UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

TERBUKA

COURSE NAME	:	ELECTROMECHANICAL AND CONTROL SYSTEM
COURSE CODE	:	BDU 20302
PROGRAMME	:	2 BDC/2 BDM
EXAMINATION DATE	:	DECEMBER 2016/JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FOUR (4)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

Q1 (a) Describe the short period mode of an aircraft when disturbed slightly from steady flight.

(4 marks)

(b) Consider an aircraft model in a wind tunnel setup where the aircraft is constrained at its center of gravity. The aircraft is free to perform a pitching motion about its center of gravity. The governing equation of this simple motion is obtained from the Newton's second law and is given as:

$$\Delta\ddot{\alpha} - \left(M_q + M_{\dot{\alpha}}\right)\Delta\dot{\alpha} - M_\alpha \Delta\alpha = M_{\delta e}\Delta\delta_e$$

where $\Delta\alpha$ is the change in angle of attack (the change in angle of attack and pitch angles are identical), $\Delta\delta e$ is the change in elevator angle. M_q and M_α are longitudinal derivatives due to pitching velocity and angle of attack. Find the transfer function relating the change in angle of attack, $\Delta\alpha(s)$ and the change in elevator angle $\Delta\delta e(s)$. Use the Laplace transform theorem in Figure **Q1.**

(4 marks)

(c) Determine the solution, $y(t)$ for the governing equation in (b) if a step input for elevator is applied to the dynamic system using the following assumptions:

$$M_q = -2.0500\, s^{-1}$$
$$M_\alpha = -8.8000\, s^{-2}$$
$$M_{\dot{\alpha}} = -0.8976\, s^{-2}$$
$$M_{\delta e} = -5.5000\, s^{-2}$$

Use partial fraction and inverse Laplace theorem to solve the problem.

(15 marks)

(d) Determine the steady state value of the solution, $y(t)$.

(2 marks)

Q2 (a) The short period response characteristics of an aircraft are of particular importance in flying and handling quality. The reduced order state space model corresponding to short period mode approximation for the Ling-Temco-Vought A-7A Corsair II aircraft is given as follows:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -3.1245 & -1.2109 \\ -0.2169 & -1.2732 \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} -0.3741 \\ -0.5581 \end{bmatrix} \delta_e$$

Find the solution to the given state space model using Paynter's numerical method. Use time interval, $\Delta t = 0.01$ to solve the numerical problem.

(16 marks)

(b) If the input of the system, u_1 is applied with 5° elevator step input with output equation and initial condition are given as follows:

$$q_k = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} w_k \\ q_k \end{bmatrix} \qquad \begin{bmatrix} w_0 \\ q_0 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0.15 \end{bmatrix}$$

Determine the output response, q_k of the state equation up to 3 iterations.

(9 marks)

Q3 (a) State **TWO (2)** reasons for using feedback (closed-loop) control system and at least **ONE (1)** reason for not using them.

(3 marks)

(b) The Dutch roll motion can be approximated using the following equation:

$$\begin{bmatrix}\Delta\dot{\beta}\\ \Delta\dot{r}\end{bmatrix}=\begin{bmatrix}\frac{Y_\beta}{u_0} & -\left(1-\frac{Y_r}{u_0}\right)\\ N_\beta & N_r\end{bmatrix}\begin{bmatrix}\Delta\beta\\ \Delta r\end{bmatrix}+\begin{bmatrix}\frac{Y_{\delta r}}{u_0}\\ N_{\delta r}\end{bmatrix}\delta r$$

Assume the aircraft has the following stability derivative characteristics as follows:

$Y_\beta = -7.5$ ft/s^2 $\quad$ $Y_r = 2.5$ ft/s

$N_\beta = 1.25$ s^{-2} $\quad$ $N_r = -0.325$ s^{-1}

$Y_{\delta r} = -4.8$ ft/s^2 $\quad$ $N_{\delta r} = 0.615$ s^{-2}

$u_0 = 150$ ft/s

(i) Determine the characteristic equation of the Dutch Roll mode.

(4 marks)

(ii) Determine the eigenvalues of the Dutch Roll mode.

(2 marks)

(iii) Determine the damping ratio, natural frequency, period, time to half amplitude and number of cycles to half amplitude for the Dutch Roll mode.

(5 marks)

(c) Compare your Dutch Roll mode natural frequency and damping ratio calculation for this particular aircraft to the handling quality criteria in Table **Q3.** Assume that the aircraft under consideration is a Class IV aircraft (High Maneuvering Aircraft) performing CAT A mission (Precision Tracking). Determine the minimum state feedback gain so that the damping ratio achieved Level 1 handling qualities. Use feedback control design based on rudder deflection proportional only to the yaw rate state, i.e.:

$$u=-K^T x=-[0 \quad K]\begin{bmatrix}\Delta\beta\\ \Delta r\end{bmatrix}.$$

(11 marks)

Q4 The open loop pitch rate response to elevator transfer function for the Lockheed F-104 Starfighter is given by the following transfer function:

$$\frac{q(s)}{\delta_e(s)}=\frac{-4.66s(s+0.133)(s+0.269)}{(s^2+0.015s+0.021)(s^2+0.911s+4.884)}$$

(a) The root locus plot of the transfer function is given in **Figure Q4**. With the aid of the root locus plot, explain how the root locus plot can be used to evaluate the effect of feedback on the characteristics modes of motion?

(4 marks)

(b) Determine the damping ratio and undamped natural frequency for short period and phugoid mode.

(4 marks)

(c) Design a pitch rate feedback controller, K_q to bring the closed loop short period mode in agreement with minimum specification for damping ratio and natural frequency. Assume the following **Level 1** flying qualities are used in the analysis:

Phugoid damping ratio $\zeta_p \geq 0.04$
Short period damping ratio $\zeta_s \geq 0.5$
Short period undamped natural frequency $0.8 \leq \omega_s \leq 3.0$ rad/s

(11 marks)

(d) Compare the augmented short period damping ratio and natural frequency with those of the unaugmented aircraft. How does pitch rate feedback to elevator input improve the longitudinal flying qualities? Explain your answer based from your findings and the given root locus plot.

(6 marks)

Q5 (a) Given a unity feedback system that has the forward transfer function:

$$G(s) = \frac{K(s-1)(s-5)}{s^2+6s+25}$$

(i) Sketch the root locus plot for the uncompensated system.

(3 marks)

(ii) Find the exact point and gain where the locus intersect imaginary axis.

(6 marks)

(iii) Find the break-in point at the real axis.

(6 marks)

(iv) Find the point where the locus crosses the 0.5 damping ratio line.

(3 marks)

(v) Find the gain at the point where the locus crosses the 0.5 damping ratio line.

(5 marks)

(vi) Find the range of gain, K for which the system is stable.

(2 marks)

-END OF QUESTION-

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Laplace transforms – Table			
$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$
$\frac{1}{2}t^2e^{-at}$	$\frac{1}{(s+a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$	$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2-\omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1+at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1,2,3..$	$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
t^ne^{at}	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s}F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t)$ *unit impulse*	$1 \quad$ *all s*
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2f}{df^2}$	$s^2F(s) - sf(0) - f'(0)$
$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{n-1}(0)$		

FIGURE Q1 The Laplace transform theorems.

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TABLE Q3 Minimum Dutch-Roll frequency and damping

Aircraft Class	Flight Phase	Minimum Values							
		Level 1			Level 2			Level 3	
		ζ	$\zeta\omega_n$	ω_n	ζ	$\zeta\omega_n$	ω_n	ζ	ω_n
I, IV	CAT A	0.19	0.35	1.0	0.02	0.05	0.5	0	0.4
II, III	CAT A	0.19	0.35	0.5	0.02	0.05	0.5	0	0.4
All	CAT B	0.08	0.15	0.5	0.02	0.05	0.5	0	0.4
I, IV	CAT C	0.08	0.15	1.0	0.02	0.05	0.5	0	0.4
II, III	CAT C	0.08	0.10	0.5	0.02	0.05	0.5	0	0.4

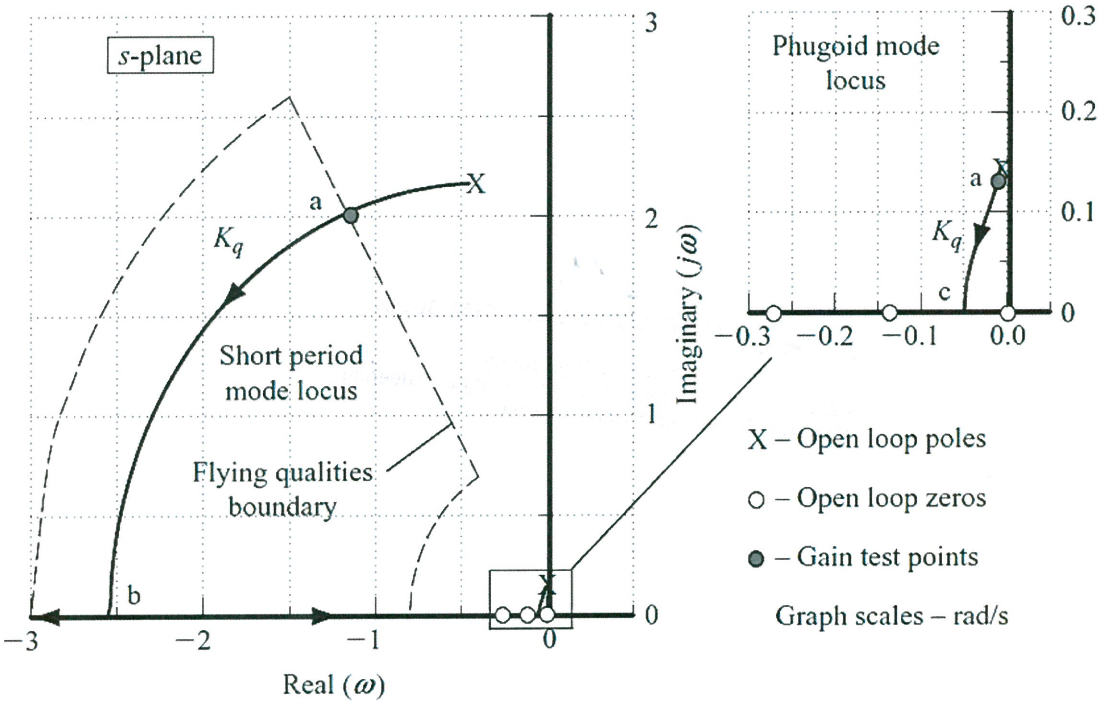

FIGURE Q4 Root locus plot showing pitch rate feedback to elevator.

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A Key Equations

The relevant equations used in this examination are given as follows:

1. Determinant of 3 × 3 matrix:
$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} \quad (1)$$
2. Partial fraction for $F(s)$ with real and distinct roots in denominator:
$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2}{(s+p_2)} + \cdots + \frac{K_m}{(s+p_m)} \quad (2)$$
3. Partial fraction for $F(s)$ with complex or imaginary roots in denominator:
$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2 s + K_3}{(s^2+as+b)} + \cdots \quad (3)$$
4. General first order transfer function:
$$G(s) = \frac{a}{s+a} \quad (4)$$
5. General second order transfer function:
$$G(s) = \frac{K}{s^2 + 2\xi\omega_n s + \omega_n} \quad (5)$$
6. The closed loop transfer function:
$$T(s) = \frac{G(s)}{1+G(s)H(s)} \quad (6)$$
where $G(s)$ is the transfer function of the open loop system and $H(s)$ is the transfer function in the feedback loop.
7. The final value theorem:
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \quad (7)$$
8. Time response:
$$T_r = \frac{2.2}{a} \quad (8)$$
$$T_s = \frac{4}{a} \quad (9)$$
$$\%OS = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\% \quad (10)$$
$$\xi = \frac{-\ln\left(\%\frac{OS}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\%\frac{OS}{100}\right)\right)^2}} \quad (11)$$

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$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\omega} \qquad (12)$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{\eta} \qquad (13)$$

$$P = \frac{2\pi}{\omega} \qquad (14)$$

$$t_{1/2} = \frac{0.693}{|\eta|} \qquad (15)$$

$$N_{1/2} = 0.110\frac{|\omega|}{|\eta|} \qquad (16)$$

9. Estimation of parameter q (Paynter Numerical Method)

$$q = \max|A_{ij}\Delta t| \qquad (17)$$

10. Estimation of integer value of p (Paynter Numerical Method)

$$\frac{1}{p!}(nq)^p e^{nq} \leq 0.001 \qquad (18)$$

11. Numerical solution of state equation (Paynter Numerical Method):

$$\mathbf{x}_{k+1} = \boldsymbol{M}\mathbf{x}_k + \boldsymbol{N}\boldsymbol{\eta}_k$$

with matrix $\boldsymbol{M}$ and $\boldsymbol{N}$ are given by the following matrix expansion:

$$\boldsymbol{M} = e^{A\Delta t} = \boldsymbol{I} + \boldsymbol{A}\Delta t + \frac{1}{2!}\boldsymbol{A}^2\Delta t^2 \ldots \qquad (19)$$

$$\boldsymbol{N} = \Delta t\left(\boldsymbol{I} + \frac{1}{2!}\boldsymbol{A}\Delta t + \frac{1}{3!}\boldsymbol{A}^2\Delta t^2 + \cdots\right)\boldsymbol{B}$$

12. Finding eigenvalues from state space model:

$$|\lambda\boldsymbol{I} - \boldsymbol{A}| = 0 \qquad (20)$$

13. Characteristic equation of the closed loop system:

$$1 + KG(s)H(s) = 0 \qquad (21)$$

14. Asymptotes: angle and real-axis intercept:

$$\sigma = \frac{[\sum Real\ parts\ of\ the\ poles - \sum Real\ parts\ of\ the\ zeros]}{n-m} \qquad (22)$$

$$\phi_a = \frac{180°[2q+1]}{n-m} \qquad (23)$$

15. Solution to find real axis break-in and breakaway points:

$$\frac{dK(\sigma)}{d\sigma} = 0 \qquad (24)$$

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16. Alternative solution to find real axis break-in and breakaway points:

$$\sum \frac{1}{\sigma + z_i} = \sum \frac{1}{\sigma + p_i} \qquad (25)$$

17. Angle of departure of the root locus from a pole of $G(s)H(s)$:

$$\theta = 180° + \sum (angles\ to\ zeros) - \sum (angles\ to\ poles) \qquad (26)$$

18. Angle of arrival at a zeros:

$$\theta = 180° - \sum (angles\ to\ zeros) + \sum (angles\ to\ poles) \qquad (27)$$

19. The steady state error:

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) \qquad (28)$$

where the error signal is given as:

$$E(s) = \frac{1}{1 + K(s)G(s)H(s)} \times U(s) \qquad (29)$$

20. Linear system (SISO) with state feedback:

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}^T)\boldsymbol{x} + \boldsymbol{B}u$$

$$\text{or, } \dot{\boldsymbol{x}} = \boldsymbol{A}_{\boldsymbol{new}}\boldsymbol{x} + \boldsymbol{B}u \qquad (30)$$

where $\boldsymbol{A}_{\boldsymbol{new}}$ is the augmented matrix and $u = \boldsymbol{K}^T\boldsymbol{x} + \delta_{ref}$

21. The characteristic equation for the standard form of the second-order differential equation:

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0$$

The roots of the characteristic equation are: (31)

$$\lambda_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{1 - \xi^2} \cdot i$$

$$\lambda_{1,2} = \sigma \pm \omega_d$$