Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BDA 14103/ BWM 10203
PROGRAMME	:	1 BDD
EXAMINATION DATE	:	JUNE 2016/JULY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

PART A

Q1 (a) Under what conditions for the constant A,B,C,D is exact equation?

$$(Ax + By)dx + (Cx + Dy)dy = 0$$

Solve the equation.

(7 marks)

(b) Assume that the level of a certain hormone in the blood of a patient varies with time. Suppose that the time rate of change $y(t)$ is the difference between a sinusoidal input of a 24 hour period from the thyroid gland ($A + Bcos(\frac{\pi t}{12})$) and a continuous removal rate proportional to the level present $Ky(t)$.

(i) Set up a model for the hormone level in the blood.
(ii) Find its general solution.
(iii) Find the particular solution for the initial condition $y(0) = 0$.

(10 marks)

(c) For the equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + cy = 0$$

where c is constant, tell which value of c correspond to each of the three cases

(i) Two real roots,
(ii) Repeated real root,
(iii) Complex roots.

(3 marks)

Q2 (a) If $L\{f(t)\} = F(s)$ and a is a constant, prove the First Shifting Theorem that

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

(4 marks)

(b) By using the First Shifting Theorem, determine the Laplace transforms of the following function.

$$f(t) = e^{-2t}\cos(3t)$$

(4 marks)

(c) By using the Convolution Theorem or Partial Fraction, determine the inverse Laplace transforms of the following function.

$$\frac{2}{s^2(s-2)}$$

(12 marks)

PART B

Q3 (a) By using the method of **Variation Parameter**, obtain the general solution for:

$$y'' + y = tanx$$

(*Note*: $tanx = sinx/cosx$ and $sin^2x + cos^2x = 1$)

(10 marks)

(b) By using the method of **Undetermined Coefficients**, obtain the general solution for:

$$y'' + y' - 2y = \sin x$$

(10 marks)

Q4 (a) Obtain the general solution for the differential equation:

$$3(x^2 + y^2 + x)dx + 3xy\,dy = 0$$

(8 marks)

(b) Based on the Newton's Law of Cooling, the rate of changes of the temperature, *T*, of a body is proportional to the difference between *T* and the temperature of the surrounding medium, T_s, multiplies to thermal conductivity, *k*.

(i) Interpret this cooling law in the form of first order ordinary differential equation, and subsequently find its general solution.

(ii) If a thermometer with a reading of 15 °C is brought into a room which temperature is 25 °C, and the reading of the thermometer is 20 °C after two minutes later; how long will it take until the reading is 24 °C?

(12 marks)

Q5 A 3 cm length silver bar with a constant cross section area 1 cm^2 (density 10 g/cm^3, thermal conductivity 3 cal/(cm sec °C), specific heat 0.15 cal/(g °C)), is perfectly insulated laterally, with ends kept at temperature 0 °C and initial uniform temperature *f(x)* = 25 °C.

The heat equation is:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$$

(a) Show that $c^2 = 2$.

(2 marks)

(b) By using the method of separation of variable, and applying the boundary condition, prove that

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{3} e^{-\frac{2n^2\pi^2 t}{9}}$$

where b_n is an arbitrary constant.

(12 marks)

(c) By applying the initial condition, find the value of b_n.

(6 marks)

Q6 Let *f(x)* be a function of period 2π such that

$$f(x) = \frac{x}{2} \qquad 0 < x < 2\pi$$

(a) Sketch the graph of *f(x)* in the interval $0 < x < 4\pi$

(2 marks)

(b) Prove that the Fourier series for *f*(*x*) in the interval $0 < x < 2\pi$ is:

$$\frac{\pi}{2} - \left[\sin(x) + \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x) + \cdots\right]$$

(12 marks)

(c) By giving an appropriate value for *x*, demonstrate that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

(6 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

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FORMULAS

First Order Differential Equation

Type of ODEs	General solution
Linear ODEs: $y' + P(x)y = Q(x)$	$y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx}Q(x)dx + C\right\}$
Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$	$F(x,y) = \int f(x,y)dx$ $F(x,y) - \int\left\{\frac{\partial F}{\partial y} - g(x,y)\right\}dy = C$
Inexact ODEs: $M(x,y)dx + N(x,y)dy = 0$ $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Integrating factor; $i(x) = e^{\int f(x)dx}$ where $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $i(y) = e^{\int g(y)dy}$ where $g(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$	$\int iM(x,y)dx - \int\left\{\frac{\partial\left(\int iM(x,y)dx\right)}{\partial y} - iN(x,y)\right\}dy = C$

Characteristic Equation and General Solution for Second Order Differential Equation

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1e^{m_1x} + c_2e^{m_2x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1e^{mx} + c_2xe^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$

Method of Undetermined Coefficient

$g(x)$	y_p
Polynomial: $P_n(x) = a_nx^n + \ldots + a_1x + a_0$	$x^r(A_nx^n + \ldots + A_1x + A_0)$
Exponential: $e^{\alpha x}$	$x^r(Ae^{\alpha x})$
Sine or Cosine: $\cos\beta x$ or $\sin\beta x$	$x^r(A\cos\beta x + B\sin\beta x)$

Note: r is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

Method of Variation of Parameters

The particular solution for $y''+by'+cy=g(x)$ (b and c constants) is given by $y(x)=u_1y_1+u_2y_2$, where

$$u_1=-\int\frac{y_2g(x)}{W}dx,$$

$$u_2=\int\frac{y_1g(x)}{W}dx,$$

where

$$W=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

Laplace Transform

$$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}dt=F(s)$$

$f(t)$	$F(s)$
a	$\dfrac{a}{s}$
$t^n, n=1,2,3,...$	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
$\sin at$	$\dfrac{a}{s^2+a^2}$
$\cos at$	$\dfrac{s}{s^2+a^2}$
$\sinh at$	$\dfrac{a}{s^2-a^2}$
$\cosh at$	$\dfrac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t), n=1,2,3,...$	$(-1)^n\dfrac{d^nF(s)}{ds^n}$
$H(t-a)$	$\dfrac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$\dot{y}(t)$	$sY(s)-y(0)$
$\ddot{y}(t)$	$s^2Y(s)-sy(0)-y'(0)$

Fourier Series

Fourier series expansion of periodic function with period 2π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Half Range Series

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$