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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BDA 24003
PROGRAMME	:	BACHELOR DEGREE IN MECHANICAL ENGINEERING WITH HONOURS
EXAMINATION DATE	:	JUNE 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE** (3) QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **TWELVE** (12) PAGES

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PART A

Q1 (a) Find and sketch the domain of $f(x,y)=\ln(4-x^2-4y^2)$.

(5 marks)

(b) Given the function

$$f(x,y)=\begin{cases}\dfrac{x-y}{x+y}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

(i) Show that along the x-axis, $\lim\limits_{(x,y)\to(0,0)} f(x,y)=1$ and along the y-axis, $\lim\limits_{(x,y)\to(0,0)} f(x,y)=-1$.

(ii) Is the function $f(x,y)$ continuous at $(0,0)$? Give your reason.

(6 marks)

(c) A rectangular steel tank of length x, width y and height z is heated. If length x, width y and height z change from 10, 7 and 5 to 10.02, 6.97 and 5.01, respectively,

(i) Approximate the change in volume V by using the total differential.

(ii) Calculate the exact change in volume V.

(9 marks)

Q2 (a) Given the following double integrals

$$\int_0^1 \int_x^1 e^{y^2}\, dy dx$$

(i) Sketch the region of integration, R.

(ii) Interchange the order of integration to $dxdy$, and subsequently evaluate the double integrals in terms of $dxdy$.

(8 marks)

(b) A solid G is bounded above by the upper hemisphere $x^2+y^2+z^2=9$, and bounded below by the cone $z=\sqrt{x^2+y^2}$.

If the solid has density $\delta(x,y,z)=\dfrac{z}{x^2+y^2+z^2}$,

(i) By changing Cartesan coordinates to spherical coordinates, show that the density function:

$$\delta(x,y,z)=\frac{z}{x^2+y^2+z^2}=\frac{\cos\phi}{\rho}$$

(ii) By using the result in part (i), find the mass of the solid.

(12 marks)

PART B

Q3 (a) The position vector of a particle in the space is described by the parametric equations $x = e^{-t}, y = 2\cos 3t$ and $z = 2\sin 3t$.

(i) Find the velocity of the particle.

(ii) Find the acceleration of the particle.

(iii) Find the speed of the particle at $t = 0$.

(5 marks)

(b) Given the vector-valued function $\mathbf{r}(t) = 3\cos t\mathbf{i} + 3\sin t\mathbf{j} + 4t\mathbf{k}$.

(i) Find its unit tangent vector, $\mathbf{T}(t)$.

(ii) Find its principal unit normal vector, $\mathbf{N}(t)$.

(iii) Find its binomial vector, $\mathbf{B}(t)$.

(iv) Find its curvature κ.

(15 marks)

Q4 (a) Given that $\mathbf{F}(x,y,z) = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$.

(i) Show that $\mathbf{F}(x,y,z)$ is a conservative field.

(ii) Find its potential function ϕ which satisfies $\nabla\phi = \mathbf{F}$.

(iii) Subsequently, find the work done by force field $\mathbf{F}(x,y,z)$ on a particle moves from point $(1,-2,1)$ to $(3,1,4)$.

(10 marks)

(b) Verify the Green's theorem for line integral $\oint_C -2y\,dx + 3x\,dy$, where C is the close path defined by the semicircle, as shown in **FIGURES Q4**.

(*Note*: $\cos 2x = 2\cos^2 x - 1, \quad \cos 2x = 1 - 2\sin^2 x$)

(10 marks)

Q5 (a) Given that $w = e^{xy} + e^{-xy}$. Show that

$$\frac{\partial^2 w}{\partial x^2} + 2\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2} = (x+y)^2 w + 2(e^{xy} - e^{-xy})$$

(7 marks)

(b) Evaluate the surface integral

$$\iint_S xy\,dS$$

where S is part of the plane $x + y + z = 1$ which lies in the first octant.

(7 marks)

(c) By using double integrals, find the surface area of the portion of the surface $2x + 3y + z = 12$ that lies above the region $R = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le 3\}$.

(6 marks)

Q6 (a) Evaluate

$$\int_C xydx + (2x + y)dy$$

where C is part of the parabola $y = x^2$ from $(-1,1)$ to $(2,4)$.

(6 marks)

(b) By using Gauss's Theorem, evaluate $\iint_\sigma \mathbf{F} \cdot \mathbf{n} dS$ where $\mathbf{F}(x, y, z) = x\mathbf{i} + x^2 y\mathbf{j} + y^2 z\mathbf{k}$ and σ is the surface enclosed by cylinder $x^2 + y^2 = 4$ lying in the first octant, and between plane $z = 0$ and $z = 4$.

(7 marks)

(c) Find the volume of the solid bounded by paraboloid $z = x^2 + y^2$, below by xy-plane and the side by cylinder $x^2 + y^2 = 9$.

(7 marks)

- END OF QUESTION -

FINAL EXAMINATION

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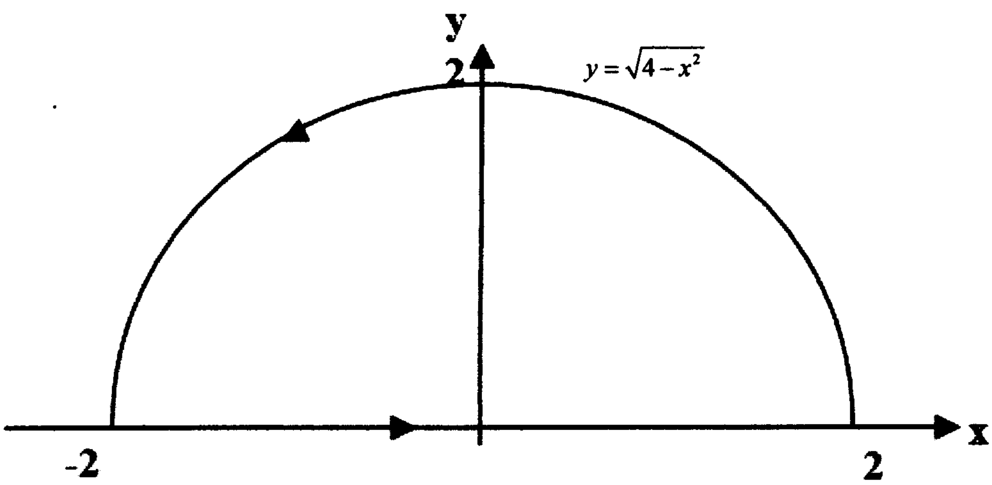
COURSE	: ENGINEERING MATHEMATICS III	COURSE CODE	: BDA24003



FIGURES Q4

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FORMULAE

Total Differential

For function $w = f(x,y,z)$, the total differential of *w*, *dw* is given by:

$$dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

Implicit Differentiation

Suppose that z is given implicitly as a function $z = f(x,y)$ by an equation of the form $F(x,y,z) = 0$, where $F(x,y,f(x,y)) = 0$ for all (x,y) in the domain of *f*, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \quad \text{and} \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
   $f(x,y)$ has a local maximum value at (a,b)
b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
   $f(x,y)$ has a local minimum value at (a,b)
c. If $D < 0$
   $f(x,y)$ has a saddle point at (a,b)
d. If $D = 0$
   The test is inconclusive.

Surface Area

$$\textit{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\,dA$$

Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$$\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)rdzdrd\theta$$

Spherical Coordinates:

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi d\rho d\phi d\theta$$

In 2-D: Lamina

Mass, $m = \iint_R \delta(x,y)dA$, where $\delta(x,y)$ is a density of lamina.

Moment of Mass

a. About y-axis, $M_y = \iint_R x\delta(x,y)dA$,

b. About x-axis, $M_x = \iint_R y\delta(x,y)dA$,

Centre of Mass

Non-Homogeneous Lamina:

$$(\bar{x},\bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

Centroid

Homogeneous Lamina:

$$\bar{x} = \frac{1}{Area\, of\ \ R}\iint_R xdA \qquad \text{and} \qquad \bar{y} = \frac{1}{Area\, of\ \ R}\iint_R ydA$$

Moment Inertia:

a. $I_y = \iint\limits_R x^2 \delta(x,y)dA$

b. $I_x = \iint\limits_R y^2 \delta(x,y)dA$

c. $I_o = \iint\limits_R (x^2 + y^2)\delta(x,y)dA$

In 3-D: Solid

Mass, $m = \iiint\limits_G \delta(x,y,z)dV$

If $\delta(x,y,z) = c$, where c is a constant, $m = \iiint\limits_G dA$ is volume.

Moment of Mass

a. About yz-plane, $M_{yz} = \iiint\limits_G x\delta(x,y,z)dV$

b. About xz-plane, $M_{xz} = \iiint\limits_G y\delta(x,y,z)dV$

c. About xy-plane, $M_{xy} = \iiint\limits_G z\delta(x,y,z)dV$

Centre of Gravity

$$(\bar{x},\bar{y},\bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m}\right)$$

Moment Inertia

a. About x-axis, $I_x = \iiint\limits_G (y^2 + z^2)\delta(x,y,z)dV$

b. About y-axis, $I_y = \iiint\limits_G (x^2 + z^2)\delta(x,y,z)dV$

c. About z-axis, $I_z = \iiint\limits_G (x^2 + y^2)\delta(x,y,z)dV$

Directional Derivative

$D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Gradient of $\phi = \nabla\phi$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector field, hence,

The **Divergence** of $\mathbf{F} = \nabla.\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is smooth curve defined by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The **Binormal Vector,** $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

Curvature

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

Radius of Curvature

$$\rho = \frac{1}{\kappa}$$

Green Theorem

$$\oint_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

Gauss Theorem

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS = \iiint_G \nabla \cdot \mathbf{F} dV$$

Stoke's Theorem

$$\oint_C \mathbf{F} \cdot dr = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$

Arc Length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}, t \in [a,b]$, hence, the **arc length,** $s = \int_a^b \|\mathbf{r}'(t)\| dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} dt$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}, t \in [a,b]$, hence, the **arc length,**

$$s = \int_a^b \|\mathbf{r}'(t)\| dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} dt$$