UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BDA 24003/BWM 20403
PROGRAMME	:	2 BDD
EXAMINATION DATE	:	JUNE 2015/ JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **SECTION A** B) ANSWER **TWO (2)** QUESTIONS IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

SECTION A

Q1 (a) A closed box has a dimension of 30 cm, 40 cm and 50 cm, respectively with the possible error 0.1 cm. Use partial derivatives to estimate the maximum possible error in calculating the following:

(i) Surface of the box

(5 marks)

(ii) Volume of the box

(5 marks)

(b) Find the derivative of $g(x) = \ln (x^{-4} + x^{+4})$.

(5 marks)

(c) Differentiate the following function $y = \sqrt[3]{x^2}(2x - x^2)$

(5 marks)

Q2 (a) Find domain and the range of each of the following function:

(i) $z = \cos(xy)$

(2 marks)

(ii) $z = 2 \sin (x + y)$

(2 marks)

(iii) $z = xy$

(2 marks)

(iv) $w = \ln (9 - x^2 - y^2 - z^2)$

(2 marks)

(b) By using double integrals, find the area of the regions enclosed by $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \frac{\pi}{4}$

(4 marks)

(c) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x+y)\,dydx$ by using polar coordinates.

(4 marks)

(d) Solve the following double integral $\int_0^2 \int_1^3 (xy^2)\,dydx$

(4 marks)

Q3 (a) Find the directional derivative of $f(x,y) = x^2 - 2xy + 3y^4$ at (2,0) in the direction of vector $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$.

(6 marks)

(b) Given the vector-valued function $r(t) = 4\cos t\ \mathbf{i} + 4\sin t\ \mathbf{j}$. Find its unit tangent vector and principal unit normal vector at $t = \pi/4$. Then, sketch the graph of $\mathbf{r}(t)$, $\mathbf{T}(\pi)$ and $\mathbf{N}(\pi)$ in the same axis.

(8 marks)

(c) Find the velocity, speed and acceleration of the particle at $t = \pi$ with the position vector $\mathbf{r}(t) = \cos t\ \mathbf{i} + e^t\ \mathbf{j} + 2t\ \mathbf{k}$

(6 marks)

SECTION B

Q4 (a) Solve the surface area of the part of the plane 3x + 2y + z = 6 that lies in the first octant.

(7 marks)

(b) Evaluate $\oint_C y^3 dx - x^3 dy$ (using Green's Theorem) where C are the two circles of radius 2 and radius 1 centered at the origin with positive orientation.

(6 marks)

(c) Evaluate $\int_C x^2 dx + xydy + z^2 dz$, where C is given by x = cos t, y= sin t and $z = t^2$, $0 < t < 2\pi$

(7 marks)

Q5 (a) Solve the velocity and position vector of a particle that has the given acceleration and the given initial velocity and position.

$\mathbf{a}(t) = \mathbf{i} + \sin 2t\mathbf{k}$, $\mathbf{v}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{r}(0) = 0$

(7 marks)

(b) Prove that the curvature of a circle of radius r is 1/r.

(7 marks)

(c) Solve arc length of helix, $\mathbf{r}(t) = 2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j} + t\,\mathbf{k}$ from (0, 2, 0) to (0, -2, π)

(6 Marks)

Q6 (a) Use Stokes' Theorem to evaluate $\oint_C F.dr$ where $F(x,y,z) = z^2 i + y^2 j + xk$ and C is the triangle with vertices (1,0,0), (0,1,0) and (0,0,1) with counter clockwise rotation.

(10 marks)

(b) Use Green's Theorem to evaluate $\oint_C \{(xy)dx + (x^2 y^3)dy\}$ where C is the triangle with vertices (0,0), (1,0) and (1,2) with positive orientation.

(10 marks)

- END OF QUESTION –

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FORMULAE

Total Differential

For function $z = f(x, y)$, the total differential of z, dz is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Relative Change

For function $z = f(x, y)$, the relative change in z is given by:

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

Implicit Differentiation

Suppose that z is given implicitly as a function $z = f(x, y)$ by an equation of the form $F(x, y, z) = 0$, where $F(x, y, f(x, y)) = 0$ for all (x, y) in the domain of f, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
 $f(x, y)$ has a local maximum value at (a,b)

b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
 $f(x, y)$ has a local minimum value at (a,b)

c. If $D < 0$
 $f(x, y)$ has a saddle point at (a,b)

d. If $D = 0$
 The test is inconclusive

Surface Area

$$\text{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$$

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Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$$\iint_R f(x,y)dA = \iint_R f(r,\theta) r\,dr\,d\theta$$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z) r\,dz\,dr\,d\theta$$

Spherical Coordinates:

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

In 2-D: Lamina

Given that $\delta(x,y)$ is a density of lamina

Mass, $m = \iint_R \delta(x,y)dA$, where

Moment of Mass

a. About x-axis, $M_x = \iint_R y\delta(x,y)dA$

b. About y-axis, $M_y = \iint_R x\delta(x,y)dA$

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Centre of Mass

Non-Homogeneous Lamina:

$$(\bar{x},\bar{y})=\left(\frac{M_y}{m},\frac{M_x}{m}\right)$$

Centroid

Homogeneous Lamina:

$$\bar{x}=\frac{1}{Area\,of\quad R}\iint_R x\,dA \text{ and } \bar{y}=\frac{1}{Area\,of\quad R}\iint_R y\,dA$$

Moment Inertia:

a. $I_y=\iint_R x^2\delta(x,y)\,dA$

b. $I_x=\iint_R y^2\delta(x,y)\,dA$

c. $I_o=\iint_R (x^2+y^2)\delta(x,y)\,dA$

In 3-D: Solid

Given that $\delta(x,y,z)$ is a density of solid

Mass, $m=\iiint_G \delta(x,y,z)\,dV$

If $\delta(x,y,z)=c$, where c is a constant, $m=\iiint_G dA$ is volume.

Moment of Mass

a. About *yz*-plane, $M_{yz}=\iiint_G x\delta(x,y,z)\,dV$

b. About *xz*-plane, $M_{xz}=\iiint_G y\delta(x,y,z)\,dV$

c. About *xy*-plane, $M_{xy}=\iiint_G z\delta(x,y,z)\,dV$

Centre of Gravity

$$(\bar{x},\bar{y},\bar{z})=\left(\frac{M_{yz}}{m},\frac{M_{xz}}{m},\frac{M_{xy}}{m}\right)$$

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Moment Inertia

a. About x-axis, $I_x = \iiint_G (y^2 + z^2)\delta(x,y,z)dV$

b. About y-axis, $I_y = \iiint_G (x^2 + z^2)\delta(x,y,z)dV$

c. About z-axis, $I_z = \iiint_G (x^2 + y^2)\delta(x,y,z)dV$

Directional Derivative

$D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Del Operator

$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$

Gradient of $\phi = \nabla\phi$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla.\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

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Let C is smooth curve defined by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The **Binormal Vector,** $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

Curvature

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

Radius of Curvature

$$\rho = \frac{1}{\kappa}$$

Green's Theorem

$$\oint_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

Gauss's Theorem

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS = \iiint_G \nabla \cdot \mathbf{F} dV$$

Stoke's Theorem

$$\oint_C \mathbf{F} \cdot dr = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$

Arc Length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}, t \in [a,b]$, hence, the **arc length,**

$$s = \int_a^b \|\mathbf{r}'(t)\| dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} dt$$