Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2017/2018

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY III
COURSE CODE	:	BWM 22403
PROGRAMME CODE	:	BND / BNE / BNF / BNG / BNH / BNL / BNM
EXAMINATION DATE	:	DECEMBER 2017 / JANUARY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS 2. ALL CALCULATIONS MUST BE IN **3 DECIMAL** PLACES

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 (a) Determine if the function

$$f(x,y)=\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$$

is continuous at (0,0).

(4 marks)

(b) By using double integrals, evaluate

$$\iint_R (x+y)\,dA$$

given that R is the region bounded by $y=\sqrt{x}$ and $y=x^2$.

(6 marks)

(c) By using spherical coordinates, calculate the volume of the solid bounded above by sphere $x^2+y^2+z^2=2z$ and below by cone $z=\sqrt{x^2+y^2}$.

(10 marks)

Q2 (a) Given the graph of the function $f(x)=x^2+4\sin(2x)-2$ over the interval $-3\le x\le 3$ as in **Figure Q2(a)**. Compute the **largest positive root** of the equation by using bisection method. Iterate until $f(c_i)<0.005$.

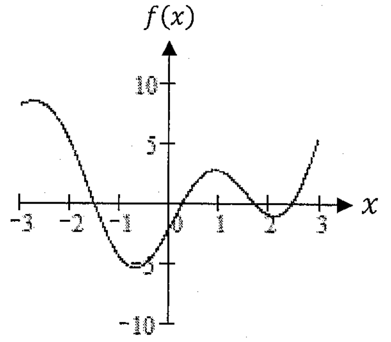

Figure Q2(a)

(10 marks)

(b) An oscillating current in an electric circuit is described by the following equation

$$i=9e^{-t}\cos(2\pi t),$$

where t is in seconds. By assuming $i=3.7$ ampere and taking the value of $t_0=0.3$, determine the value of t by using Newton-Raphson method. Iterate until $|f(t_i)|<0.005$.

(10 marks)

Q3 (a) Given a system of linear equations as below.

$$-7p-2q-4r=-3$$
$$4p+q+6r=9$$
$$2p-5q-r=-9$$

Solve the above system by using Gauss-Seidel iteration method. Use initial guess as $(-0.6 \quad 1.4 \quad 2.5)^T$.

(10 marks)

(b) Consider the following vectors

$$\vec{A}=p\underset{\sim}{i}+q\underset{\sim}{j},\ \vec{B}=-2\underset{\sim}{i}+\underset{\sim}{j} \text{ and } \vec{C}=\underset{\sim}{i}+3\underset{\sim}{j},$$

where $\vec{A}$ is an unknown vector. Given that

$$(\vec{A}\times\vec{B})+(\vec{A}\times\vec{C})=(2p-3q)\underset{\sim}{i}+14\underset{\sim}{k}.$$

Compute the vector $\vec{A}$ by using Gauss elimination method.

[Hint: $\vec{a}\times\vec{b}=\begin{vmatrix}\underset{\sim}{i} & \underset{\sim}{j}\\ a_1 & a_2\\ b_1 & b_2\end{vmatrix}=(a_1b_2-a_2b_1)\underset{\sim}{k}$]

(10 marks)

Q4 (a) A rod is rotating in a plane one of its ends. **Table Q4(a)** gives the angle θ radian through which the rod has turned for different values of time, t seconds. Calculate its angular velocity when $t=0.7$ seconds by using Newton's divided difference method.

Table Q4(a) : The angle, θ through which the rod has turned at time, t

t (seconds)	0.2	0.4	0.8	1.0
θ (radian)	0.12	0.48	2.0	3.20

(7 marks)

(b) A culture of cells in lab has a population of 100 cells when nutrients are added at $t=0$ $(N(0)=100)$. Given the population at time t, $N(t)$ as

$$N(t)=N(0)+\int_0^t N'(x)\,dx.$$

Suppose the population, $N(t)$ increases at a rate given by $N'(t)=90e^{-0.1t}$ cells/hr. Compute $N(t)$ when $t=4.5$ hours by applying $\frac{3}{8}$ Simpson's rule. [Hint: $h=0.5$]

(13 marks)

Q5 (a) Estimate the dominant eigenvalue and the corresponds eigenvector for

$$A = \begin{pmatrix} 2 & -3 & 3 \\ 12 & -4 & -6 \\ -6 & -3 & 11 \end{pmatrix}$$

by using power method. Use $\mathbf{v}^{(0)} = (0.4 \quad 1 \quad 1)^T$.

(10 marks)

(b) Consider the heat conduction equation as

$$\frac{\partial}{\partial t}T(x,t) = \alpha \frac{\partial^2}{\partial x^2}T(x,t), \quad 0 < x < 10, \; t > 0,$$

where $\alpha = 10$ is the thermal diffusity. Given the boundary conditions as

$$T(0,t) = 0, \; T(10,t) = 100$$

and initial condition

$$T(x,0) = x^2.$$

By using implicit Crank-Nicolson method, develop the system of linear equations to determine $T(x, 0.055)$ with 5 grid intervals on the x coordinate. Please DO NOT SOLVE THE SYSTEM.

(10 marks)

- END OF QUESTIONS -

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Formulas

Spherical coordinates:

$$x = \rho \sin\phi \cos\theta \qquad y = \rho \sin\phi \sin\theta \qquad z = \rho \cos\phi$$

$$V = \iiint_G dV = \iiint_G \rho^2 \sin\phi \; d\rho \, d\phi \, d\theta$$

$$x^2 + y^2 + z^2 = \rho^2, \quad 0 \le \theta \le 2\pi, \quad 0 \le \phi \le \pi$$

Nonlinear equations

Bisection method : $c_i = \dfrac{a_i + b_i}{2}, \; i = 0,1,2,\ldots$

Newton-Raphson method : $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}, i = 0,1,2,\ldots$

System of linear equations

Gauss-Seidel iteration : $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, \; \forall i = 1, 2, 3,.., n.$

Interpolation

Newton divided difference:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \ldots + f_0^{[n]}(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

Numerical integration

Simpson $\frac{3}{8}$ rule : $\int_a^b f(x)\,dx \approx \dfrac{3}{8}h\left[\begin{array}{c}(f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) \\ + 2(f_3 + f_6 + \cdots + f_{n-3})\end{array}\right]$

Eigenvalue

Power Method : $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \qquad k = 0,1,2,.....$

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Partial differential equations

Heat equation- Implicit Crank-Nicolson:

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = \frac{c^2}{2}\left(\frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{h^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}\right)$$