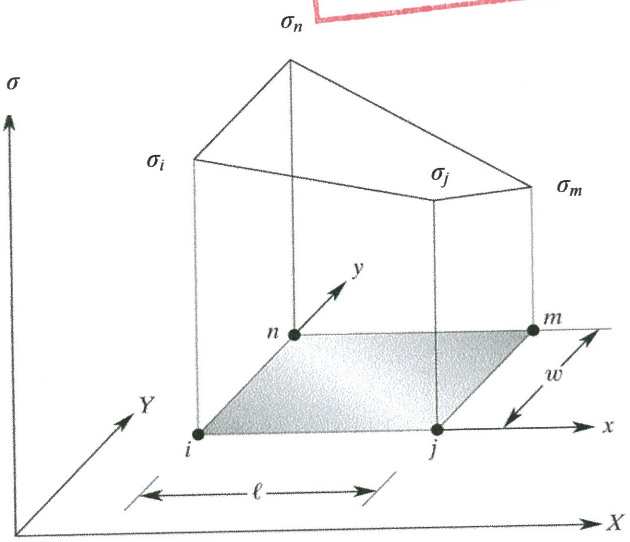

Figure Q3

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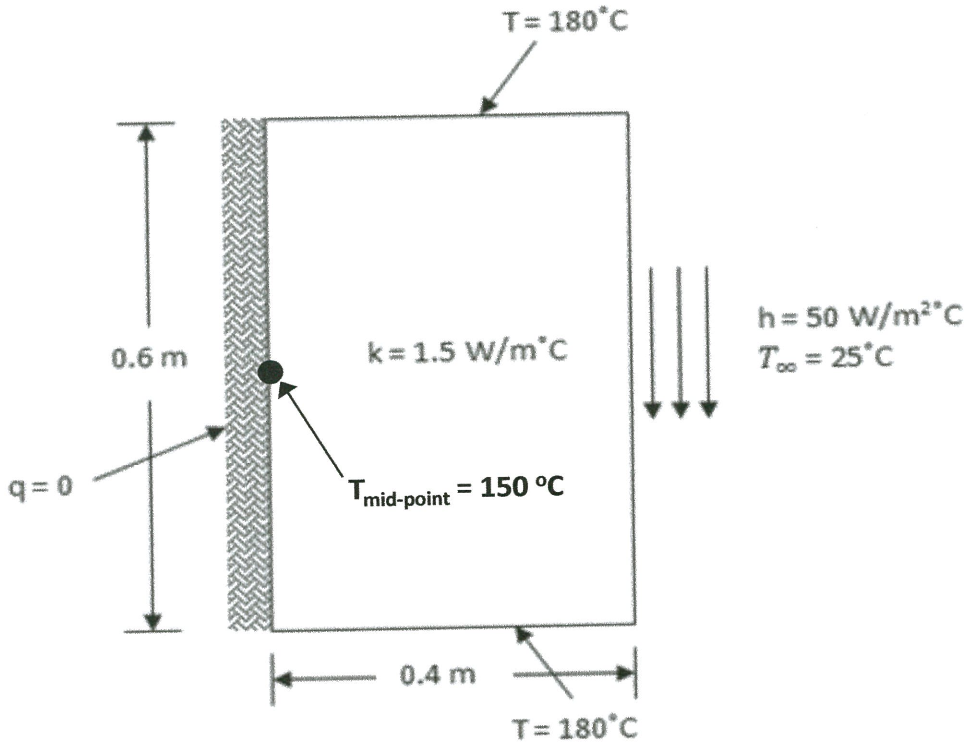

Figure Q4

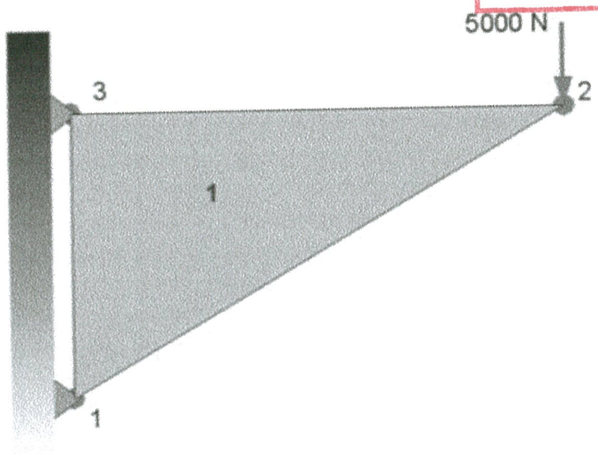

Table 1: Node data

Node	x(m)	y(m)
1	0	0
2	0.13	0.05
3	0	0.05

Figure Q5

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USEFUL FORMULA

FLUID FLOW NETWORK PROBLEM

Element's flow-resistance matrix

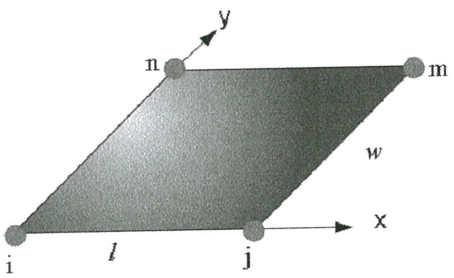
$$[\mathbf{R}]^{(e)} = \begin{bmatrix} \dfrac{\pi D^4}{128L\mu} & -\dfrac{\pi D^4}{128L\mu} \\ -\dfrac{\pi D^4}{128L\mu} & \dfrac{\pi D^4}{128L\mu} \end{bmatrix}$$

Flow rates

$$Q_i = C(P_i - P_{i+1})$$
$$Q_{i+1} = C(P_{i+1} - P_i)$$

BILINEAR RECTANGULAR HEAT TRANSFER:

Conductance matrix due to conduction



$$[K^e] = \frac{k_x w}{6l}\begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{k_y l}{6w}\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

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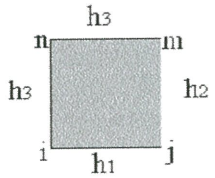
Conductance matrix due to convection

$$[K^e]=\frac{h_3 L_{mn}}{6}\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&2&1\\0&0&1&2\end{bmatrix}$$

$$[K^e]=\frac{h_4 L_{ni}}{6}\begin{bmatrix}2&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&2\end{bmatrix}$$



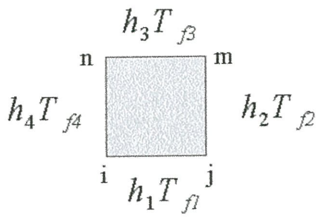
$$[K^e]=\frac{h_2 L_{jm}}{6}\begin{bmatrix}0&0&0&0\\0&2&1&0\\0&1&2&0\\0&0&0&0\end{bmatrix}$$

$$[K^e]=\frac{h_1 L_{ij}}{6}\begin{bmatrix}2&1&0&0\\1&2&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

Thermal load due to heat loss by convection along the edges

$$\{F^e\}=\frac{h_3 T_{f3} L_{mn}}{2}\begin{Bmatrix}0\\0\\1\\1\end{Bmatrix}$$

$$\{F^e\}=\frac{h_4 T_{f4} L_{ni}}{2}\begin{Bmatrix}1\\0\\0\\1\end{Bmatrix}$$



$$\{F^e\}=\frac{h_2 T_{f2} L_{jm}}{2}\begin{Bmatrix}0\\1\\1\\0\end{Bmatrix}$$

$$\{F^e\}=\frac{h_1 T_{f1} L_{ij}}{2}\begin{Bmatrix}1\\1\\0\\0\end{Bmatrix}$$

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LINEAR TRIANGULAR HEAT TRANSFER ELEMENT:

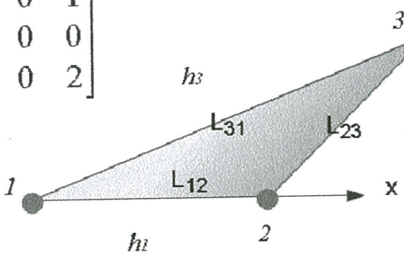
Conductance matrix due to conduction

$$[K^e]=\frac{k_X}{4A}\begin{bmatrix} y_{23}^2 & y_{31}y_{23} & y_{12}y_{23} \\ y_{23}y_{31} & y_{31}^2 & y_{12}y_{31} \\ y_{23}y_{12} & y_{31}y_{12} & y_{12}^2 \end{bmatrix}+\frac{k_Y}{4A}\begin{bmatrix} x_{32}^2 & x_{13}x_{32} & x_{21}x_{32} \\ x_{32}x_{13} & x_{13}^2 & x_{21}x_{13} \\ x_{32}x_{21} & x_{13}x_{21} & x_{21}^2 \end{bmatrix}$$

$$y_{ij}=y_i-y_j \qquad x_{ij}=x_i-x_j$$

Conductance matrix due to convection

$$[K^e]=\frac{h_3L_{31}}{6}\begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$



$$[K^e]=\frac{h_2L_{23}}{6}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$[K^e]=\frac{h_1L_{12}}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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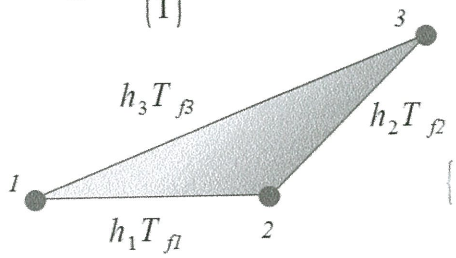
Thermal load due to heat loss by convection along the edges

$$\{F^e\}=\frac{h_3 T_{f3} L_{31}}{2}\begin{Bmatrix}1\\0\\1\end{Bmatrix}$$



$$\{F^e\}=\frac{h_2 T_{f2} L_{23}}{2}\begin{Bmatrix}0\\1\\1\end{Bmatrix}$$

$$\{F^e\}=\frac{h_1 T_{f1} L_{12}}{2}\begin{Bmatrix}1\\1\\0\end{Bmatrix}$$

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CST ELEMENT

$$[K^e]= t\ A^e\ [B^e]^T [E^e][B^e]$$

$$[B]=\frac{1}{2A}\begin{bmatrix}y_{23} & 0 & y_{31} & 0 & y_{12} & 0\\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21}\\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12}\end{bmatrix}$$

$$A=\frac{1}{2}\begin{vmatrix}1 & x_1 & y_1\\ 1 & x_2 & y_2\\ 1 & x_3 & y_3\end{vmatrix}$$

$$x_{ij}=x_i-x_j \qquad y_{ij}=y_i-y_j$$

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Plane Strain:

$$[E]=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

Plane Stress:

$$[E]=\frac{E}{(1-\nu^2)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$