UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

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COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BDA24003
PROGRAMME	:	2 BDD
EXAMINATION DATE	:	DISEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **SECTION A** B) ANSWER **TWO (2)** QUESTIONS IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

SECTION A

Q1 (a) A closed box has a dimension of 30 cm, 41 cm and 53 cm, respectively with the possible error of 0.2 cm. Use partial derivatives to estimate the maximum possible error by calculating the following:

(i) Surface area of the box

(5 marks)

(ii) Volume of the box

(5 marks)

(b) Sketch the domain of the following function f(x,y) = $\dfrac{x-2y}{x+2y}$.

(5 marks)

(c) Sketch the graph of the following function
$f(x,y) = 8 - \sqrt{4x^2 + y^2}$.

(5 marks)

Q2 (a) Evaluate $\int_1^3 \int_0^{\frac{\pi}{4}} 2x\sec^2 y\,dy\,dx$

(5 marks)

(b) Evaluate $\iint 2xy + y^2 dA$ if R is a region in a triangle with vertices (0,0), (1,0) and (1,2).

(5 marks)

(c) Find area of region R between y = cos x and y = sin x over interval
$0 \ll x \ll \frac{\pi}{4}$.

(5 marks)

(d) A lamina bounded by x-axis, x=1 and the curve $y^2 = x$ has density $\delta(x,y) = x + y$. Calculate its total mass.

(5 marks)

Q3 (a) Sketch the graph of the vector function $\mathbf{r}(t) = (t-1)\,\mathbf{i} + t^2\,\mathbf{j}$ by indicating the direction of the vector.

(6 marks)

(b) Obtain the tangent vector at $t = \pi$ if $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + \sin t\,\mathbf{k}$.

(8 marks)

(c) Sketch the curvature of a line $\mathbf{r}(s) = (3 - 3s/5)\,\mathbf{i} + 4s/5\,\mathbf{j}$.

(6 marks)

SECTION B

Q4 (a) Integrate $\int_0^{\frac{\pi}{3}}\int_0^{\cos\theta}\int_0^{\phi} \rho^2 \sin\theta \, d\rho \, d\phi \, d\theta$.

(7 marks)

(b) Evaluate the volume of the solid that enclosed by y = x2, z + y = 1, z = 0 and xz-plane and yz-plane.

(6 marks)

(c) The solid bounded above by plane z = 1 and below by right circular cone $z = \sqrt{x^2 + y^2}$ has density $\delta(x, y, z) = 2$. Evaluate the moment about the xy-plane of the solid.

(7 marks)

Q5 (a) Given **F** = y^2z^3 **i** + $2xyz^3$ **j** + $3xy^2z^2$ **k** is a vector field. Prove that **F** is conservative.

(6 marks)

(b) Evaluate outward unit normal for the plane, x + y + z = 1.

(6 marks)

(c) By using Green Theorem, evaluate $\oint_C x^4 dx + xy dy$ where C is the triangular curve consisting of the line segments from (0,0) to (1,0), from (1,0) to (0,1) and from (0,1) to (0,0).

(8 Marks)

Q6 (a) By using Gauss Theorem, evaluate surface integral of **F** over all surfaces of the cube x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1 if **F** = x **i** + y **j** + 2z **k**.

(10 marks)

(b) Evaluate $\int_C F.dr$ by using Stokes Theorem for **F** = z^2 **i** + 2x **j** – y^3 **k** with C is the circle $x^2 + y^2 = 1$ in the xy-plane with counterclockwise orientation looking down the positive z-axis.

(10 marks)

- END OF QUESTION –

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FORMULAE

Total Differential
For function $z = f(x,y)$, the total differential of z, dz is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Relative Change
For function $z = f(x,y)$, the relative change in z is given by:

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

Implicit Differentiation
Suppose that z is given implicitly as a function $z = f(x,y)$ by an equation of the form $F(x,y,z)=0$, where $F(x,y,f(x,y))=0$ for all (x,y) in the domain of f, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
   $f(x,y)$ has a local maximum value at (a,b)
b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
   $f(x,y)$ has a local minimum value at (a,b)
c. If $D < 0$
   $f(x,y)$ has a saddle point at (a,b)
d. If $D = 0$
   The test is inconclusive

Surface Area

$$\textit{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\,dA$$

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Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)rdzdrd\theta$$

Spherical Coordinates:

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2\sin\phi d\rho d\phi d\theta$$

In 2-D: Lamina

Given that $\delta(x,y)$ is a density of lamina

Mass, $m = \iint_R \delta(x,y)dA$, where

Moment of Mass

a. About x-axis, $M_x = \iint_R y\delta(x,y)dA$

b. About y-axis, $M_y = \iint_R x\delta(x,y)dA$

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Centre of Mass
Non-Homogeneous Lamina:
$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m} \right)$$

Centroid
Homogeneous Lamina:
$$\bar{x} = \frac{1}{Area\,of \quad R} \iint_R x\,dA \text{ and } \bar{y} = \frac{1}{Area\,of \quad R} \iint_R y\,dA$$

Moment Inertia:

a. $I_y = \iint_R x^2 \delta(x,y)\,dA$

b. $I_x = \iint_R y^2 \delta(x,y)\,dA$

c. $I_o = \iint_R (x^2 + y^2) \delta(x,y)\,dA$

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In 3-D: Solid
Given that $\delta(x,y,z)$ is a density of solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$

If $\delta(x,y,z) = c$, where c is a constant, $m = \iiint_G dA$ is volume.

Moment of Mass

a. About *yz*-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

b. About *xz*-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

c. About *xy*-plane, $M_{xy} = \iiint_G z\delta(x,y,z)\,dV$

Centre of Gravity
$$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m} \right)$$

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Moment Inertia

a. About x-axis, $I_x = \iiint_G (y^2 + z^2)\delta(x,y,z)dV$

b. About y-axis, $I_y = \iiint_G (x^2 + z^2)\delta(x,y,z)dV$

c. About z-axis, $I_z = \iiint_G (x^2 + y^2)\delta(x,y,z)dV$

Directional Derivative

$D_u f(x,y) = (f_x \mathbf{i} + f_y \mathbf{j}) \cdot \mathbf{u}$

Del Operator

$\nabla = \dfrac{\partial}{\partial x}\mathbf{i} + \dfrac{\partial}{\partial y}\mathbf{j} + \dfrac{\partial}{\partial z}\mathbf{k}$

Gradient of $\phi = \nabla \phi$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla.\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

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Let C is smooth curve defined by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The **Binormal Vector,** $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

Curvature

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

Radius of Curvature

$$\rho = \frac{1}{\kappa}$$

Green's Theorem

$$\oint_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

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Gauss's Theorem

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS = \iiint_G \nabla \cdot \mathbf{F} dV$$

Stoke's Theorem

$$\oint_C \mathbf{F} \cdot dr = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$

Arc Length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}, t \in [a,b]$, hence, the **arc length,**

$$s = \int_a^b \|\mathbf{r}'(t)\| dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} dt$$