UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BDA 34003
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS IN **SECTION A** 2. ANSWER **TWO (2)** QUESTIONS IN **SECTION B** 3. PERFORM ALL CALCULATION IN **4 DECIMAL PLACES**

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THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

SECTION A

Q1 Consider the vibrating system with 2 masses and 3 springs, as illustrated in **Figure Q1**. The masses are constrained to move only in the horizontal direction. Determining the equations of motion for each mass using its free-body diagram results in the following system of equations:

$$\begin{pmatrix} \dfrac{(k_1+k_2)}{m_1} & -\dfrac{k_2}{m_1} \\ -\dfrac{k_2}{m_2} & \dfrac{(k_2+k_3)}{m_2} \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

(a) Consider that $m_1 = m_2 = k_1 = k_2 = k_3 = 1$. Use the characteristic equation to find the eigenvalues of this vibrating system.

(4 marks)

(b) Determine the largest eigenvalue and the corresponding eigenvector of this vibrating system by using an appropriate numerical method. Use the initial eigenvector $v^{(0)} = (0 \quad 1)^T$. Iterate until $|\lambda_{i+1} - \lambda_i| < 0.05$.

(6 marks)

(c) Compare your answer in **Q1(a)** and **Q1(b)** in terms of relative error.

(4 marks)

(d) What conclusion can you draw for the motion of this system, if the system vibrates at 1.7321rad/s?

(6 marks)

Q2 Take the case of a pressure vessel that is being tested in the laboratory to check its ability to withstand pressure. For a thick pressure vessel of inner radius a and outer radius b, the differential equation for the radial displacement u of a point along the thickness is given by:

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}=0$$

The inner radius $a = 125$ mm and the outer radius $b = 200$ mm. The material of the pressure vessel is ASTM A 36 steel. The boundary conditions are:

$$u\big|_{r=a}=0.0968275\,\text{mm}$$

$$u\big|_{r=b}=0.0769240\,\text{mm}$$

(a) By dividing the radial thickness of the pressure vessel into four (4) equidistant nodes, derive the differential equation in numerical (finite difference) form. An example of the model is illustrated in **Figure Q2**.

(5 marks)

(b) Determine the radial displacement profile by using the Finite Difference Method.

(5 marks)

(c) The exact expression for radial displacement is given by

$$u=C_1 r+\frac{C_2}{r}$$

where C_1 and C_2 can be found by using boundary conditions at $r=a$ and $r=b$. Compare your answer in **Q2(b)** with the exact solution in terms of absolute error.

(5 marks)

(d) How is the radial displacement profile related to the boundary conditions, if the boundary conditions are:

$$u\big|_{r=a}=1.5\,\text{mm}$$

$$u\big|_{r=b}=1\,\text{mm}$$

(5 marks)

Q3 A straight metal bar of length 2.5 cm is illustrated in **Figure Q3**. The temperatures on its ends are maintained for 2 seconds. The initial temperatures of the bar are shown in the figure. The temperature at point A is maintained at 100°C while point F is maintained at 5°C. The distribution of initial temperature is as shown in **Figure Q3**. This bar is fully insulated on its surface so the heat transfer occurs only in its longitudinal axis.

The unsteady state heating equation follows a heat equation

$$\frac{\partial T}{\partial t} - K\frac{\partial^2 T}{\partial x^2} = 0$$

where K is thermal diffusivity of material and x is the longitudinal coordinate of the bar. The thermal diffusivity of the silver is given as 1.8 cm^2/s.

(a) Draw clearly finite difference grid to predict the temperature of all points up to 2 seconds and label all unknown temperatures in the grid.

(5 marks)

(b) Based on your grid illustration in **Q3(a)**, propose simultaneous equations based on Implicit Crank Nicolson method to determine the temperature of point A, B, C, D, E and F for every 1 second.

(10 marks)

(c) Determine the unknown temperatures of each point in your grid illustration when time is 1 second. You have to use Thomas Algorithm table.

(5 marks)

SECTION B

Q4 (a) You have conducted an experiment and obtained a set of data points as in **Table Q4(a)**:

Table Q4(a)

x	8	11	15	18
$f(x)$	5	9	10	8

Predict the value for $f(12.7)$ by using Newton's Divided Difference.

(6 marks)

(b) A new bullet test in a laboratory has detected the history of the velocity in 9 seconds as shown in **Table Q4(b)**.

Table Q4(b)

Time (s)	0	1	2	3	4	5	6	7	8	9
Velocity (m/s)	0	10	20	30	40	50	55	60	70	100

(i) Analyze the distance travel (in meter) of the bullet in 9 seconds from idle by using quick calculation of trapezoidal rule from the data records.

(4 marks)

(ii) Comparing trapezoidal rule and Simpson's method, which method gives more accurate answer for approximating definite integral in general? Justify your answer.

(3 marks)

(iii) Improve the accuracy of the distance prediction by using the appropriate Simpson's method.

(7 marks)

Q5 (a) Given

$$\frac{dy}{dt} = yt^3 - 1.5y, \quad y(0) = 1$$

(i) Solve the initial value problem using Midpoint method over the interval from $t = 0$ to $t = 2$. Use $h = 0.5$.

(5 marks)

(ii) Analyze the difference between the approximated values and actual values given by $y = e^{\left(\frac{t^4}{4} - 1.5t\right)}$.

(5 marks)

(b) Given

$$f(x) = \sin x + \cos(1 + x^2) - 1$$

(i) Locate the first positive root of $f(x)$ using Secant method, where x is in radians. Do your calculation up to four iterations with the initial guess $x_0 = 1.5$ and $x_1 = 2.5$.

(5 marks)

(ii) Examine the answer in **Q5(b)(i)** and the graph of $f(x)$ in **Figure Q5**, analyze and justify the correctness of your answer.

(5 marks)

Q6 (a) Given the function $f(x)=\dfrac{2^x}{x}$. From analytical differentiation, the value of the second order derivative $f''(2)=0.5746$.

(i) Evaluate $f''(2)$ numerically with the three-point central difference formula using points $x=1.8$, $x=2$ and $x=2.2$

(4 marks)

(ii) How accurate are your answers in (i) in comparison with the exact derivative?

(2 marks)

(iii) Improve the accuracy of your approximation answer in **Q6(a)(i)** with suitable method.

(4 marks)

(b) Given the following system of linear equations:

$$\text{A:}\quad \begin{pmatrix} -1 & 6 & 0 & 0 \\ 5 & -2 & 7 & 0 \\ 0 & 2 & -3 & 8 \\ 0 & 0 & 3 & -4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}=\begin{pmatrix} 4 \\ 0 \\ 9 \\ 5 \end{pmatrix}$$

$$\text{B:}\quad \begin{pmatrix} 4 & -1 & 1 \\ -1 & 7 & 3 \\ 1 & 3 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$\text{C:}\quad \begin{pmatrix} 1 & -1 & 0 \\ 3 & 2 & -2 \\ 0 & 8 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 6 \\ 2 \\ 6 \end{pmatrix}$$

(i) Examine which system of linear equations can be solved by Gauss-Seidel iteration method.

(4 marks)

(ii) Subsequently, solve the selected system of linear equations in **Q6(b)(i)** using Gauss-Seidel iteration method. Iterate until tolerance error $\max\{|x_{i+1}-x_i|\}<0.05$.

(6 marks)

-END OF QUESTIONS –

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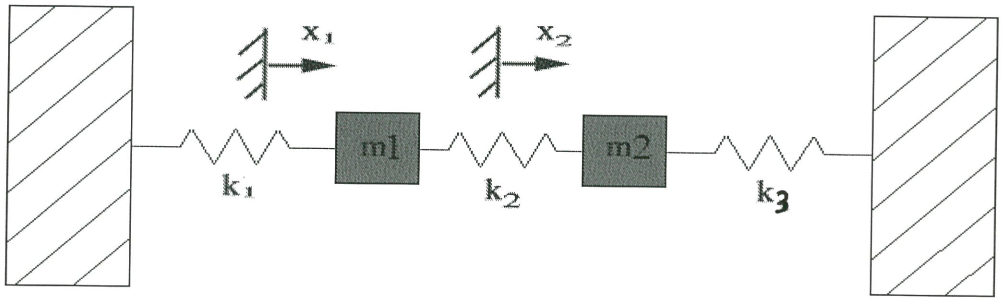

Figure Q1

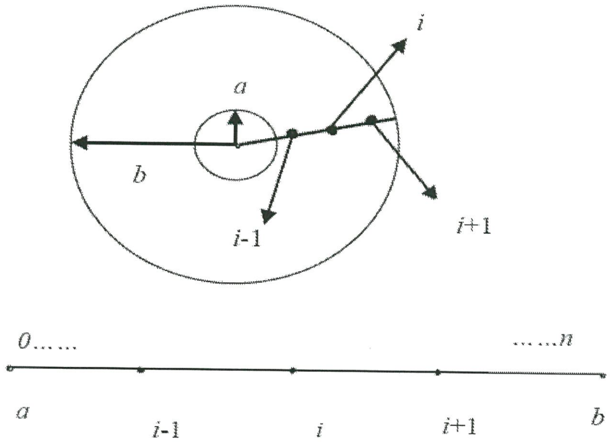

Figure Q2

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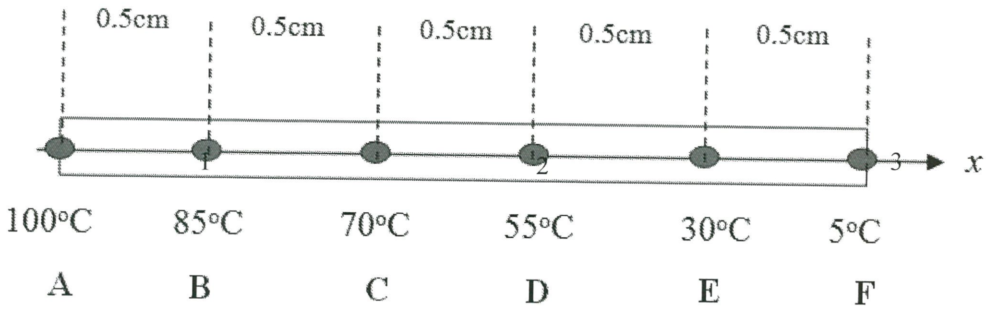

Figure Q3

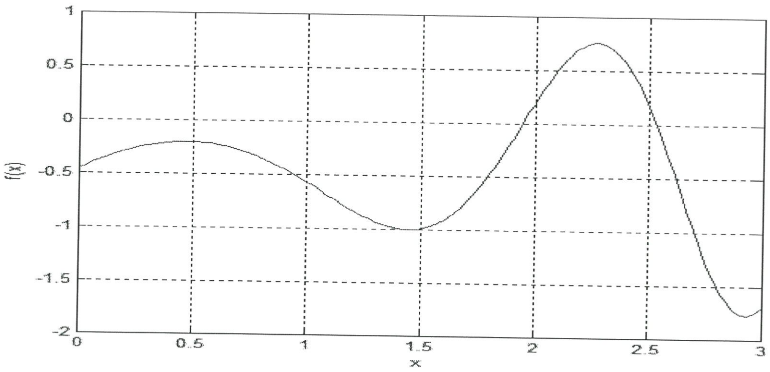

Figure Q5

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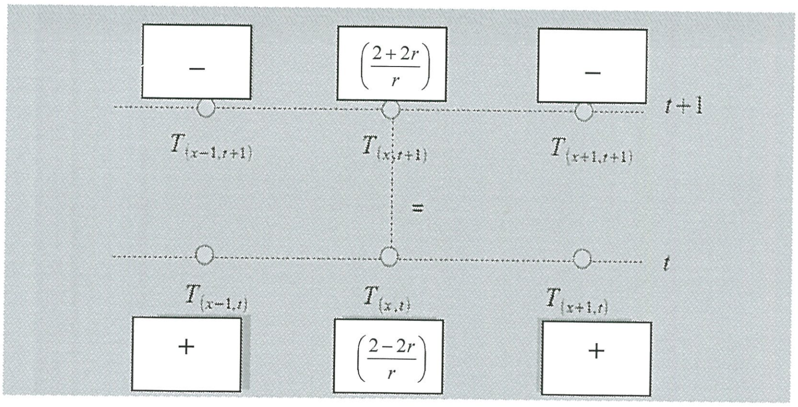
FORMULA

Implicit Crank Nicolson Method



Thomas Algorithm

i	1	2	...	n
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \frac{e_i}{\alpha_i}$				
$y_1 = \frac{b_1}{\alpha_1}$ $y_i = \frac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

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Midpoint Method

$$f(x_i, y_i) = y'(x_i)$$

$$y_{i+1} = y_i + k_2$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$$

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