UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

TERBUKA

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS III
COURSE CODE	:	BDU 21103
PROGRAMME CODE	:	BDC / BDM
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **5 QUESTIONS** ONLY 2. ALL CALCULATIONS MUST BE IN **3 DECIMAL** PLACES

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) The total surface area, S of a cone of base radius, r and perpendicular height, h is given by

$$S = \pi r^2 + \pi r\sqrt{r^2 + h^2}.$$

If r and h are each increasing at the rate of 0.25 cm s^{-1}, find the rate at which S is increasing at the instant when $r = 3$ cm and $h = 4$ cm.

(6 marks)

(b) Given

$$z = x^2(1 + y)^3.$$

Use total differential to approximate

$$(2.03)^2(1 + 8.9)^3 - 2^2(1 + 9)^3.$$

(6 marks)

(c) Evaluate

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (4 - y)\, dy\, dx$$

by changing to polar coordinate.

(8 marks)

Q2 (a) Let $f(x) = x^3 - 10x + 10$.

(i) Verify that $f(x)$ has a root on the interval $(1,2)$.

(2 marks)

(ii) Find the root of $f(x)$ by using secant method. Iterate until $|f(x_i)| < \varepsilon = 0.005$.

(4 marks)

(iii) Given that the exact value of the root is $x = 1.153$. Compute the absolute error in the approximation in **Q2**(a)(ii).

(2 marks)

(b) Solve the system of linear equations by using Thomas algorithm method.

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 9 & 1 & 0 \\ 0 & 1 & 9 & 4 \\ 0 & 0 & 4 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 2 \\ 8 \end{pmatrix}.$$

(12 marks)

Q3 (a) Find the volume of paraboloid $z = x^2 + y^2$ bounded by the plane $z = 4$ in first octant.

(10 marks)

(b) A cube is defined by three inequalities $0 \le x \le 1$, $0 \le y \le 1$ and $0 \le z \le 1$. The cube has a density function $\delta(x, y, z) = k(x^2 + y^2 + z^2)$. Given that the mass of the cube is k. Find its center of gravity.

(10 marks)

Q4 (a) The monthly payment on a 30 year mortgage of RM 100,000, for four different annual interest rates is given in Table 1. Use Lagrange interpolation to estimate the monthly payment corresponding to an interest rate of 8.25% per year.

Table 1: Monthly payment on a 30 year mortgage of RM 100,000

Data Point Number k	Annual Interest Rate	Monthly Payment
0	7%	RM 665.30
1	10%	RM 877.57
2	8%	RM 733.76
3	9%	RM 804.62

(10 marks)

(b) Given a matrix

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 4 \end{bmatrix}.$$

(i) Find the inverse of matrix A by calculator.

(1 mark)

(ii) Find the smallest (in absolute) eigenvalue and its corresponding eigenvector by inverse power method.

Let $v^{(0)} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T$ and $\varepsilon = 0.005$.

(9 marks)

Q5 (a) A violin string of 1m unit length is stretched out horizontally with both ends fixed which satisfies the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0,$$

with the initial conditions, $u(x,0) = \sin(4\pi x)$, $u_t(x,0) = 0$ for $0 \le x \le 1$. By taking $h = \Delta x = 0.2$ and $k = \Delta t = 0.1$, find the displacement of the violin string up to level 2 only by using the explicit finite difference method.

(10 marks)

(b) Given that $y' + 2y = xe^{3x}$ with initial condition $y(0) = 0$.

(i) Approximate the solution for $x = 0(0.2)1$ by using classical fourth-order Runge-Kutta (RK4) method.

(8 marks)

(iii) The exact solution for **Q5(b)** is given by

$$y(x) = \frac{1}{5}xe^{3x} - \frac{1}{25}e^{3x} + \frac{1}{25}e^{-2x}.$$

Hence, find its error.

(2 marks)

Q6 (a) The upward velocity of a rocket, measured at 3 different times, is shown in the table below.

Table 2: Upward velocity of a rocket

Time , t (seconds)	Velocity, v (meters/second)
5	106.8
8	177.2
12	279.2

The velocity over the time interval $5 \le t \le 12$ is approximated by a quadratic expression as

$$v(t) = a_1 t^2 + a_2 t + a_3 .$$

Find the values of a_1, a_2 and a_3 by using Gauss – Elimination method.

(10 marks)

(b) Torque-speed data for an electric motor is given table below:

Table 3: Torque-speed for an electric motor

Speed ω (rpm $\times$ 1000)	0.5	1.0	1.5	2.0	2.5
Torque, T_i (ft-lb)	31	28	24	14	2

(i) Find the equation of the Newton divided–difference interpolating polynomial that passes through each data point.

(8 marks)

(ii) Use the obtained interpolating polynomial to estimate the torque at 1800 rpm.

(2 marks)

-END OF QUESTIONS –

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Formulas

Partial differential equations

Wave equation: Finite-difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \Leftrightarrow \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$

System of linear equations

Thomas Algorithm:

i	**1**	**2**	**...**	n
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \frac{e_i}{\alpha_i}$				
$y_1 = \frac{b_1}{\alpha_1}$ $y_i = \frac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

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Eigenvalue

Power Method: $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \qquad k = 0,1,2,.....$

Inverse Power Method: $\lambda_{\text{smallest}} = \dfrac{1}{\lambda_{\text{Shifted}}}$

Cylindrical coordinates:

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z, \qquad x^2 + y^2 = r^2, \quad 0 \le \theta \le 2\pi$$

$$V = \iiint_G dV = \iiint_G dz\, r\, dr\, d\theta$$

Fourth-order Runge-Kutta Method

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = hf(x_i, y_i),$$

$$k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right),$$

$$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right),$$

$$k_4 = hf(x_i + h, y_i + k_3).$$

Mass

$$m = \iiint_G \delta(x,y,z)\, dV$$

Center of Gravity $(\bar{x}, \bar{y}, \bar{z})$

$$\bar{x} = \frac{1}{m}\iiint_G x\,\delta(x,y,z)\, dV, \ \bar{y} = \frac{1}{m}\iiint_G y\,\delta(x,y,z)\, dV, \ \bar{z} = \frac{1}{m}\iiint_G z\,\delta(x,y,z)\, dV.$$