UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER 1
SESSION 2010/2011

COURSE	:	FINITE ELEMENT METHODS
COURSE CODE	:	BDA 4033
PROGRAM	:	3 BDD / 4BDD
DATE	:	November/December 2010
DURATION	:	2 HOURS and 30 MINUTES
INSTRUCTION	:	PART A: ANSWER ALL QUESTIONS PART B: ANSWER 2 QUESTIONS

THIS PAPER CONSISTS OF (9) PAGES

PART A - Basic Comprehension and Understanding
(3 Questions, ANSWER ALL Questions)

Q1 The finite element model of a structural component is represented by the following simultaneous equations

$$\begin{bmatrix} 47628 & -12096 & -16128 & 12096 & -31500 & 0 \\ -12096 & 9072 & 12096 & -9072 & 0 & 0 \\ -16128 & 12096 & 16128 & -12096 & 0 & 0 \\ 12096 & -9072 & -12096 & 9072 & 0 & 0 \\ -31500 & 0 & 0 & 0 & 3150 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} -1000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

(a) If the specified conditions (constraints) are $v_1 = -0.1$ and $u_2 = u_3 = v_3 = 0$, reduce the number of equations (the order of the matrix) by using direct elimination method:

i. Write the form of matrix equation after considering the specified conditions (constraints)

ii. Find the displacement of unconstrained nodes

(b) By using Penalty Method,

i. Write the form of matrix equation after considering the specified conditions (constraints)

ii. Explain (in brief) how to find the displacement vector

(15 marks)

Q2 Figure Q2 shows a beam structure. The beam is loaded by distributed load along the span and at the middle of the beam, a concentrated load is also applied. This structure is made of Aluminum with Young's modulus of $70000\,\mathrm{N/mm^2}$.
By dividing the structure into 2 elements as shown Figure Q2,

(a) Write the stiffness matrix and the force vector of each element

(b) Write the global assembled stiffness matrix and the force vector before considering any constraints

(c) Write the global assembled stiffness matrix and the force vector after considering any constraints, using either Direct Elimination Method or Penalty Method

(d) Explain (in brief) how to find the displacement vector

(20 marks)

Q3 Illustrate SOLID elements: Hexahedron 8 nodes, Tetrahedron 4 nodes, Pyramid 5 nodes, and Wedge 6 nodes. Explain in brief the degrees of freedom of every node in solid elements.

(5 marks)

PART B - Analysis and Applications
(Answer 2 Questions ONLY)

Q4 Aluminum fins of a rectangular profile, as shown in the Figure Q4, are used to remove heat from a surface whose temperature is $100°C$. The temperature of the ambient air is $20°C$. The thermal conductivity of the aluminum is $k = 160\,{}^{W}/_{m°C}$. The natural convective heat transfer coefficient associated with the surrounding air is $h = 30\,{}^{W}/_{m^2\,°C}$. The fins are 80 mm long, 5 mm wide, and 1 mm thick.
Considering the finite element model as shown in Figure Q4,

(a) Calculate the conductance matrix and the thermal load vector of each element

(b) Write the global conductance matrix and the global thermal load vector after considering all constraints, by implementing either Direct Elimination Method or Penalty Method

(c) Explain how to determine the temperature distribution vector of the fin (no calculation required!)

(30 marks)

Q5 A two-dimensional plate is isolated in upper and lower edges, so no convection heat transfer is allowed. This plate has been modelled as 2 quadrilateral elements as illustrated in Figure Q5. The left edge is maintained at $50°C$ and heat flux $q = 100\,{}^{W}/_{m^2}$ is applied on this edge. The right edge is expose to ambient temperature of $25°C$ with heat transfer convection coefficient, $h = 30\,{}^{W}/_{m^2\,°C}$. The thermal conductivity of the plate is $k = 25\,{}^{W}/_{m°C}$
By following the node and element definitions as seen in Figure Q5,

(a) Calculate the conductance matrix and the thermal load vector of every element

(b) Do you have any thermal constraints in this problem?

(c) Considering your answer in (b), write the global conductance matrix and the global thermal load vector that will be used to solve the temperature vector. You can use either elemination method or penalty method

(d) Explain how to determine the temperature distribution vector of the fin (no calculation required!)

(30 marks)

Q6 A simplified diagram of piping network is illustrated in Figure Q6. In this figure, the definition of elements and corresponding nodes are also drawn. If the dynamic viscosity of the fluid is known as $\mu = 0.25\,{}^{Ns}/_{m^2}$ and the pressures at the node 1 and node 6 are also known as 40000 Pa and −4000 Pa, respectively,

(a) Write the flow-resistance matrix and the flow rate vector of each element

(b) Write the global flow-resistance matrix and the global flow rate vector before considering any constraints

(c) Considering the constraints, modify the global flow-resistance matrix and the global flow rate vector by implementing either direct elimination method or penalty method. You need to specify what is the method you are using!

(d) Explain (and write the equation, if necessary) how to calculate pressure of the nodes (no calculation required!)

(e) Explain how to calculate the flow distribution in every element (no calculation required!)

(30 marks)

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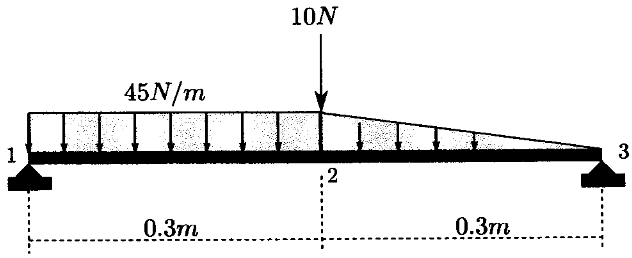
No figure is required for Q1

FIGURE Q1: Empty figure



Element	Node i	Node j
1	1	2
2	2	3

Cross section profile

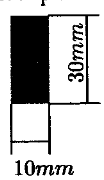


FIGURE Q2: Beam structure

No figure is required for Q3

FIGURE Q3: Empty figure

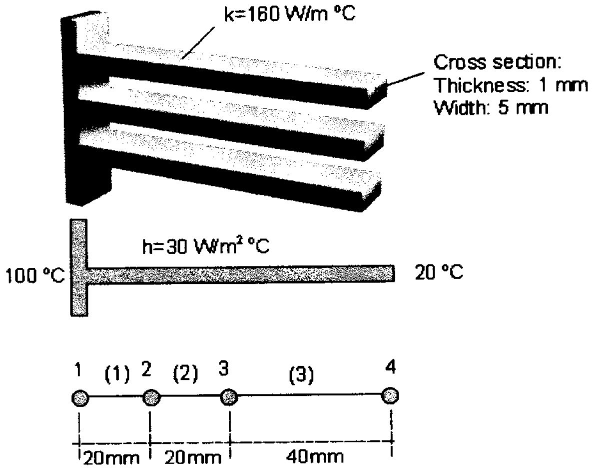


FIGURE Q4: Fin problem

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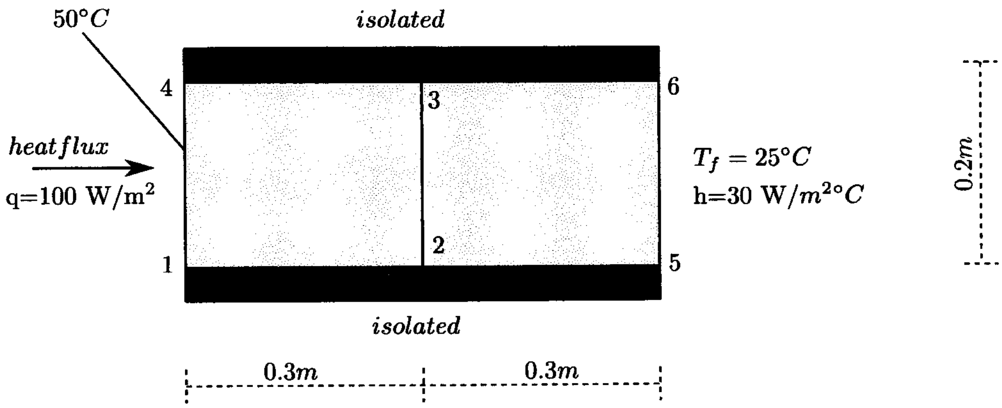

FIGURE Q5: Heat transfer 2D

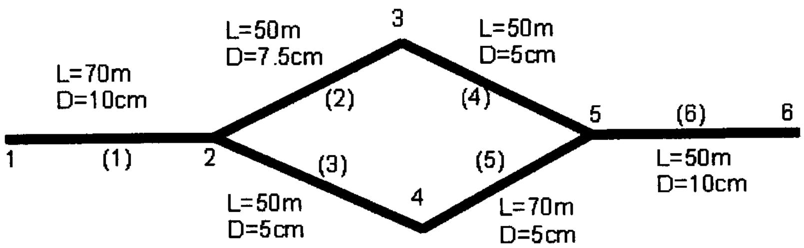

FIGURE Q6: Pipe network

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USEFUL EQUATIONS

Axial element: element e, node i and j, degree of freedom: u

$$[k^e] = \begin{array}{cc} \begin{array}{cc} u_i & u_j \end{array} & \\ \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} & \begin{array}{c} u_i \\ u_j \end{array} \end{array}$$

$$k = \frac{A^e E^e}{L^e}$$

Truss 2D element: element e, node i and j, degrees of freedom: u and v

$$[K^e] = \frac{A^e E^e}{L^e} \begin{array}{cc} \begin{array}{cccc} u_i & v_i & u_j & v_j \end{array} & \\ \begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} & \begin{array}{c} u_i \\ v_i \\ u_j \\ v_j \end{array} \end{array}$$

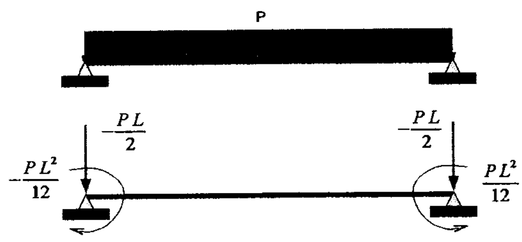
$$C = \frac{x_j - x_i}{L^e} \qquad S = \frac{y_j - y_i}{L^e} \qquad L^e = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

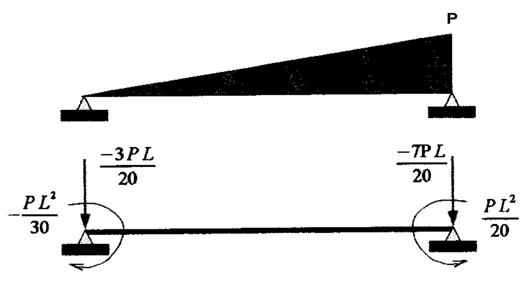
$$u_i^{'} = C\,u_i + S\,v_i$$

$$u_j^{'} = C\,u_j + S\,v_j$$

Beam element: element e, node i and j, degrees of freedom: v and θ

$$[K^e] = \frac{E^e I^e}{(L^e)^3} \begin{array}{cc} \begin{array}{cccc} v_i & \theta_i & v_j & \theta_j \end{array} & \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} & \begin{array}{c} v_i \\ \theta_i \\ v_j \\ \theta_j \end{array} \end{array}$$





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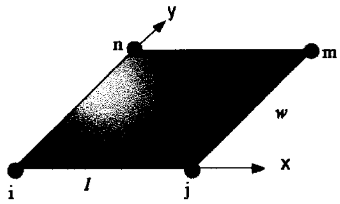
USEFUL EQUATIONS

BILINEAR RECTANGULAR HEAT TRANSFER

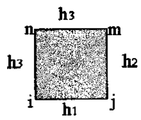


$$[K^e]=\frac{k_x w}{6l}\begin{bmatrix}2 & -2 & -1 & 1\\ -2 & 2 & 1 & -1\\ -1 & 1 & 2 & -2\\ 1 & -1 & -2 & 2\end{bmatrix}+\frac{k_y l}{6w}\begin{bmatrix}2 & 1 & -1 & -2\\ 1 & 2 & -2 & -1\\ -1 & -2 & 2 & 1\\ -2 & -1 & 1 & 2\end{bmatrix}$$

Additional conductance matrix due to convection

$$[K^e]=\frac{h_3 L_{mn}}{6}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 2 & 1\\ 0 & 0 & 1 & 2\end{bmatrix}$$

$$[K^e]=\frac{h_4 L_{ni}}{6}\begin{bmatrix}2 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 2\end{bmatrix}$$



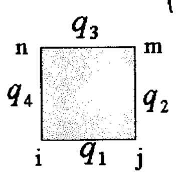
$$[K^e]=\frac{h_2 L_{jm}}{6}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 2 & 1 & 0\\ 0 & 1 & 2 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$[K^e]=\frac{h_1 L_{ij}}{6}\begin{bmatrix}2 & 1 & 0 & 0\\ 1 & 2 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Thermal load heat flux

$$\{F^e\}=\frac{q_3 l_{mn}}{2}\begin{Bmatrix}0\\ 0\\ 1\\ 1\end{Bmatrix}$$

$$\{F^e\}=\frac{q_4 l_{ni}}{2}\begin{Bmatrix}1\\ 0\\ 0\\ 1\end{Bmatrix}$$



$$\{F^e\}=\frac{q_2 l_{jm}}{2}\begin{Bmatrix}0\\ 1\\ 1\\ 0\end{Bmatrix}$$

$$\{F^e\}=\frac{q_1 l_{ij}}{2}\begin{Bmatrix}1\\ 1\\ 0\\ 0\end{Bmatrix}$$

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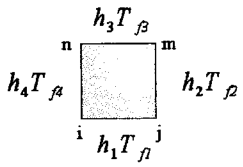
USEFUL EQUATIONS

Thermal load due to heat loss

$$\{F^e\} = \frac{h_3 T_{f3} L_{mn}}{2}\begin{Bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{Bmatrix}$$

$$\{F^e\} = \frac{h_4 T_{f4} L_{ni}}{2}\begin{Bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{Bmatrix}$$



$$\{F^e\} = \frac{h_2 T_{f2} L_{jm}}{2}\begin{Bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{Bmatrix}$$

$$\{F^e\} = \frac{h_1 T_{f1} L_{ij}}{2}\begin{Bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$

PIPING NETWORK:
Flow resistance matrix

$$[R] = \frac{\pi D^4}{128 L\mu}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

ONE DIMENSIONAL HEAT TRANSFER
Conductance matrix:

$$[k]^{(e)} = \underbrace{\begin{bmatrix} h_{\infty i}A & 0 \\ 0 & 0 \end{bmatrix}}_{i-\text{end convection}} + \underbrace{\frac{kA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}_{\text{conduction}} + \underbrace{\frac{h_\infty pL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}_{\text{convection}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & h_{\infty j}A \end{bmatrix}}_{j-\text{end convection}}$$

Thermal load vector:

$$\{f\}^{(e)} = \underbrace{h_{\infty i}AT_{\infty i}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{i-\text{end heat loss}} + \underbrace{\frac{QAL}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\text{heat source}} + \underbrace{\frac{qPL}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\text{heat flux}} + \underbrace{\frac{h_\infty TPL}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\text{convection}} + \underbrace{h_{\infty j}AT_{\infty j}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{j-\text{end heat loss}}$$