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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BDA 14103
PROGRAMME	:	BDD
EXAMINATION DATE	:	JUNE 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

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PART A

Q1 (a) By using method of variation parameter, obtain the general solution for:

$$y''+y=2\sin x$$

(*Note*: $\sin 2x = 2\cos x \sin x \qquad \cos 2x = 1-2\sin^2 x$)

(8 marks)

(b) A model for forced spring mass system is given by:

$$y''+8y'+8y=4\cos t$$

(i) State the value of the mass, spring constant and damping constant for this system.

(ii) If another 1 kg mass is added to this system, what will happen to this differential equation?

(iii) Find the steady state solution for case with 2kg mass.

(iv) Find the particular solution for the answer in Q2(b)(iii) that satisfies $y(0)=1$ and $y'(0)=2$.

(12 marks)

Q2 (a) Find the Laplace transform of $f(t) = \cos 2t$.

(2 marks)

(b) Express $\dfrac{3}{s(s^2+4)}$ in partial fraction.

(8 marks)

(c) By using the obtained result in Q3(b), show that

$$\mathcal{L}^{-1}\left\{\frac{3}{s(s^2+4)}e^{-4s}\right\} = \frac{3}{4}H(t-4) - \frac{3}{4}H(t-4)\cos(2(t-4)).$$

(3 marks)

(d) By using the obtained results in Q3(a)-(c), solve the following initial value problem.

$$y'' + 4y = f(t), \quad y(0) = 1, \quad y'(0) = 0, \text{ with}$$

$$f(t) = \begin{cases} 0, & for \quad 0 \le t < 4 \\ 3, & for \quad t \ge 4 \end{cases}.$$

(7 marks)

PART B

Q3 A periodic function is defined as:

$$f(x) = \begin{cases} -1 & , \quad -\pi < x < 0 \\ 0 & , \quad x = 0 \\ 1 & , \quad 0 < x < \pi \end{cases}$$

$$f(x) = f(x + 2\pi)$$

(a) Determine whether the given function is even, odd or neither.

(2 marks)

(b) Prove that the corresponding Fourier series to this periodic function is given by:

$$\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin[(2n-1)x]}{2n-1}, \qquad -\pi < x < \pi$$

(12 marks)

(c) By choosing an appropriate value for x, show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

(6 marks)

Q4 A bar of silver of length 3 cm and constant across section of area 1cm^2 (density 10g/cm^3, thermal conductivity 1.5 cal/(cm sec°C), specific heat 0.075 cal/(g °C), is perfectly insulated laterally, with ends kept at temperature 0°C and initial uniform temperature $f(x) = 25°C$.

Given the heat equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$$

(a) Show that $c^2 = 2$.

(2 marks)

(b) By using the method of separation of variable, and applying the boundary condition, prove that

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{3} e^{-\frac{2n^2\pi^2 t}{9}}$$

where b_n is an arbitrary constant.

(12 marks)

(c) By applying the initial condition, find the value of b_n.

(6 marks)

Q5 (a) Obtain the general solution for the differential equation:

$$(x^2 + y^2 + x)dx + xydy = 0$$

(8 marks)

(b) Newton's Law of Cooling states that the rate of changes of the temperature, T, of a body is proportional to the difference between T and the temperature of the surrounding medium, T_s, multiplies to thermal conductivity k.

(i) Interpret this cooling law in the form of first order ordinary differential equation, and subsequently find its general solution.

(ii) If a thermometer with a reading of 10°C, is brought into a room whose temperature is 23°C, and the reading of the thermometer is 18°C after two minutes later, how long will it take until the reading is 23°C?

(12 marks)

Q6 (a) Solve the differential equation:

$$x\frac{dy}{dx} - y = \frac{x}{x+1}$$

(6 marks)

(b) By using the method of Laplace transform, solve the initial value problem of:

$$y'' + 5y' + 6y = e^{-t}$$

with the initial condition $y(0) = y'(0) = 0$.

(14 marks)

FINAL EXAMINATION

SEMESTER / SESSION	: SEM II /20122013	PROGRAMME	: 1 BDD
COURSE	: ENGINEERING MATHEMATICS II	COURSE CODE	: BDA14103

FORMULAS

Characteristic Equation and General Solution for Second Order Differential Equation

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1e^{m_1x} + c_2e^{m_2x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1e^{mx} + c_2xe^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

Method of Undetermined Coefficient

$g(x)$	y_p
Polynomial: $P_n(x) = a_nx^n + \ldots + a_1x + a_0$	$x^r(A_nx^n + \ldots + A_1x + A_0)$
Exponential: e^{ax}	$x^r(Ae^{ax})$
Sine or Cosine: $\cos \beta x$ or $\sin \beta x$	$x^r(A\cos \beta x + B\sin \beta x)$

Note: r is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

Method of Variation of Parameters

The particular solution for $y'' + by' + cy = g(x)$ (b and c constants) is given by $y(x) = u_1y_1 + u_2y_2$, where

$$u_1 = -\int \frac{y_2g(x)}{W}dx,$$

$$u_2 = \int \frac{y_1g(x)}{W}dx,$$

where

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

Laplace Transform

$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$	
$f(t)$	$F(s)$
a	$\frac{a}{s}$
$t^n, n = 1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t), n = 1,2,3,\ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$\dot{y}(t)$	$sY(s)-y(0)$
$\ddot{y}(t)$	$s^2Y(s)-sy(0)-y'(0)$

Fourier Series

Fourier series expansion of periodic function with period 2π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Half Range Series

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

- END OF QUESTION -