UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2011/2012

COURSE NAME	:	FINITE ELEMENT METHOD
COURSE CODE	:	BDA 4033
PROGRAMME	:	BACHELOR OF MECHANICAL ENGINEERING WITH HONOURS
EXAMINATION DATE	:	JUNE 2012
DURATION	:	2 HOURS and 30 MINUTES
INSTRUCTION	:	**PART A:** ANSWER <u>ALL</u> QUESTIONS **PART B:** ANSWER **<u>TWO (2)</u>** QUESTIONS ONLY

THIS PAPER CONTAINS ELEVEN (11) PAGES

PART A - Basic Comprehension and Understanding
(ANSWER ALL Questions)

Q1 The finite element model of a structural component is represented by the following simultaneous equations

$$\begin{aligned}&4u_1 - 4u_2 = 0\\&-4u_1 + 16u_2 - 8u_3 = -30\\&-8u_2 + 15u_3 - 3u_4 = 30\\&-3u_3 - 3u_4 = 0\end{aligned}$$

(a) Rewrite the simultaneous equations in matrix form.

(5 marks)

(b) If the specified conditions (constraints) are $u_1 = 0$ and $u_4 = 0.3$, reduce the number of equations (the order of the matrix) by using direct elimination method:

  i. Modify the form of matrix equation after considering the specified conditions (constraints).
  ii. Solve the displacement of u_2 and u_3.

(10 marks)

(c) Use Penalty Method:

  i. Modifies the form of matrix equation after considering the specified conditions (constraints).
  ii. From the matrix equation, show that $u_4 = 0.3$.
  iii. Solve the reaction forces of the structure.

(10 marks)

Q2 For all the structural problems below, list all their geometrical and mechanical properties.

(a) Axial structure.

(3 marks)

(b) Beam structure.

(3 marks)

(c) Torsional structure.

(3 marks)

(d) Space frame structure.

(6 marks)

PART B - Analysis and Applications
(ANSWER TWO Questions ONLY)

Q3 A plane truss shown in the **FIGURE Q3** is supported by the spring with a spring constant k = 2000 kN/m at node 1. In every element, the modulus of elasticity is E = 210 GPa and the cross section is $A = 5 \times 10^{-4}$ m^2. The structure is vertically loaded by 50 kN at node 1. The data of element numbers and the node numbers are as follows:

Element	Node	Node
1	1	2
2	1	3
3	1	4

(a) Generate the element stiffness matrix for each element.

(10 marks)

(b) Construct the global stiffness matrix and the global force vector before considering any constraints and present it in the form of $[K_{\text{global}}]\ [x] = [F_{\text{global}}]$.

(5 marks)

(c) Using the elimination approach, solve for displacement matrix $[x]$.

(5 marks)

(d) Evaluate the stress in element 1 & 2.

(10 marks)

Q4 **FIGURE Q4** shows a two dimensional structure isolated on two edges; edge 3-5 and edge 5-4. Temperature of node 4 is maintained at 30°C and temperature of node 5 is maintained at 50°C. The edge 4-2-1-3 is exposed to the air with temperatures and convection coefficients as shown in **FIGURE Q4**. The conductivity of the materials is as shown in **FIGURE Q4.** The data of element numbers and the node numbers are as follows:

Element	Connecting Nodes
1	1-3-2
2	2-3-5-4

(a) Generate the conductance matrix of each element.

(10 marks)

(b) Compute the thermal load vector of each element.

(5 marks)

(c) Construct the global system matrix equation $[Kc]\ [T] = [F]$ after considering all constraints.

(5 marks)

(d) Evaluate the temperature distribution of the structure.

(10 marks)

Q5 A thin plate with thickness 0.013 m was loaded as shown in **FIGURE Q5**. The material's modulus of elasticity, E is 200 GPa and Poisson ratio, υ of 0.3.

(a) Determine whether this is a plane stress or plane strain problem and generate the stiffness matrix, $[K]$ from elasticity matrix, $[E]$ and the strain displacement matrix, $[B]$.

(10 marks)

(b) Construct the global stiffness matrix and the global force vector before considering any constraints and present it in the form of $[K]\ [x] = [F]$.

(5 marks)

(c) Solve the displacement vector $[x]$

(5 marks)

(d) Evaluate the elemental stress of the plate.

(10 marks)

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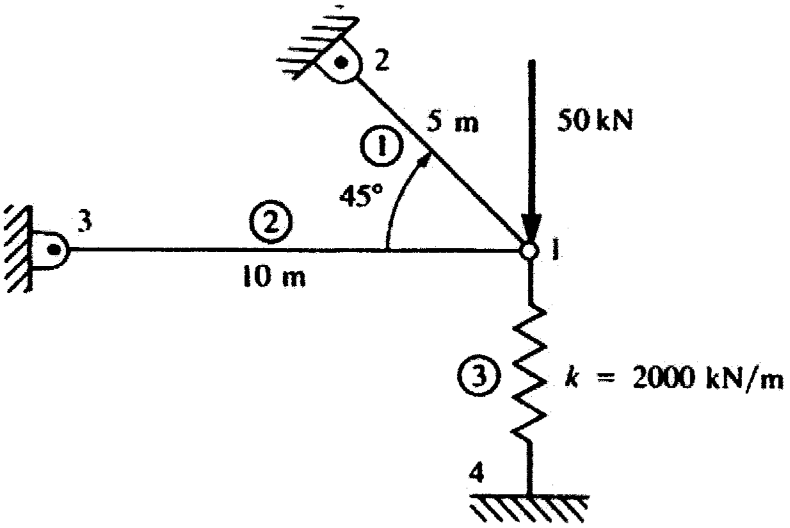

FIGURE Q3

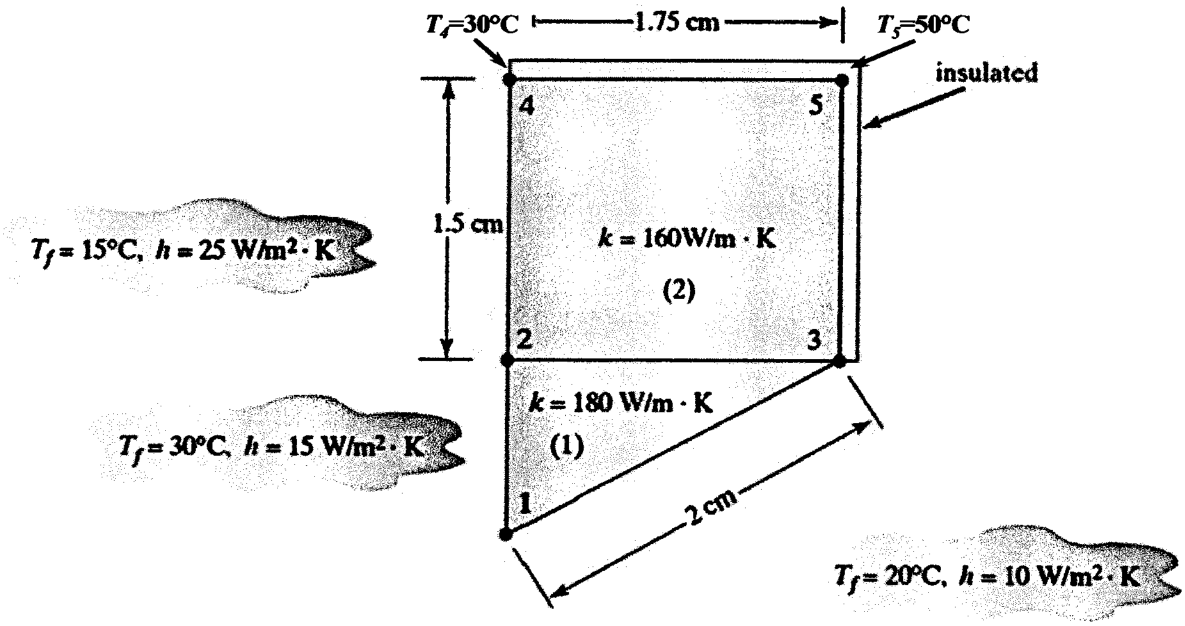

FIGURE Q4

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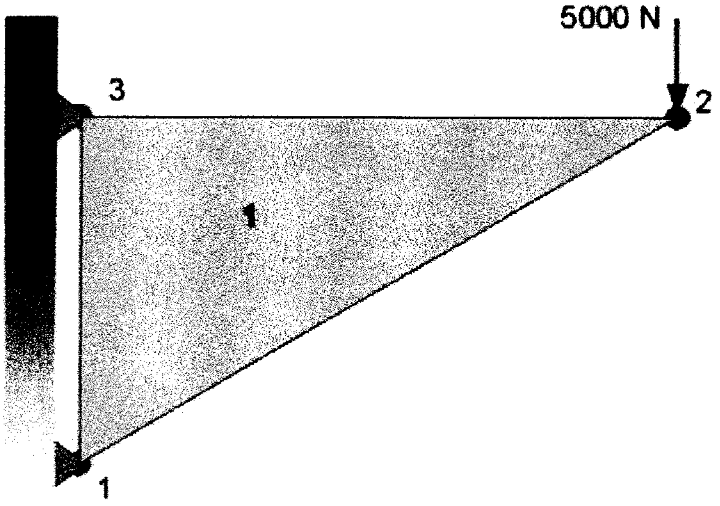

Table 1: Node data

Node	x(m)	y(m)
1	0	0
2	0.13	0.05
3	0	0.05

FIGURE Q5

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USEFUL EQUATIONS

TRUSS ELEMENT:

$$[K^e] = \frac{A^e E^e}{L^e} \begin{array}{c} \\ \left[\begin{array}{cccc} u_i & v_i & u_j & v_j \\ C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{array} \right] \end{array} \begin{array}{c} \\ u_i \\ v_i \\ u_j \\ v_j \end{array}$$

$$C = \frac{x_j - x_i}{L^e} \qquad S = \frac{y_j - y_i}{L^e} \qquad L^e = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

$$u_i' = C\,u_i + S\,v_i$$

$$u_j' = C\,u_j + S\,v_j$$

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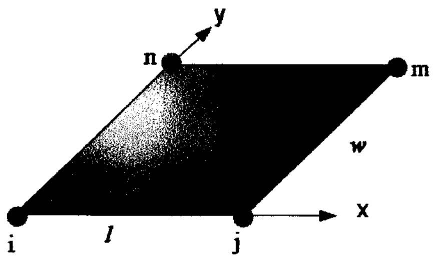
USEFUL EQUATIONS

BILINEAR RECTANGULAR HEAT TRANSFER:

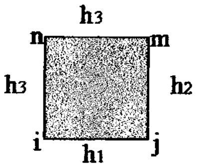


$$[K^e]=\frac{k_x w}{6l}\begin{bmatrix}2 & -2 & -1 & 1\\ -2 & 2 & 1 & -1\\ -1 & 1 & 2 & -2\\ 1 & -1 & -2 & 2\end{bmatrix}+\frac{k_y l}{6w}\begin{bmatrix}2 & 1 & -1 & -2\\ 1 & 2 & -2 & -1\\ -1 & -2 & 2 & 1\\ -2 & -1 & 1 & 2\end{bmatrix}$$

Additional conductance matrix due to convection

$$[K^e]=\frac{h_3 L_{mn}}{6}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 2 & 1\\ 0 & 0 & 1 & 2\end{bmatrix}$$

$$[K^e]=\frac{h_4 L_{ni}}{6}\begin{bmatrix}2 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 2\end{bmatrix}$$



$$[K^e]=\frac{h_2 L_{jm}}{6}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 2 & 1 & 0\\ 0 & 1 & 2 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$[K^e]=\frac{h_1 L_{ij}}{6}\begin{bmatrix}2 & 1 & 0 & 0\\ 1 & 2 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

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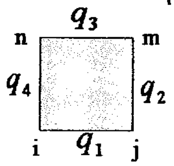
USEFUL EQUATIONS

Thermal load heat flux

$$\{F^e\} = \frac{q_3 l_{mn}}{2}\begin{Bmatrix}0\\0\\1\\1\end{Bmatrix}$$

$$\{F^e\} = \frac{q_4 l_{ni}}{2}\begin{Bmatrix}1\\0\\0\\1\end{Bmatrix}$$



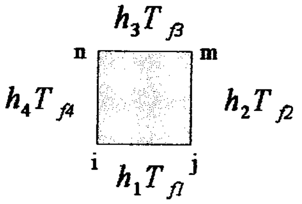
$$\{F^e\} = \frac{q_2 l_{jm}}{2}\begin{Bmatrix}0\\1\\1\\0\end{Bmatrix}$$

$$\{F^e\} = \frac{q_1 l_{ij}}{2}\begin{Bmatrix}1\\1\\0\\0\end{Bmatrix}$$

Thermal load due to heat loss

$$\{F^e\} = \frac{h_3 T_{f3} L_{mn}}{2}\begin{Bmatrix}0\\0\\1\\1\end{Bmatrix}$$

$$\{F^e\} = \frac{h_4 T_{f4} L_{ni}}{2}\begin{Bmatrix}1\\0\\0\\1\end{Bmatrix}$$



$$\{F^e\} = \frac{h_2 T_{f2} L_{jm}}{2}\begin{Bmatrix}0\\1\\1\\0\end{Bmatrix}$$

$$\{F^e\} = \frac{h_1 T_{f1} L_{ij}}{2}\begin{Bmatrix}1\\1\\0\\0\end{Bmatrix}$$

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USEFUL EQUATIONS

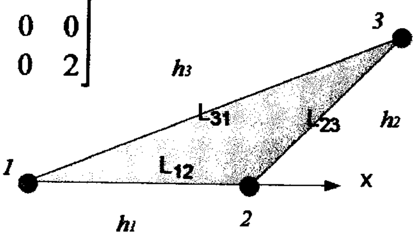
TRIANGULAR HEAT TRANSFER:

$$[K^e]=\frac{k_X}{4A}\begin{bmatrix} y_{23}^2 & y_{31}y_{23} & y_{12}y_{23} \\ y_{23}y_{31} & y_{31}^2 & y_{12}y_{31} \\ y_{23}y_{12} & y_{31}y_{12} & y_{12}^2 \end{bmatrix}+\frac{k_Y}{4A}\begin{bmatrix} x_{32}^2 & x_{13}x_{32} & x_{21}x_{32} \\ x_{32}x_{13} & x_{13}^2 & x_{21}x_{13} \\ x_{32}x_{21} & x_{13}x_{21} & x_{21}^2 \end{bmatrix}$$

$$y_{ij}=y_i-y_j \qquad x_{ij}=x_i-x_j$$

Additional conductance matrix due to convection

$$[K^e]=\frac{h_3L_{31}}{6}\begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

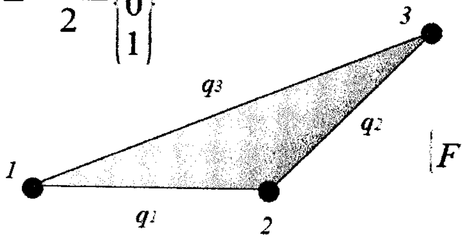


$$[K^e]=\frac{h_2L_{23}}{6}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$[K^e]=\frac{h_1L_{12}}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thermal load heat flux

$$\{F^e\}=\frac{q_3L_{31}}{2}\begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix}$$



$$\{F^e\}=\frac{q_2L_{23}}{2}\begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix}$$

$$\{F^e\}=\frac{q_1L_{12}}{2}\begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

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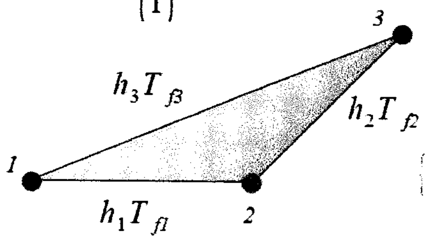
USEFUL EQUATIONS

Thermal load due to heat loss

$$\{F^e\} = \frac{h_3 T_{f3} L_{31}}{2} \begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix}$$



$$\{F^e\} = \frac{h_2 T_{f2} L_{23}}{2} \begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix}$$

$$\{F^e\} = \frac{h_1 T_{f1} L_{12}}{2} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

CST ELEMENT
Plane Stress:

$$[E] = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Plane Strain:

$$[E] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

$$[B] = \frac{1}{2A} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \qquad A = \frac{1}{2} \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

$$x_{ij} = x_i - x_j \qquad y_{ij} = y_i - y_j$$