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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BDA 14103/ BWM 10203
PROGRAMME	:	1 BDD
EXAMINATION DATE	:	JUNE/JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

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PART A

Q1 (a) Show that the function

$$y = (x + 1) - \frac{1}{3}e^x$$

is a solution to the first-order initial value problem

$$\frac{dy}{dx} = y - x, \qquad y(0) = \frac{2}{3}$$

(6 marks)

(b) What are the different between equations (i) and (ii) below.

(i) $y'' - 2y' + y = 0$

(ii) $y'' + 4y = 2\sin(2x)$

(2 marks)

(c) Determine the particular solution of the differential equations (b)(i) that satisfies the given condition.

$$y(0) = 1, \qquad y'(0) = -1$$

(4 marks)

(d) Find the general solution of the differential equations for (b)(ii).

(8 marks)

Q2 (a) Determine whether the following differential equation are homogeneous or not.

$$\frac{dy}{dx} = \frac{x^2 + y^2}{(x - y)(x + y)}$$

(3 marks)

(b) If $L\{f(t)\} = F(s)$ and a is a constant, prove the First Shift Theorem that

$$\mathcal{L}\{e^{at} f(t)\} = F(s - a)$$

(5 marks)

(c) By using the Convolution Theorem, determine the inverse Laplace transforms of the following function.

$$\frac{2}{s^2(s^2 - 2)}$$

(12 marks)

PART B

Q3 (a) A model for forced spring mass system is given by:

$$my'' + cy' + ky = r(t)$$

(i) Find the steady state solution for case with value of $m = 1$, $c = 4$, $k = 4$ and $r(t) = 2\cos t$.

(6 marks)

(ii) Find the particular solution for the answer in Q3 (a)(i) that satisfies $y(0) = 1$ and $y'(0) = 2$.

(6 marks)

(b) An object of mass m is dropped from a certain elevated level. Find its velocity as a function of time t, assuming that the air resistance is proportional to the velocity of the object.

(8 marks)

Q4 According to Newton's Law of cooling, the rate at which body cools is proportional to the difference between the temperature of the body and that of the surrounding medium. Let θ represent the temperature of a body in a room (in ^{o}C). The initial body temperature is 70^{o}C. The room temperature is kept at a constant of 18^{o}C. The relation between body temperature and time are shown in the following function

$$\frac{d\theta}{dt} = -k(\theta - 18)$$

Where k is a constant.

(a) By using Laplace transform, determine the solution of $\theta(t)$

(10 marks)

(b) If the body cools from 70^{o}C to 57^{o}C in 5 minutes, calculate the value of constant k

(5 marks)

(c) Analyze how much times will it take for the body temperature to drop to 40^{o}C

(5 marks)

Q5 A 3 cm length silver bar with a constant cross section area 1 cm^2 (density 10 g/cm^3, thermal conductivity 3 cal/(cm sec°C), specific heat 0.15 cal/(g °C)), is perfectly insulated laterally, with ends kept at temperature 0 °C and initial uniform temperature $f(x) = 25°C$.

The heat equation is:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$$

(a) Show that $c^2 = 2$.

(2 marks)

(b) By using the method of separation of variable, and applying the boundary condition, prove that

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{3} e^{-\frac{2n^2\pi^2 t}{9}}$$

where b_n is an arbitrary constant.

(12 marks)

(c) By applying the initial condition, find the value of b_n.

(6 marks)

Q6 Let *f(x)* be a function of period 2π such that

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ \pi, & \pi < x < 2\pi \end{cases}$$

(a) Sketch the graph of *f(x)* in the interval $-2\pi < x < 2\pi$

(2 marks)

(b) Prove that the Fourier series for *f(x)* in the interval $0 < x < 2\pi$ is:

$$\frac{3\pi}{4} - \frac{2}{\pi}\left[\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right]$$
$$-\left[\sin x + \frac{1}{2}\sin x + \frac{1}{3}\sin 3x + \cdots\right]$$

(12 marks)

(c) By giving an appropriate value for x, demonstrate that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

(6 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

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FORMULAS

First Order Differential Equation

Type of ODEs	General solution
Linear ODEs: $y' + P(x)y = Q(x)$	$y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx} Q(x)dx + C\right\}$
Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$	$F(x,y) = \int f(x,y)dx$ $F(x,y) - \int \left\{\frac{\partial F}{\partial y} - g(x,y)\right\} dy = C$
Inexact ODEs: $M(x,y)dx + N(x,y)dy = 0$ $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Integrating factor; $i(x) = e^{\int f(x)dx}$ where $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $i(y) = e^{\int g(y)dy}$ where $g(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$	$\int iM(x,y)dx - \int \left\{ \frac{\partial\left(\int iM(x,y)dx\right)}{\partial y} - iN(x,y) \right\} dy = C$

Characteristic Equation and General Solution for Second Order Differential Equation

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1 e^{mx} + c_2 x e^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

Method of Undetermined Coefficient

$g(x)$	y_p
Polynomial: $P_n(x) = a_n x^n + \ldots + a_1 x + a_0$	$x^r(A_n x^n + \ldots + A_1 x + A_0)$
Exponential: e^{ax}	$x^r(Ae^{ax})$
Sine or Cosine: $\cos \beta x$ or $\sin \beta x$	$x^r(A\cos \beta x + B \sin \beta x)$

Note: r is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

Method of Variation of Parameters

The particular solution for $y''+by'+cy=g(x)$ (b and c constants) is given by $y(x)=u_1y_1+u_2y_2$, where

$$u_1=-\int\frac{y_2g(x)}{W}dx,$$

$$u_2=\int\frac{y_1g(x)}{W}dx,$$

where

$$W=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

Laplace Transform

$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}dt=F(s)$	
$f(t)$	$F(s)$
a	$\frac{a}{s}$
$t^n, n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t), n=1,2,3,\ldots$	$(-1)^n\frac{d^nF(s)}{ds^n}$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$\dot{y}(t)$	$sY(s)-y(0)$
$\ddot{y}(t)$	$s^2Y(s)-sy(0)-y'(0)$

Fourier Series

Fourier series expansion of periodic function with period 2 π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Half Range Series

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$