UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2017/2018

COURSE NAME	:	RISK THEORY
COURSE CODE	:	BWA 40803
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2017 / JANUARY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **EIGHT(8)** PAGES

Q1 (a) Let the loss random variable X have a *p.d.f.* given by

$$f(x) = 0.1e^{-0.1x}, \qquad x > 0.$$

(i) Calculate $E[X]$ and $Var(X)$.

(3 marks)

(ii) If premium, $P = 5$ is to be spent for insurance to be purchased by the payment of the pure premium, show that

$$I(x) = \frac{x}{2}$$

and

$$I_d(x) = \begin{cases} 0, & x < d, \\ x - d, & x \geq d, \end{cases} \text{ where } d = 10 \log 2,$$

both represent feasible insurance policies with pure premium $P = 5$ and $I(x)$ is called proportional insurance.

(3 marks)

(b) Jennifer's von Neumann-Morgensterm utility of money function is

$$u(w) = \sqrt{w}.$$

Consider the following lottery:

$$L = \begin{pmatrix} \$30 & \$28 & \$24 & \$18 & \$8 \\ \\ \frac{2}{10} & \frac{1}{10} & \frac{1}{10} & \frac{2}{10} & \frac{4}{10} \end{pmatrix}.$$

(i) Compute the expected value and expected utility of L.

(3 marks)

(ii) Calculate the premium associated with L?

(3 marks)

(iii) Calculate $\frac{du(w)}{dw}$ and $\frac{d^2u(w)}{dw^2}$.

(2 marks)

(iv) Explain, whether Jennifer is risk-averse or risk neutral or risk-loving?

(2 marks)

(c) Johan owns a house worth RM200,000. The value of the building is RM75,000 and the value of the land is RM125,000. In the area where he lives there is a 0.1% probability that a fire will completely destroy the building in a given year. In the other hand, the land would not be affected by a fire. An insurance company offers a policy that covers the replacement cost of the building in the event of fire. There is no deductible. The premium for this policy is RM7,500 per year. What attitude to risk must Johan have in order to buy the insurance policy? Explain your answer.

(4 marks)

Q2 (a) Let X be the number of heads observed in seven tosses of a true coin. Then, X true dice are thrown. Let Y be the sum of the numbers showing on the dice. Determine the mean and variance of Y.

(6 marks)

(b) Consider a portfolio of 32 policies. For each policy, the probability q of a claim is $\frac{1}{6}$ and B, the benefit amount given that there is a claim, has *p.d.f.*

$$f(y) = \begin{cases} 2(1-y), & 0 < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Let S be the total claims for portfolio. Using a normal approximation, estimate $\mathbb{P}(S > 4)$.

(8 marks)

(c) The number of claims in a period has a geometric distributions with mean 4. The amount of each claim X follows

$$\mathbb{P}(X = x) = 0.25, \quad x = 1,2,3,4.$$

The number of claims and the claim amounts are independent. S is the aggregate claim amount in the period. Calculate $F_S(3)$.

(6 marks)

Q3 (a) You are given:

- An aggregate loss distribution has a compound Poisson distribution with expected number of claims equal to 1.25.
- Individual claim amounts can take only the values 1, 2 or 3, with equal probability.

Determine the probability that aggregate losses exceed 3 and calculate the expected aggregate losses if an aggregate deductible of 1.6 is applied.

(15 marks)

(b) You own a fancy bulb factory. Your workforce is a bit clumsy they keep dropping boxes of light bulbs. The boxes have varying numbers of light bulbs in them, and when dropped, the entire box is destroyed. You are given:

- Expected number of boxes dropped per month is 50.
- Variance of the number of boxes dropped per month is 100.
- Expected value per box is 200.
- Variance of the value per box is 400.

You pay your employees a bonus if the value of light bulbs destroyed in a month is less than 8000. Assuming independence and using the normal approximation, calculate the probability that you will pay your employees a bonus next month.

(5 marks)

Q4 (a) The claim variable X has the following distributions:

$$f_X(0) = 0.5, \quad f_X(1) = f_X(2) = 0.2 \quad \text{and} \quad f_X(3) = 0.1.$$

Calculate the probability of ultimate ruin $\psi(u)$ for $u \geq 0$.

(10 marks)

(b) Let $U(t;u)$ be a compound Poisson surplus function with $X \sim \mathcal{G}(3, 0.5)$. Compute the adjustment coefficient and its approximate value using equation

$$r^* \simeq \frac{2\theta\mu_X}{\sigma_X^2 + (1+\theta)^2\mu_X^2},$$

for $\theta = 0.1$ and 0.2. Calculate the upper bounds for the probability of ultimate ruin for $u = 5$ and $u = 10$.

(10 marks)

Q5 Jim is considering different types of insurances to cover against a random loss X, which is distributed as an *exponential* with mean 100. His utility function is given by

$$u(w) = -e^{-0.001w}, \quad \forall w \geq 0.$$

(a) For an insurance with deductible d,

$$i.e., \quad I_d(X) = (X - d)_+$$

Estimate the deductible value which makes the pure premium equal to \$10.

(5 marks)

(b) For a proportional insurance with proportion k, examine the value of k which makes the pure premium equal to \$10.

(3 marks)

(c) For an insurance with an ordinary deductible of d and reimbursing only 75% of the loss in excess of the deductible, identify d which makes the pure premium equal to \$10.

(5 marks)

(d) Which of the two insurances in part **Q5**(b) or **Q5**(c) would be preferred by Jim?

(7 marks)

- END OF QUESTIONS -

FORMULAE

Probability Distributions

Discrete Distributions	p.f.	Restrictions on Parameters	Moment Generating Function, $M(s)$	Moments: Mean	Moments: Variance
Binomial	$\binom{n}{x}p^x q^{n-x},\ x = 0, 1, \ldots n$	$0 < p < 1$, $q = 1 - p$	$(pe^s + q)^n$	np	npq
Bernoulli	Special case $n = 1$				
Negative Binomial	$\binom{r+x-1}{x}p^r q^x,\ x = 0, 1, 2, \ldots$	$0 < p < 1$, $q = 1 - p$, $r > 0$	$\left(\frac{p}{1 - qe^s}\right)^r,\ qe^s < 1$	$\frac{rq}{p}$	$\frac{rq}{p^2}$
Geometric	Special case $r = 1$				
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!},\ x = 0, 1, 2, \ldots$	$\lambda > 0$	$e^{\lambda(e^s - 1)}$	λ	λ
Uniform	$\frac{1}{n},\ x = 1, \ldots, n$	n, a positive integer	$\frac{e^s(1 - e^{sn})}{n(1 - e^s)},\ s \neq 0$; $1,\ s = 0$	$\frac{n + 1}{2}$	$\frac{n^2 - 1}{12}$

Continuous Distributions	p.d.f.	Restrictions on Parameters	Moment Generating Function, $M(s)$	Moments: Mean	Moments: Variance
Uniform	$\frac{1}{b - a},\ a < x < b$	—	$\frac{e^{bs} - e^{as}}{(b - a)s},\ s \neq 0$; $1,\ s = 0$	$\frac{b + a}{2}$	$\frac{(b - a)^2}{12}$
Normal	$\frac{1}{\sigma\sqrt{2\pi}}\exp[-(x - \mu)^2/2\sigma^2]$, $-\infty < x < \infty$	$\sigma > 0$	$\exp(\mu s + \sigma^2 s^2/2)$	μ	σ^2
Gamma	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x},\ x > 0$	$\alpha > 0, \beta > 0$	$\left(\frac{\beta}{\beta - s}\right)^\alpha,\ s < \beta$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Exponential	Special case $\alpha = 1$				
Chi-square	Special case $\alpha = \frac{k}{2}, \beta = \frac{1}{2}$	k, a positive integer			
Inverse Gaussian	$\frac{\alpha}{\sqrt{2\pi\beta}}x^{-3/2}\exp\left[-\frac{(\beta x - \alpha)^2}{2\beta x}\right]$, $x > 0$	$\alpha > 0, \beta > 0$	$\exp\left[\alpha\left(1 - \sqrt{1 - \frac{2s}{\beta}}\right)\right]$, $s < \frac{\beta}{2}$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Pareto	$\alpha x_0^\alpha / x^{\alpha+1},\ x > x_0$	$x_0 > 0, \alpha > 0$		$\frac{\alpha x_0}{\alpha - 1}$, $\alpha > 1$	$\frac{\alpha x_0^2}{(\alpha - 2)(\alpha - 1)^2}$, $\alpha > 2$
Lognormal	$\frac{1}{x\sigma\sqrt{2\pi}}\exp[-(\log x - m)^2/2\sigma^2]$, $x > 0$	$-\infty < m < \infty$, $\sigma > 0$		$e^{m+\sigma^2/2}$	$(e^{\sigma^2} - 1)e^{2m+\sigma^2}$

Utility Theory

- An insurer with utility $u(\cdot)$ and wealth w needs π or more to cover X if $E[u(w = \pi - X)] = u(w)$.
- Expected value: $E[w] = pw_1 + qw_2$
- Expected utility function: $E[u(w)] = pu(w_1) + qu(w_2)$

Individual Risk Model

- Aggregate claim is $S = X_1 + X_2 + \cdots + X_n$ where n is number of risk unit insured and X_i is the distribution of amount of claims

➤ X, the claim random variable. Its p.f is

$$f_X(x) = Pr(X = x) = \begin{cases} 1-q, & x = 0 \\ q, & x = b \\ 0, & \text{elsewhere,} \end{cases}$$

where q, the probability of a claim during the year and b, insurer pay amount if the insured dies with in a year of policy issue and nothing if insured survives the year.

➤ The distribution function is

$$F_X(x) = Pr(X \leq x) = \begin{cases} 0, & x < 0 \\ 1-q, & 0 \leq x \leq b \\ 1, & x \geq b \end{cases}$$

➤ Mean, $E[X] = bq$ and Variance, $Var(X) = b^2q(1-q)$.

➤ Conditional Expectations and Variance:

$$E[X] = E[E[X \mid I]] = \mu q,$$

$$Var[X] = Var(E[X \mid I]) + E[Var(X \mid]) = \mu^2 q(1-q) + \sigma^2 q.$$

➤ Distribution function of sum of two independent random variables is,

$$F_S(s) = \sum_{all\ y \leq s} F_X(s-y) f_Y(y),$$

and the probability function is

$$f_S(s) = \sum_{all\ y \leq s} f_X(s-y) f_Y(y).$$

➤ MGF is $M_S(t) = E[e^{tS}]$.

Normal Approximation

➤ The approximate distribution of S is,

$$Pr(S \leq s) = Pr\left(\frac{S - E(S)}{\sqrt{Var(S)}} \leq \frac{s - E(S)}{\sqrt{Var(S)}}\right)$$

Collective Risk Model

$S = \sum_{j=1}^{N} X_j$

$N, X_1, X_2, \ldots$ are independent random variables.
Each X_j has d.f. $P(x)$,
m.g.f. $M_X(t)$, and
$p_k = E[X^k] \quad k = 1, 2, \ldots$

$P^{*0}(x) = \begin{cases} 1 & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$

$P^{*n}(x) = \begin{cases} \sum_{j=0}^{x} p(x-j)P^{*(n-1)}(j), \text{ or} \\ \int_0^x p(x-y)P^{*(n-1)}(y)dy \end{cases}$

Definitions	Distribution Function, $F_S(x)$	Restrictions on Parameters	Moment Generating Function, $M_S(t)$	Mean	Variance
General	$\sum_{n=0}^{\infty} \Pr(N=n)P^{*n}(x)$	—	$M_N[\log M_X(t)]$	$p_1 E[N]$	$E[N](p_2 - p_1^2) + p_1^2 \text{Var}(N)$
Compound Poisson	$\sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} P^{*n}(x)$	$\lambda > 0$	$e^{\lambda[M_X(t)-1]}$	λp_1	λp_2
Compound Negative Binomial	$\sum_{n=0}^{\infty} \binom{r+n-1}{n} p^r q^n P^{*n}(x)$	$0 < p < 1$ $q = 1 - p$ $r > 0$	$\left[\frac{p}{1-qM_X(t)}\right]^r \quad qM_X(t) < 1$	$\frac{rqp_1}{p}$	$\frac{rqp_2}{p} + \frac{rq^2p_1^2}{p^2}$
Compound Poisson Inverse Gaussian	no known closed form	$\alpha > 0$ $\beta > 0$	$\exp\left\{\alpha\left[1 - \left\{1 - \frac{2[M_X(t)-1]}{\beta}\right\}^{1/2}\right]\right\}$	$\frac{\alpha}{\beta} p_1$	$\frac{\alpha}{\beta}\left(p_2 + \frac{p_1^2}{\beta}\right)$

TERBUKA

Ruin Theory

- Surplus process is $U(t) = u + ct - S(t), t \geq 0$ where $U(t)$, the insurer's random capital at time t; $u = U(0)$, the initial surplus; c, the constant premium income per unit of time; $S(t) = X_1 + X_2 + \cdots + X_{N(t)}$.
- Ruin probability, $\psi(u) = Pr(T < \infty)$ where

$$T = \begin{cases} min\{t \mid t \geq 0 \& U(t) < 0\}; \\ \infty, \ if \ U(t) \geq 0 \ \forall t. \end{cases}$$

- Adjustment coefficient (continuos case): $1 + (1+\theta)\mu R = M_X(R)$
- Loading factor, $c = (1+\theta)\lambda\mu$.