UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BDA 14103
PROGRAMME CODE	:	1 BDD
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

PART A

Q1 (a) By using method of variation parameter, obtain the general solution for:

$$4y'' + 4y = 8sinx$$

(*Note*: $\sin 2x = 2\cos x \sin x \qquad \cos 2x = 1 - 2\sin^2 x$)

(8 marks)

(b) A model for forced spring mass system is given by:

$$my'' + cy' + ky = r(t)$$

(i) Find the steady state solution for case with value of $m = 1$, $c = 4$, $k = 4$ and $r(t) = sin2t/sint$

(ii) Find the particular solution for the answer in **Q1** (b) (i) that satisfies $y(0) = 1$ and $y'(0) = 2$.

(12 marks)

Q2 (a) Express $\dfrac{3}{s(s^2+4)}$ in partial fraction.

(8 marks)

(b) Find the Laplace transform of $f(t) = \cos 2t$.

(2 marks)

(c) By using the obtained result in **Q2**(b), show that

$$\mathcal{L}^{-1}\left\{\frac{3}{s(s^2+4)}e^{-4s}\right\} = \frac{3}{4}H(t-4) - \frac{3}{4}H(t-4)\cos(2(t-4)).$$

(3 marks)

(d) By using the obtained results in **Q2** (a)-(c), solve the following initial value problem.

$$y'' + 4y = f(t), \qquad y(0) = 1, \quad y'(0) = 0, \text{ with}$$

$$f(t) = \begin{cases} 0, & for \quad 0 \le t < 4 \\ 3, & for \quad t \ge 4 \end{cases}.$$

(7 marks)

PART B

Q3 (a) Solve the differential equation:

$$4x\,\frac{dy}{dx} - 4y = \frac{4x}{x+1}$$

(6 marks)

(b) By using the method of Laplace transform, solve the initial value problem of;

$$y'' + 5y' + 6y = e^{-t}$$

with the initial condition $y(0) = y'(0) = 0$.

(14 marks)

Q4 (a) A model for forced spring mass system is given by:

$$y'' + 4y' + 4y = r(t)$$

Find the steady state solution (y_p) for case with external force $r(t) = 2\cos t$.

(8 marks)

(b) A birth rate $\left(\frac{dP}{dt}\right)$ of insects in a region is proportional to their current population (P) as shown below. Where r, is the positive population constant.

$$\frac{dP}{dt} = rP$$

In the absence of any outside factors the population will triple in two week's time. If there are initially 100 insects in the area, identify the value of r, the positive population constant.

(8 marks)

(c) A corpse was discovered in a motel room at midnight and its temperature was 27°C. The temperature of the room is kept constant at 15°C. Two hours later the temperature of the corpse dropped to 24°C. Identify the time of death if the temperature of a corpse at time of death is assumed as 37°C.

(6 marks)

Q5 A 3 cm length silver bar with a constant cross section area 1 cm^2 (density 10 g/cm^3, thermal conductivity 3 cal/(cm sec°C), specific heat 0.15 cal/(g°C)), is perfectly insulated laterally, with ends kept at temperature 0°C and initial uniform temperature $f(x) = 25$°C.

The heat equation is:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$$

(a) Show that $c^2 = 2$.

(2 marks)

(b) By using the method of separation of variable, and applying the boundary condition, prove that

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{3} e^{-\frac{2n^2\pi^2 t}{9}}$$

where b_n is an arbitrary constant.

(12 marks)

(c) By applying the initial condition, find the value of b_n.

(6 marks)

Q6 Let $f(x)$ be a function of period 2π such that

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ \pi, & \pi < x < 2\pi \end{cases}$$

(a) Sketch the graph of $f(x)$ in the interval $-2\pi < x < 2\pi$.

(2 marks)

(b) Prove that the Fourier series for $f(x)$ in the interval $0 < x < 2\pi$ is:

$$\frac{3\pi}{4} - \frac{2}{\pi}\left[\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right]$$
$$-\left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right]$$

(12 marks)

(c) By giving an appropriate value for x, demonstrate that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

(6 marks)

-END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2016/2017
COURSE NAME : ENGINEERING MATHEMATICS II
PROGRAMME CODE : 1BDD
COURSE CODE : BDA14103

FORMULAS

First Order Differential Equation

Type of ODEs	General solution
Linear ODEs: $y' + P(x)y = Q(x)$	$y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx}Q(x)dx + C\right\}$
Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$	$F(x,y) = \int f(x,y)dx$ $F(x,y) - \int\left\{\frac{\partial F}{\partial y} - g(x,y)\right\}dy = C$
Inexact ODEs: $M(x,y)dx + N(x,y)dy = 0$ $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Integrating factor; $i(x) = e^{\int f(x)dx}$ where $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $i(y) = e^{\int g(y)dy}$ where $g(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$	$\int iM(x,y)dx - \int\left\{\frac{\partial\left(\int iM(x,y)dx\right)}{\partial y} - iN(x,y)\right\}dy = C$ TERBUKA

Characteristic Equation and General Solution for Second Order Differential Equation

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1e^{m_1x} + c_2e^{m_2x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1e^{mx} + c_2xe^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$

Method of Undetermined Coefficient

$g(x)$	y_p
Polynomial: $P_n(x) = a_nx^n + \ldots + a_1x + a_0$	$x^r(A_nx^n + \ldots + A_1x + A_0)$
Exponential: $e^{\alpha x}$	$x^r(Ae^{\alpha x})$
Sine or Cosine: $\cos\beta x$ or $\sin\beta x$	$x^r(A\cos\beta x + B\sin\beta x)$

Note: r is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2016/2017
COURSE NAME : ENGINEERING MATHEMATICCS II
PROGRAMME CODE : 1BDD
COURSE CODE : BDA 14103

Method of Variation of Parameters

The particular solution for $y''+by'+cy=g(x)$ (b and c constants) is given by $y(x)=u_1y_1+u_2y_2$, where $u_1=-\int\frac{y_2g(x)}{W}dx$, $u_2=-\int\frac{y_1g(x)}{W}dx$ and $W=\begin{vmatrix}y_1 & y_2\\ y_1' & y_2'\end{vmatrix}$

Laplace Transform

$$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}dt=F(s)$$

$f(t)$	$F(s)$
a	$\frac{a}{s}$
$t^n, n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t), n=1,2,3,\ldots$	$(-1)^n\frac{d^nF(s)}{ds^n}$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$\dot{y}(t)$	$sY(s)-y(0)$
$\ddot{y}(t)$	$s^2Y(s)-sy(0)-y'(0)$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2016/2017
COURSE NAME : ENGINEERING MATHEMATICS II
PROGRAMME CODE : 1BDD
COURSE CODE : BDA 14103

Fourier Series

Fourier series expansion of periodic function with period 2 π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Half Range Series

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L} dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L} dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$