UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER 2
SESSION 2009/2010

SUBJECT	:	FINITE ELEMENT METHOD
SUBJECT CODE	:	BDA 4033
COURSE	:	4 BDD
DATE	:	____APRIL/MAY 2010
DURATION	:	2 HOURS and 30 MINUTES
INSTRUCTION	:	PART A: ANSWER ALL QUESTIONS PART B: ANSWER ALL QUESTIONS

THIS PAPER CONSISTS OF 8 PAGES INCLUDING COVER PAGE

PART A - Basic Comprehension and Understanding - 40 Marks
(2 Questions, Answer All Questions)

Q1 A structure consists of three nodes (node 1, 2 and 3) and every node has two-degree of freedom. A global static equation of $[K]\{u\} = \{F\}$ of the structure before any constraints applied to the static equation is shown below

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{Bmatrix}$$

The constraints of the structure is as follows:
Node 1 is fixed, the node cannot move to any direction.
Node 2 is constrained in x direction (must be displaced β_1).
Node 3 is constrained in y direction (must be displaced β_2) .

(a) Write the matrix equation above after all constraints applied, by implementing penalty method.

(b) Write the matrix equation above after all constraints applied, by implementing direct elimination method.

(25 marks)

Q2 Assuming that the global matrix structural equation of a structure before constraints applied is $[K]\{u\} = \{F\}$ and after all boundary conditions (constraints) applied is $[K^c]\{u\} = \{F^c\}$

(a) Write the equation to calculate the displacement vector $\{u\}$

(b) Write the equation to calculate the reaction force vector when the structure has been displaced by the displacement vector $\{u\}$

(15 marks)

PART B - Analysis, Synthesis and Applications - 60 Marks
(3 Questions, Answer All Questions)

Q3 In general one dimensional heat transfer problem, the conductance matrix of the element can be expressed into four parts,

$$[k]^{(e)} = \underbrace{\begin{bmatrix} h_{\infty i}A & 0 \\ 0 & 0 \end{bmatrix}}_{X} + \underbrace{\frac{kA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}_{Y} + \underbrace{\frac{h_{\infty}pL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}_{Z} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & h_{\infty j}A \end{bmatrix}}_{Q}$$

and the the elemental thermal load vector can be written into five parts,

$$\{f\}^{(e)} = \underbrace{h_{\infty i}AT_{\infty i}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{x} + \underbrace{\frac{QAL}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{y} + \underbrace{\frac{qPL}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{z} + \underbrace{\frac{h_{\infty}TPL}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{q} + \underbrace{h_{\infty j}AT_{\infty i}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{m}$$

In a case of isolated cylinder (diameter 2 cm, divided into four elements) shown in FIGURE Q3,

(a) Draw clearly your one dimensional finite element model and indicate the element numbers and the nodes. You have to use your definition consistently in answering the next questions.

(b) Calculate the conductance matrix and the elemental thermal load vector of every element.

(c) Write the system complete matrix equation $[K^c]\{T\} = \{F^c\}$ after applying the boundary condition T_b. Either penalty method or elimination method can be chosen in implementing the boundary condition (constraint) into the equation.

(d) Explain how to find the distribution of the temperature. One sentence answer should be sufficient, or just write the equation to find $\{T\}$.

(20 marks)

Q4 A two dimensional structure is isolated in two edges. The upper edge is exposed to the air with temperature of $T_f = 20°C$ and the convection coefficient $h = 50\,\mathrm{W/m^2C}$. The right edge is maintained the temperature at $T = 80°C$. The conductivity of the material is uniform, $k = 168\,\mathrm{W/mC}$
This two dimensional heat transfer problem is modelled by using Bilinear Rectangular elements as shown in FIGURE Q4.

(a) Firstly you are requested to write clearly your definition of the element and the corresponding nodes. Use a table with columns of Element, Node i, Node j, Node k, Node l.

(b) Calculate the conductance matrix of each element and the thermal load vector of each element

(c) Write the the global system matrix equation $[K^c]\{T\} = \{F^c\}$ after considering all constraints.

(20 marks)

Q5 A beam structure is shown in FIGURE Q5. The beam has distributed load and concentrated load at the location as indicated by the figure. The material elasticity of the beam is $E = 200\,\text{GPa}$ and the inertia cross section of the beam is $I = 0.0002\,\text{m}^4$. The beam is supported by a vertical directional spring with a spring constant $k = 200\,{}^{\text{kN}}/_{\text{m}}$. The element definition in regard to the nodes is collated in the table below

Element	Node i	Node j
1	1	2
2	2	3
3	3	4

At node 2, there is a constraint that it must be displaced vertically 5 mm

(a) Calculate the stiffness matrix of every element $[k]^{(e)}$ as well as the force vector of every element $\{f\}^{(e)}$

(b) Determine the system stiffness matrix after considering the boundary conditions.

(c) Write down your calculation approach to find the displacement of every node, simultaneously as the displacement vector $\{u\}$. You do not need to really solve!

(d) Write down your calculation approach to find the reaction force in every node.

(20 marks)

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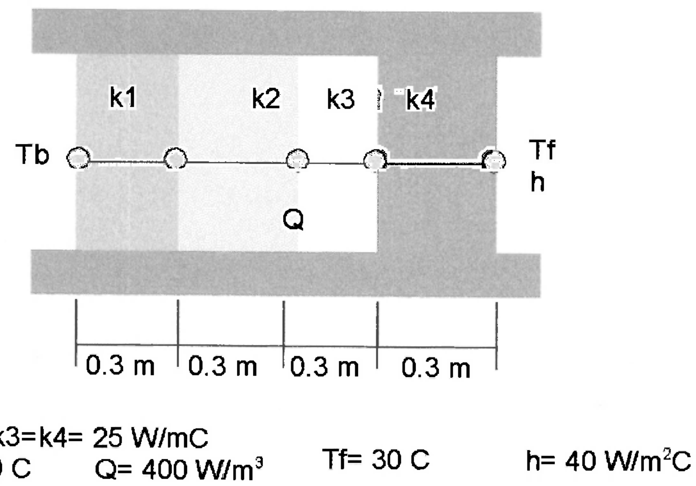

FIGURE Q3: Cylinder heat problem

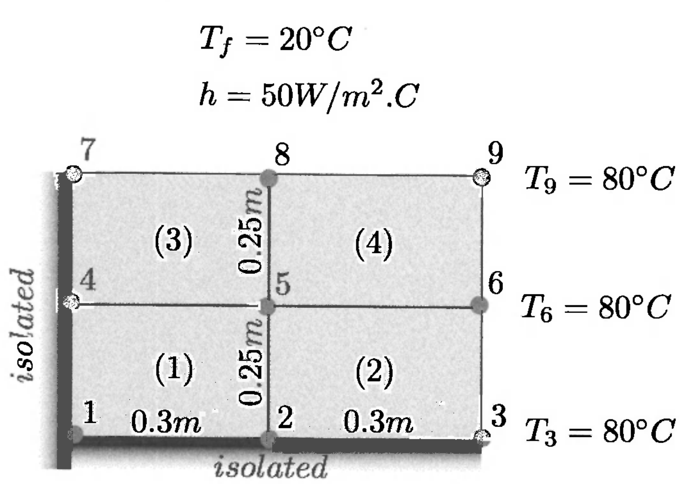

FIGURE Q4: Plate heat problem

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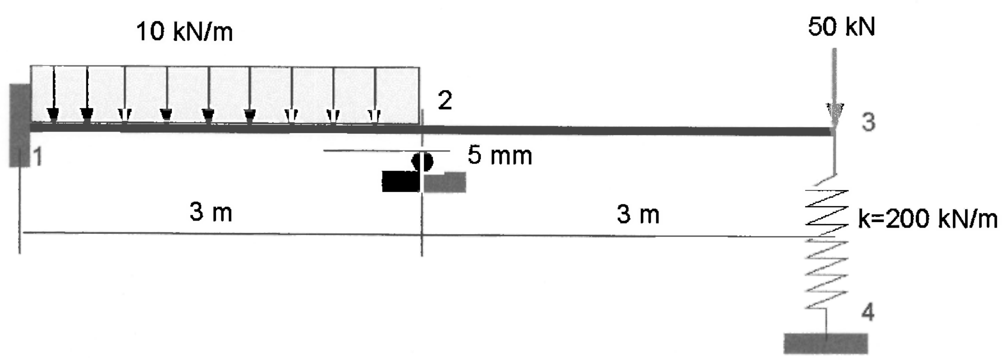

FIGURE Q5: Beam with constraint and spring supported

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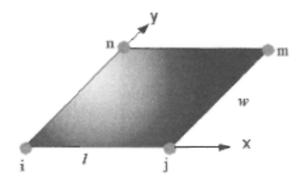
BI LINEAR RECTANGULAR HEAT ELEMENT:

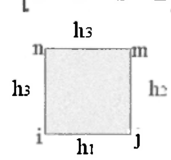


$$[K^e]=\frac{k_x w}{6l}\begin{bmatrix}2 & -2 & -1 & 1\\ -2 & 2 & 1 & -1\\ -1 & 1 & 2 & -2\\ 1 & -1 & -2 & 2\end{bmatrix}+\frac{k_y l}{6w}\begin{bmatrix}2 & 1 & -1 & -2\\ 1 & 2 & -2 & -1\\ -1 & -2 & 2 & 1\\ -2 & -1 & 1 & 2\end{bmatrix}$$

Additional of conductance matrix when exposed to free medium:

$$[K^e]=\frac{h_3 L_{mn}}{6}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 2 & 1\\ 0 & 0 & 1 & 2\end{bmatrix}$$

$$[K^e]=\frac{h_4 L_{ni}}{6}\begin{bmatrix}2 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 2\end{bmatrix}$$



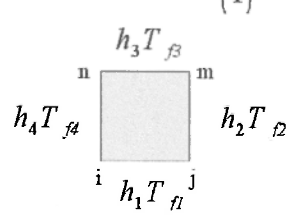
$$[K^e]=\frac{h_2 L_{jm}}{6}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 2 & 1 & 0\\ 0 & 1 & 2 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$[K^e]=\frac{h_1 L_{ij}}{6}\begin{bmatrix}2 & 1 & 0 & 0\\ 1 & 2 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Thermal load vector due to heat loss along the edge

$$\{F^e\}=\frac{h_3 T_{f3} L_{mn}}{2}\begin{Bmatrix}0\\ 0\\ 1\\ 1\end{Bmatrix}$$

$$\{F^e\}=\frac{h_4 T_{f4} L_{ni}}{2}\begin{Bmatrix}1\\ 0\\ 0\\ 1\end{Bmatrix}$$



$$\{F^e\}=\frac{h_2 T_{f2} L_{jm}}{2}\begin{Bmatrix}0\\ 1\\ 1\\ 0\end{Bmatrix}$$

$$\{F^e\}=\frac{h_1 T_{f1} L_{ij}}{2}\begin{Bmatrix}1\\ 1\\ 0\\ 0\end{Bmatrix}$$

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BEAM STIFFNESS MATRIX:

$$[k]^{(e)} = \frac{EI}{L^3} \begin{array}{c c} \begin{array}{cccc} v_i & \theta_i & v_j & \theta_j \end{array} & \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} & \begin{array}{c} v_i \\ \theta_i \\ v_j \\ \theta_j \end{array} \end{array}$$

Distributed load in beam: