Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	SIGNALS AND SYSTEMS
COURSE CODE	:	BNR 36103
PROGRAMME CODE	:	BNF
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

Q1 (a) Briefly explain continuous-time and discrete-time signals and systems with the aid of graphical representations and block diagrams.

(6 marks)

(b) The signal (t)=$cos(\pi t/4)$+$sin(\pi t/8)$ consists of two continuous-time periodic signal. Determine the period T of the signal.

(7 marks)

(c) Write the signal $x(t)$ for **Figure Q1(c)** using a single analytical expression with aid of the unit step function, $u(t)$.

(7 marks)

Q2 (a) Explain the differences between causality and invertible system

(4 marks)

(b) Consider a periodic signal $x(t)$ is defined by

$$x(t) = 3 + 4\sin(2w_o t) + 7\cos(4w_o t + 30°)$$

(i) Determine exponential Fourier Series coefficient of signal $x(t)$.

(7 marks)

(ii) Sketch the magnitude and phase spectrum of signal $x(t)$.

(2 marks)

(c) Compute the coefficients a_k using Fourier series analysis equation for the continuous-time periodic signal

$$x(t) = \begin{cases} 1.5 & 0 \leq t < 1 \\ -1.5 & 1 \leq t < 2 \end{cases}$$

with the fundamental frequency $\omega_o = \pi$

Fourier series analysis equation $a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_o t}dt$

(7 marks)

Q3 (a) Briefly explain how Fourier Transform (FT) is obtained from Fourier series and the convolution property of FT.

(4 marks)

(b) Determine the Fourier Transform of following signals using the definition of FT.

(i) $x(t) = 2e^{-\frac{t}{5}}\, u(t-2)$

(8 marks)

(ii) $x(t) = \sin(2\pi t) + \cos(4\pi t)$

(8 marks)

Q4 (a) Using the definition of Laplace transform, determine the Laplace transform of $x(t) = 3e^{-3t}u(t-2)$.

(5 marks)

(b) Sketch the zero-pole plot and region of convergence (if exist) of the signal $x(t)$ from **Q4(a)**.

(3 marks)

(c) A Linear Time Invariant (LTI) system is shown in **Figure Q4(c)** has an impulse response $h(t) = 0.25e^{-3t}u(t)$. If the system produces output $y(t) = (e^{-t} - e^{-2t})u(t)$, determine the input $x(t)$.

(12 marks)

Q5 (a) Briefly explain at least **THREE (3)** properties of the region of convergence for the z-transform.

(6 marks)

(b) Evaluate the z-transform of the following signal and specify the corresponding region of convergence.

$$x[n] = \left(\frac{1}{5}\right)^n u[n-3]$$

(5 marks)

(c) Determine the poles of the following signal and specify the corresponding region of convergence for $X(z)$.

$$x[n] = \begin{cases} (1/3)^n \cos(\frac{\pi}{4}n) & n \leq 0 \\ 0 & n > 0 \end{cases}$$

(9 marks)

- END OF QUESTIONS –

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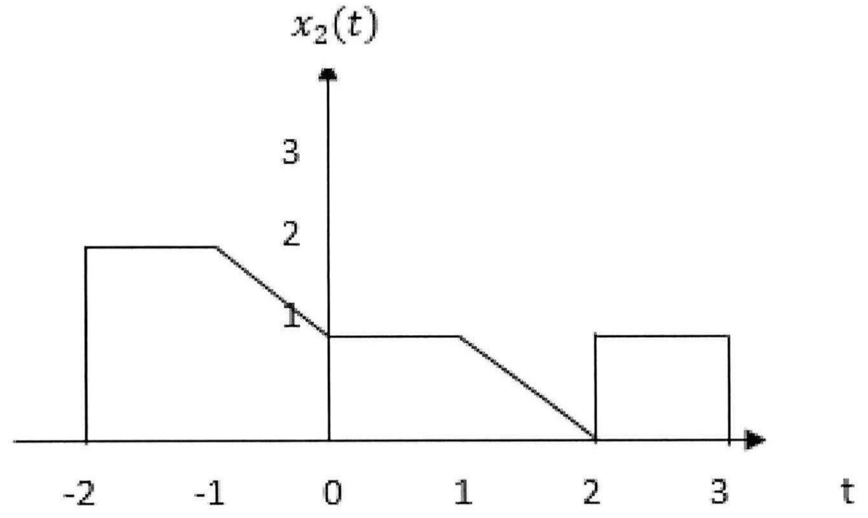

Figure Q1(c)

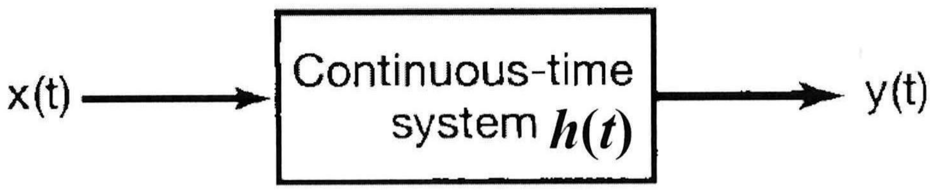

Figure Q4(c)

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TABLE 1: INDEFINITE INTEGRALS

$\int \cos at\ dt = \frac{1}{a} \sin at$	$\int \sin at\ dt = -\frac{1}{a} \cos at$
$\int t \cos at\ dt = \frac{1}{a^2} \cos at + \frac{1}{a} t \sin at$	$\int t \sin at\ dt = \frac{1}{a^2} \sin at - \frac{1}{a} t \cos at$

TABLE 2: EULER'S IDENTITY

$e^{\pm\frac{j\pi}{2}} = \pm j$	$A\angle \pm \theta = Ae^{\pm j\theta}$
$e^{\pm jn\pi} = \cos(n\pi)$	$e^{\pm j\theta} = \cos\theta \pm j \sin\theta$
$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$	$\sin\theta = \frac{1}{2}\left(e^{j\theta} - e^{-j\theta}\right)$

TABLE 3: TRIGONOMETRIC IDENTITIES

$\sin\alpha = \cos\left(\alpha - \frac{\pi}{2}\right)$	$\cos\alpha = \sin\left(\alpha + \frac{\pi}{2}\right)$
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 4: VALUES OF COSINE, SINE AND EXPONENTIAL FUNCTIONS FOR INTEGRAL MULTIPLE OF π

Function	Value	Function	Value
$\cos(2n\pi)$	1	$e^{\frac{jn\pi}{2}}$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = even \\ (-1)^{\frac{n-1}{2}} & ,n = odd \end{cases}$
$\sin(2n\pi)$	0		
$\cos(n\pi)$	$(-1)^n$	$\cos\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = even \\ 0 & ,n = odd \end{cases}$
$\sin(n\pi)$	0		
$e^{j2n\pi}$	1	$\sin\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n-1}{2}} & ,n = even \\ (-1)^{\frac{n+1}{2}} & ,n = odd \end{cases}$
$e^{jn\pi}$	$(-1)^n$		

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TABLE 5: FOURIER SERIES

Exponential	$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\frac{2\pi}{T}t}$, $x_n = \frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\, e^{-jn\frac{2\pi}{T}t}$
Trigonometric	$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\frac{2\pi}{T}t + b_n \sin n\frac{2\pi}{T}t$ $a_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t) \cos n\frac{2\pi}{T}t$ $b = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t) \sin n\frac{2\pi}{T}t$
Amplitude-phase	$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2\pi n}{T}t + \angle\phi_n\right)$

FOURIER TRANSFORM

$$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$

INVERSE FOURIER TRANSFORM

$$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega$$

LAPLACE TRANSFORM

$$\mathcal{L}[x(t)] = X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$$

INVERSE LAPLACE TRANSFORM

$$x(t) = \mathcal{L}^{-1}[X(s)] = \frac{1}{j2\pi}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$$

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TABLE 6: FOURIER TRANSFORM PAIRS

Time domain, $x(t)$	**Frequency domain, $X(\omega)$**	**Time domain, $x(t)$**	**Frequency domain, $X(\omega)$**
$\delta(t)$	1	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$
1	$2\pi\delta(\omega)$	$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	$e^{-j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$u(t-\tau)-u(t+\tau)$	$2\frac{\sin\omega\tau}{\omega}$	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$
$\|t\|$	$\frac{-2}{\omega^2}$	$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$
$\text{sgn}(t)$	$\frac{2}{j\omega}$	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+{\omega_0}^2}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$e^{-at}\cos\omega_0 t\, u(t$	$\frac{a+j\omega}{(a+j\omega)^2+{\omega_0}^2}$
$e^{at}u(-t)$	$\frac{1}{\alpha-j\omega}$		

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TABLE 7: FOURIER TRANSFORM PROPERTIES

Property	**Time domain, $x(t)$**	**Frequency domain, $X(\omega)$**
	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t-t_0)u(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
Modulation	$\cos(\omega_0 t)\,x(t)$	$\frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$
Time differentiation	$\frac{d}{dt}(x(t))$ $\frac{d^n}{dt^n}(x(t))$	$j\omega X(\omega)$ $(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{F(\omega)}{j\omega}+\pi F(0)\delta(\omega)$
Frequency differentiation	$t^n x(t)$	$(j)^n \frac{d^n}{d\omega^n}X(\omega)$
Time Reversal	$x(-t)$	$X(-\omega)$ or $X^*(\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Convolution in t	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Convolution in ω	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$

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TABLE 8: LAPLACE TRANSFORM

$x(t),\ t > 0$	$X(s)$	$x(t),\ t > 0$	$X(s)$
$\delta(t)$	1	$\cos bt$	$\frac{s}{s^2+b^2}$
$u(t)$	$\frac{1}{s}$	$\sin bt$	$\frac{b}{s^2+b^2}$
t	$\frac{1}{s^2}$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
t^n	$\frac{n!}{s^{n+1}}$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
e^{-at}	$\frac{1}{s+a}$	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$

TABLE 9: LAPLACE TRANSFORM PROPERTIES

Name	Operation in Time Domain	Operation in Frequency Domain
1. Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$
2. Differentiation	$\frac{d^nx(t)}{dt^n}$	$s^nX(s) - s^{n-1}x(0^-) - \cdots - x^{(n-1)}(0^-)$
3. Integration	$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\frac{X(s)}{s} + \frac{x^{(-1)}(0^-)}{s}$
4. s-shift	$x(t)\exp(-\alpha t)$	$X(s+\alpha)$
5. Delay	$x(t-t_0)u(t-t_0)$	$X(s)\exp(-st_0)$
6. Convolution	$x_1(t) * x_2(t) = \int_0^{\infty} x_1(\lambda)x_2(t-\lambda)\, d\lambda$	$X_1(s)X_2(s)$
7. Product	$x_1(t)x_2(t)$	$\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X_1(s-\lambda)X_2(\lambda)\, d\lambda$
8. Initial value (provided limits exist)	$\lim_{t\to 0^+} x(t)$	$\lim_{s\to\infty} sX(s)$
9. Final value (provided limits exist)	$\lim_{t\to\infty} x(t)$	$\lim_{s\to 0} sX(s)$
10. Time scaling	$x(at),\quad a > 0$	$a^{-1}X\left(\frac{s}{a}\right)$

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TABLE 10: PROPERTIES OF THE Z-TRANSFORM

10.1 PROPERTIES OF THE z-TRANSFORM

Property	Signal	z-Transform	ROC
	$x[n]$	$X(z)$	R
	$x_1[n]$	$X_1(z)$	R_1
	$x_2[n]$	$X_2(z)$	R_2
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least the intersection of R_1 and R_2
Time shifting	$x[n - n_0]$	$z^{-n_0}X(z)$	R, except for the possible addition or deletion of the origin
Scaling in the z-domain	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	R
	$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	z_0R
	$a^n x[n]$	$X(a^{-1}z)$	Scaled version of R (i.e., $\|a\|R$ = the set of points $\{\|a\|z\}$ for z in R)
Time reversal	$x[-n]$	$X(z^{-1})$	Inverted R (i.e., R^{-1} = the set of points z^{-1}, where z is in R)
Time expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$ for some integer r	$X(z^k)$	$R^{1/k}$ (i.e., the set of points $z^{1/k}$, where z is in R)
Conjugation	$x^*[n]$	$X^*(z^*)$	R
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least the intersection of R_1 and R_2
First difference	$x[n] - x[n-1]$	$(1 - z^{-1})X(z)$	At least the intersection of R and $\|z\| > 0$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - z^{-1}}X(z)$	At least the intersection of R and $\|z\| > 1$
Differentiation in the z-domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R

Initial Value Theorem
If $x[n] = 0$ for $n < 0$, then
$$x[0] = \lim_{z \to \infty} X(z)$$

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TABLE 11: THE Z-TRANSFORM PAIRS

Signal	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1 - z^{-1}}$	$\lvert z \rvert > 1$
3. $-u[-n-1]$	$\frac{1}{1 - z^{-1}}$	$\lvert z \rvert < 1$
4. $\delta[n-m]$	z^{-m}	All z, except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $\alpha^n u[n]$	$\frac{1}{1 - \alpha z^{-1}}$	$\lvert z \rvert > \lvert \alpha \rvert$
6. $-\alpha^n u[-n-1]$	$\frac{1}{1 - \alpha z^{-1}}$	$\lvert z \rvert < \lvert \alpha \rvert$
7. $n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$\lvert z \rvert > \lvert \alpha \rvert$
8. $-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$\lvert z \rvert < \lvert \alpha \rvert$
9. $[\cos \omega_0 n]u[n]$	$\frac{1 - [\cos \omega_0]z^{-1}}{1 - [2\cos \omega_0]z^{-1} + z^{-2}}$	$\lvert z \rvert > 1$
10. $[\sin \omega_0 n]u[n]$	$\frac{[\sin \omega_0]z^{-1}}{1 - [2\cos \omega_0]z^{-1} + z^{-2}}$	$\lvert z \rvert > 1$
11. $[r^n \cos \omega_0 n]u[n]$	$\frac{1 - [r\cos \omega_0]z^{-1}}{1 - [2r\cos \omega_0]z^{-1} + r^2 z^{-2}}$	$\lvert z \rvert > r$
12. $[r^n \sin \omega_0 n]u[n]$	$\frac{[r\sin \omega_0]z^{-1}}{1 - [2r\cos \omega_0]z^{-1} + r^2 z^{-2}}$	$\lvert z \rvert > r$