UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS I
COURSE CODE	:	BDU 10903
PROGRAMME CODE	:	BDC / BDM
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) Find the real and imaginary parts of the imaginary number $z+\frac{1}{z}$ for $z=\frac{4+i}{1-i}$.

(6 marks)

(b) Given the transformation equation, $w=(4-5i)z+3-5i$. Calculate

(i) the magnification involved.
(ii) the rotation involved.
(iii) the translation involved.

(6 marks)

(c) Find the image in the w plane of the straight line $y=4-3x$ in the z plane $z=x+iy$ under mapping $w=2z+1$.

(8 marks)

Q2 (a) Given a helix, $\mathbf{r}(t)=3(\cos t)\mathbf{i}+3(\sin t)\mathbf{j}+t\mathbf{k}$, $0\le t\le 4\pi$.

(i) Sketch the helix.
(ii) Find the arc length for $\mathbf{r}(t)$.

(11 marks)

(b) A particle moves in space so that at time t, its position can be written as vector $\mathbf{r}(t)=(3-2t)\mathbf{i}+(t^2+4t)\mathbf{j}+(t^3+3t^2)\mathbf{k}$.

(i) Find the velocity and acceleration of the particle when $t=2$.
(ii) Determine the unit tangent vector of $\mathbf{r}(t)$ at point $(-1,-4,20)$.

(9 marks)

Q3 (a) Show that the Maclaurin series for $f(x)=\sin x$ is

$$x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$$

Then,

(i) evaluate $\int_0^1 \sin x^3 dx$.

(ii) find the first six terms of a series for $\cos x$ and also $2\cos x\sin x$.

(12 marks)

(b) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n}}$$

is absolutely convergent, conditionally convergent or divergent.

(8 marks)

Q4 (a) Integrate the following expression with respect to x.

(i) $\dfrac{x+4}{(x-2)^2}$.

(6 marks)

(ii) $\sec^2 x \tan^5 x$.

(4 marks)

(b) Evaluate $\displaystyle\int \frac{2}{1+\cos x}\,dx$ by using $t = \tan\frac{x}{2}$ substitution.

(5 marks)

(c) Solve $\displaystyle\int \frac{3}{\sqrt{x^2-9}}\,dx$ by using trigonometric substitution.

(5 marks)

Q5 (a) Find $\dfrac{dy}{dx}$ if $y = x^{-2}\sin^2(x^3)$.

(5 marks)

(b) Determine the slope of the curve,

$$x = \frac{t^3}{1+t^2} \quad \text{and} \quad y = \frac{4t+3}{t}$$

when $t = -1$.

(6 marks)

(c) Let

$$f(x) = x^3 - 3x + 1.$$

Find all critical numbers. Then use the *Second Derivative Test* to determine the properties of all the local extreme points.

(9 marks)

Q6 (a) Evaluate the limits below.

(i) $\lim_{x\to 7} \dfrac{\frac{1}{x}-\frac{1}{7}}{x-7}$.

(ii) $\lim_{x\to +\infty} x^3 + \sqrt{x^6 - 7x^3}$.

(iii) $\lim_{x\to 0^+} \dfrac{\tan 4x}{x}$.

(13 marks)

(b) Determine whether or not the following function is continuous or not at $x = -2$.

$$f(x) = \begin{cases} \dfrac{x+2}{x^3 + 2x^2 + x + 2}, & x < -2, \\ \dfrac{1}{x} - \dfrac{7}{5x}, & x \geq -2. \end{cases}$$

(7 marks)

- END OF QUESTIONS -

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Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \csc^2 x\, dx = -\cot x + C$$

$$\int \sec x \tan x\, dx = \sec x + C$$

$$\int \csc x \cot x\, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x\, dx = \sinh x + C$$

$$\int \sinh x\, dx = \cosh x + C$$

$$\int \operatorname{sech}^2 x\, dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x\, dx = -\coth x + C$$

$$\int \operatorname{sech} x \tanh x\, dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \coth x\, dx = -\operatorname{csch} x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C, \ |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C, \ |x| < 1$$

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\, dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\, dx = \csc^{-1} x + C, \ |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\, dx = \cosh^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\, dx = \operatorname{sech}^{-1} |x| + C, \ 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\, dx = \operatorname{csch}^{-1} |x| + C, \ x \neq 0$$

$$\int \frac{1}{1-x^2}\, dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

TRIGONOMETRIC SUBSTITUTION

Expression	***Trigonometry***	***Hyperbolic***
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

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TRIGONOMETRIC SUBSTITUTION

$t=\tan\frac{1}{2}x$		$t=\tan x$	
$\sin x=\dfrac{2t}{1+t^2}$	$\cos x=\dfrac{1-t^2}{1+t^2}$	$\sin 2x=\dfrac{2t}{1+t^2}$	$\cos 2x=\dfrac{1-t^2}{1+t^2}$
$\tan x=\dfrac{2t}{1-t^2}$	$dx=\dfrac{2dt}{1+t^2}$	$\tan 2x=\dfrac{2t}{1-t^2}$	$dx=\dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$\cos^2 x+\sin^2 x=1$

$\sin 2x=2\sin x\cos x$

$\cos 2x=\cos^2 x-\sin^2 x$

$\quad =2\cos^2 x-1$

$\quad =1-2\sin^2 x$

$1+\tan^2 x=\sec^2 x$

$1+\cot^2 x=\csc^2 x$

$\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$

$\tan(x\pm y)=\dfrac{\tan x\pm\tan y}{1\mp\tan x\tan y}$

$\sin(x\pm y)=\sin x\cos y\pm\sin y\cos x$

$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$

$2\sin ax\cos bx=\sin(a+b)x+\sin(a-b)x$

$2\sin ax\sin bx=\cos(a-b)x-\cos(a+b)x$

$2\cos ax\cos bx=\cos(a-b)x+\cos(a+b)x$

Hyperbolic Functions

$\sinh x=\dfrac{e^x-e^{-x}}{2}$

$\cosh x=\dfrac{e^x+e^{-x}}{2}$

$\cosh^2 x-\sinh^2 x=1$

$\sinh 2x=2\sinh x\cosh x$

$\cosh 2x=\cosh^2 x+\sinh^2 x$

$\quad =2\cosh^2 x-1$

$\quad =1+2\sinh^2 x$

$1-\tanh^2 x=\text{sech}^2 x$

$\coth^2 x-1=\text{csch}^2 x$

$\tanh 2x=\dfrac{2\tanh x}{1+\tanh^2 x}$

$\tanh(x\pm y)=\dfrac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$

$\sinh(x\pm y)=\sinh x\cosh y\pm\sinh y\cosh x$

$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$

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Formulae

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = 2\pi \int_{x_1}^{x_2} f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\, dx$$

$$S = 2\pi \int_{y_1}^{y_2} g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\, dy$$

CURVATURE, ARC LENGTH AND TANGENT VECTORS

$$\kappa = \frac{\|d\underline{T}/dt\|}{\|d\underline{r}/dt\|}$$

$$s(t) = \int_a^b \|\underline{r}'(t)\| dt$$

$$\underline{T}(t) = \frac{\underline{r}'(t)}{\|\underline{r}'(t)\|}, \quad \underline{r}'(t) \neq 0$$