CONFIDENTIAL

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER I
## SESSION 2016/2017

| | | |
|---|---|---|
| COURSE NAME | : | ELECTROMAGNETIC TECHNOLOGY |
| COURSE CODE | : | BNR 20603 |
| PROGRAMME | : | BND/BNF |
| EXAMINATION DATE | : | DECEMBER 2016 / JANUARY 2017 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | ANSWERS **ALL** QUESTIONS. |

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THIS QUESTION PAPER CONSISTS OF **TWELVE (12)** PAGES

**Q1** (a) State the definition of Gauss's law as one of the fundamental laws of electromagnetism.

(2 marks)

(b) **Figure Q1(b)** shows a conducting spherical cable with radius $a = 4\,m, b = 6\,m,$ $c = 10\,m$, and $d = 12\,m$, have a surface charge distributions of 8, 5, -10, and 15 $\mu C/m^2$, respectively.

(i) Calculate the flux through $radius = 5\,m$ and radius $= 11\,m$.

(ii) Compute flux density at $r = 11\,m, r = 13m$.

(7 marks)

(c) Determine the total charge enclosed $Q_{enc}$ in an incremental volume $dv = 10^{-6}\,m^3$ located at the origin. The electric flux densities:

$$\boldsymbol{D} = -e^x \cos y \boldsymbol{a_x} + 4x^2 \boldsymbol{a_y} + 5z \boldsymbol{a_z}\ C/m^2$$

(4 marks)

(d) A point charge of 5 nC is located at the origin. If $V$ = 2V at (0, 6, -8), calculate

(i) The potential at $A$ (-3, 2, 6)

(ii) The potential at $B$ (1, 5, 7)

(iii) The potential difference $V_{AB}$

(7 marks)

**Q2** (a) According to the law of conservation of magnetic flux, determine the magnitude of the magnetic field B at a point on the axis midway between the coil as shown in **Figure Q2 (a)** where two identical coaxial circular coils carry the same current $I$ but in opposite directions. Justify your answer.

(4 marks)

(b) A conducting filament carries current 5A along Z axis from $0 \le Z \le 12\,m$. Calculate magnetic field intensity at points (6, 8, 0).

(6 marks)

(c) Two infinitely long solid conductor whose cross section is illustrated in **Figure Q2(c)**. Both conductors are separated by 8 m. The wire centered at (0, 0, 0) carries current of 10 A while the other centered at (8 m, 0, 0) carries the return current. Determine $\vec{H}$ at (8 m, 4 m, 0). [Hint: $\boldsymbol{H} = \frac{I}{2\pi\rho}\tilde{a}_\phi$]

(10 marks)

**Q3** (a) (i) According to Gauss's law, under static conditions, what is the value of electric field intensity $\vec{E}$, volume charge density $\rho_v$ and potential difference $V_{ab}$ for a perfect conductor.

(2 marks)

(ii) What happened to the electrons motion when a steady potential difference is applied across the ends of a conducting wire?

(2 marks)

(b) A homogeneous dielectric ($\varepsilon_r = 2.5$) fills region 1 ($x \leq 0$) while region 2 ($x \geq 0$) is free space.

(i) If $\boldsymbol{D}_1 = 12\boldsymbol{a}_x - 10\boldsymbol{a}_y + 4\boldsymbol{a}_z$ nC/m$^2$, Determine $\boldsymbol{D}_2$ and $\theta_2$

(ii) If $E_2 = 12$ V/m and $\theta_2 = 60°$, Determine $E_1$ and $\theta_1$

(8 marks)

(c) Determine the capacitance of 10 m length of each cylindrical capacitors as shown in **Figure Q3 (c)**. Given $a = 1$ mm, $b = 3$ mm, $c = 2$ mm, $\varepsilon_{r1} = 2.5$, and $\varepsilon_{r2} = 3.5$.

(8 marks)

**Q4** (a) State the generalized forms of Maxwell's Equations in the differential and integral form.

(8marks)

(b) **Figure Q4 (b)** shows a conducting loop of area $20\,cm^2$ and resistance $4\,\Omega$. If $\overline{B} = 40\cos 10^4 t\underset{\sim}{a}_z\ mWb/m^2$, determine the induced current in the loop and indicate its direction as $\overline{B}$ increasing.

(7 marks)

(c) If the area of the loop in **Figure Q4(c)** is 10 cm$^2$, calculate $V_1$ and $V_2$.

(5 marks)

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**Q5** (a) With the aid of diagrams, Explain **TWO (2)** technology applications nowadays that developed based on the electromagnetic theory.

(8 marks)

(b) A plane wave travelling in the +y-direction in a lossy medium ($\varepsilon_r = 4$, $\mu_r = 1$, $\sigma = 10^{-2}$ mhos/m ) has $\vec{E} = 30\cos\left(10^9\pi t + \frac{\pi}{4}\right)\boldsymbol{a}_z$ V/m at $y = 0$. Determine

(i) $\boldsymbol{E}$ at $y = 1$ m, $t = 2$ ns

(5 marks)

(ii) The distance traveled by the wave to have a phase shift of 10°

(1 marks)

(iii) The distance traveled by the wave to have its amplitude reduced by 40%

(1 marks)

(iv) $\boldsymbol{H}$ at $y = 2$ m, $t = 2$ ns

(5 marks)

**- END OF QUESTION –**

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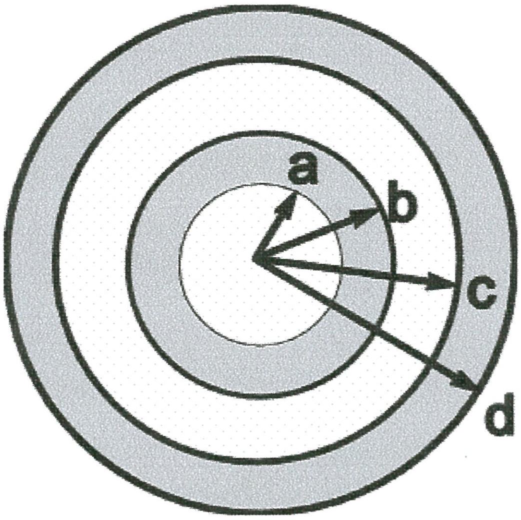


**FIGURE Q1(b)**

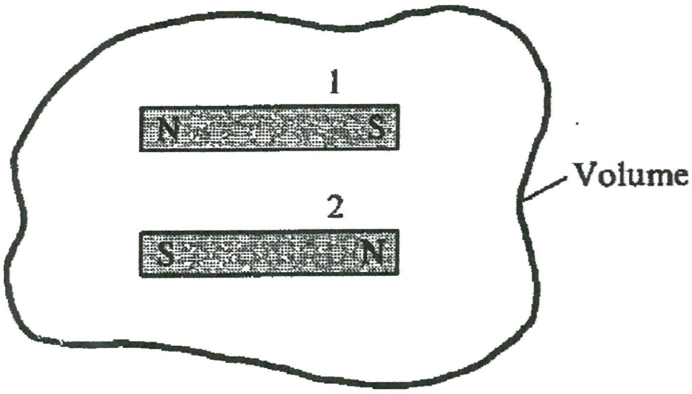


**FIGURE Q2(a)**

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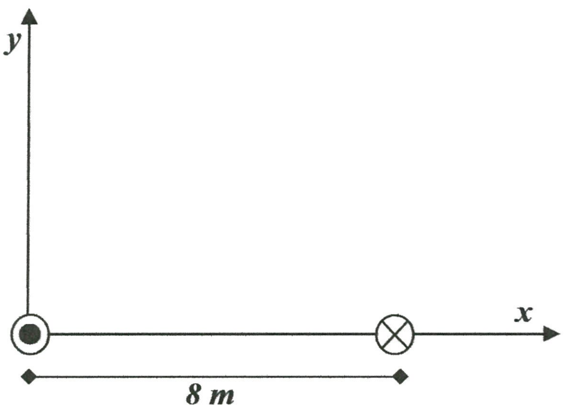
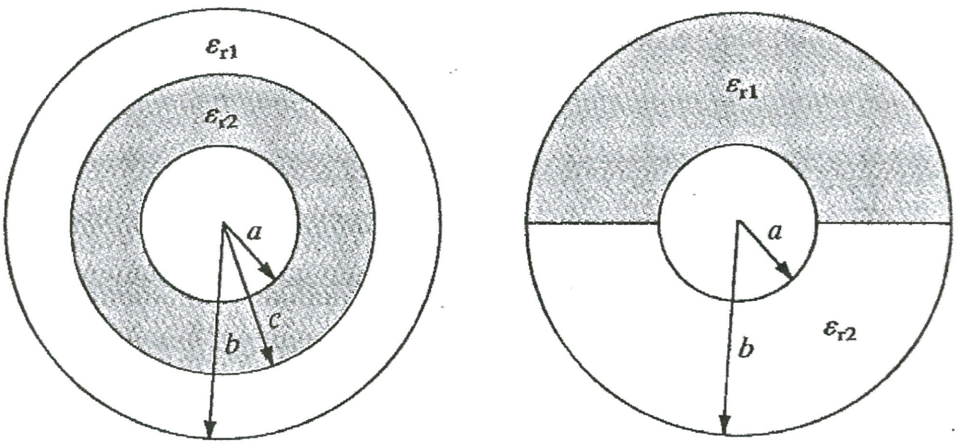



**FIGURE Q2(c)**




(i) (ii)

**FIGURE Q3(c)**

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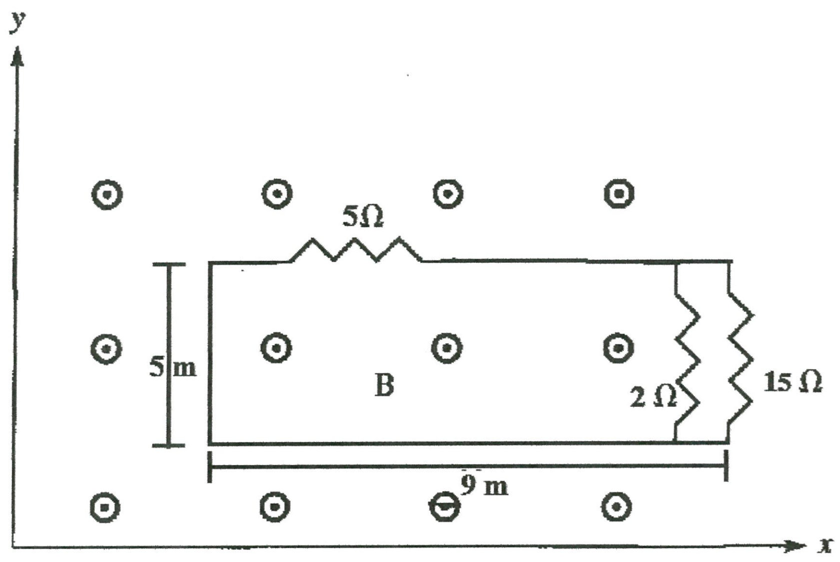


**FIGURE Q4 (b)**

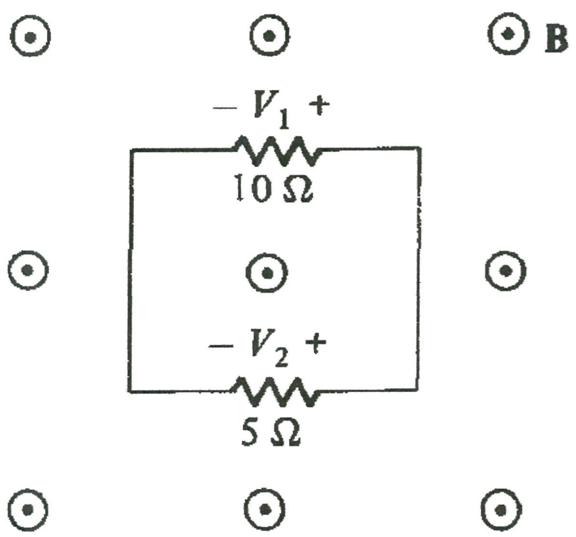




**FIGURE Q4 (c)**

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**Formula**

**Gradient of a scalar**

| Cartesian | Cylindrical | Spherical |
|---|---|---|
| $\nabla V = \frac{\partial V}{\partial x}\underset{\sim}{a}_x + \frac{\partial V}{\partial y}\underset{\sim}{a}_y + \frac{\partial V}{\partial z}\underset{\sim}{a}_z$ | $\nabla V = \frac{\partial V}{\partial \rho}\underset{\sim}{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\underset{\sim}{a}_\phi + \frac{\partial V}{\partial z}\underset{\sim}{a}_z$ | $\nabla V = \frac{\partial V}{\partial r}\underset{\sim}{a}_r + \frac{1}{r}\frac{\partial V}{\partial y}\underset{\sim}{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\underset{\sim}{a}_\varphi$ |

**Divergence of a vector**

| Cartesian | Cylindrical | Spherical |
|---|---|---|
| $\nabla \bullet \overline{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$ | $\nabla \bullet \overline{V} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho V_\rho\right) + \frac{1}{\rho}\frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z}$ | $\nabla \bullet \overline{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(V_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial \varphi}$ |

**Curl of a vector**

| Cartesian |
|---|
| $\nabla \times \overline{V} = \left[\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right]\underset{\sim}{a}_x + \left[\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right]\underset{\sim}{a}_y + \left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right]\underset{\sim}{a}_z$ |
| Cylindrical |
| $\nabla \times \overline{V} = \left[\frac{1}{\rho}\frac{\partial V_z}{\partial \phi} - \frac{\partial V_\phi}{\partial z}\right]\underset{\sim}{a}_\rho + \left[\frac{\partial V_\rho}{\partial z} - \frac{\partial V_z}{\partial \rho}\right]\underset{\sim}{a}_\phi + \frac{1}{\rho}\left[\frac{\partial\left(\rho V_\phi\right)}{\partial \rho} - \frac{\partial V_\rho}{\partial \phi}\right]\underset{\sim}{a}_z$ |
| Spherical |
| $\nabla \times \overline{V} = \frac{1}{r\sin\theta}\left[\frac{\partial\left(V_\phi \sin\theta\right)}{\partial \theta} - \frac{\partial V_\theta}{\partial \phi}\right]\underset{\sim}{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial V_r}{\partial \phi} - \frac{\partial\left(r V_\phi\right)}{\partial r}\right]\underset{\sim}{a}_\theta + \frac{1}{r}\left[\frac{\partial\left(r V_\theta\right)}{\partial r} - \frac{\partial V_r}{\partial \theta}\right]\underset{\sim}{a}_\phi$ |

**Laplacian of a scalar**

| Cartesian |
|---|
| $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$ |
| Cylindrical |
| $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$ |
| Spherical |
| $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$ |

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**Differential length** $d\bar{l}$

| Cartesian | Cylindrical | Spherical |
|---|---|---|
| $d\bar{l} = dx\underset{\sim}{a}_x + dy\underset{\sim}{a}_y + dz\underset{\sim}{a}_z$ | $d\bar{l} = d\rho\underset{\sim}{a}_\rho + \rho d\phi\underset{\sim}{a}_\phi + dz\underset{\sim}{a}_z$ | $d\bar{l} = dr\underset{\sim}{a}_r + rd\theta\underset{\sim}{a}_\theta + r\sin\theta d\phi\underset{\sim}{a}_\phi$ |

**Differential normal area** $d\bar{S}$

| Cartesian | Cylindrical | Spherical |
|---|---|---|
| $d\bar{S} = dydz\underset{\sim}{a}_x$<br>$dxdz\underset{\sim}{a}_y$<br>$dxdy\underset{\sim}{a}_z$ | $d\bar{S} = \rho d\phi dz\underset{\sim}{a}_\rho$<br>$d\rho dz\underset{\sim}{a}_\phi$<br>$\rho d\phi d\rho\underset{\sim}{a}_z$ | $d\bar{S} = r^2\sin\theta d\theta d\phi\underset{\sim}{a}_r$<br>$r\sin\theta dr d\phi\underset{\sim}{a}_\theta$<br>$rdrd\theta\underset{\sim}{a}_\phi$ |

**Differential volume** $dv$

| Cartesian | Cylindrical | Spherical |
|---|---|---|
| $dv = dxdydz_z$ | $dv = \rho d\rho d\phi dz$ | $dv = r^2\sin\theta dr d\theta d\phi$ |

**Electrostatic**

| | | |
|---|---|---|
| $Q = \int \rho_l dl$ | $Q = \int \rho_s dS$ | $Q = \int \rho_v dv$ |
| $\vec{F}_{12} = \dfrac{Q_1Q_2}{4\pi\varepsilon r^2}\underset{\sim}{a}_{12}$ | $\bar{E} = \dfrac{\bar{F}}{Q}$ | $\vec{E} = \int \dfrac{\rho_l dl}{4\pi\varepsilon r^2}\underset{\sim}{a}_r$ |
| $\vec{E} = \int \dfrac{\rho_s dS}{4\pi\varepsilon r^2}\underset{\sim}{a}_r$ | $\vec{E} = \int \dfrac{\rho_v dv}{4\pi\varepsilon r^2}\underset{\sim}{a}_r$ | $\vec{D} = \varepsilon\vec{E} = \dfrac{Q_{enc}}{S}$ |
| $\varepsilon = \varepsilon_o\varepsilon_r$ | $\psi_e = \int \bar{D} \bullet d\bar{S}$ | $\psi = Q_{enc} = \oint \bar{D} \cdot d\bar{S}$ |
| $\rho_v = \nabla \bullet \bar{D}$ | $V_{AB} = -\int_A^B \bar{E} \bullet d\bar{l} = \dfrac{W}{Q}$ | $V = \dfrac{Q}{4\pi\varepsilon r}$ |
| $V = \int \dfrac{\rho_l dl}{4\pi\varepsilon r}$ | $\oint \bar{E} \cdot d\bar{l} = 0$ | $\nabla \times \bar{E} = 0$ |
| $\bar{E} = -\nabla V$ | $\nabla^2 V = 0$ | $\nabla^2 V = -\dfrac{\rho_v}{\varepsilon}$ |
| $R = \dfrac{\ell}{\sigma S} = \dfrac{\rho_c \ell}{S}$ | $I = \int \bar{J} \bullet d\bar{S} = JS$ | $J = \sigma E = \rho_v u$ |
| $\rho_v = ne$ | $\varepsilon_r = 1 + \chi_e$ | $\bar{P} = \chi_e\varepsilon_o\bar{E}$ |
| $\rho_s = \bar{D} \bullet \underset{\sim}{a}_n = D_n$ | $\rho_{ps} = \bar{P} \bullet \underset{\sim}{a}_n = P_n$ | $V = Ed$ |

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**Electrostatic Boundary Condition (Dielectric-Dielectric)**

| | | |
|---|---|---|
| $\overline{E} = \overline{E_n} + \overline{E_t}$ | $\overline{D} = \overline{D_n} + \overline{D_t}$ | $\tan\theta = \frac{E_t}{E_n} = \frac{D_t}{D_n}$ |
| $E_{1t} = E_{2t}$ | $D_{1n} = D_{2n}$ | $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$ |
| $\frac{E_{1n}}{\varepsilon_{r2}} = \frac{E_{2n}}{\varepsilon_{r1}}$ | $\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$ | $w_e = \frac{1}{2}\varepsilon\lvert\overline{E}\rvert^2$ <br> $W_E = \int w_e dv$ |

**Electrostatic Boundary Condition (Conductor-Dielectric)**

| | | |
|---|---|---|
| $\overline{E} = 0 \qquad \rho_v = 0$ | $D_t = \varepsilon_o\varepsilon_r E_t = 0$ | $D_n = \varepsilon_o\varepsilon_r E_n = \rho_s$ |

**Electrostatic Boundary Condition (Conductor-Free Space)**

| | | |
|---|---|---|
| $\varepsilon_r = 1$ | $D_t = \varepsilon_o E_t = 0$ | $D_n = \varepsilon_o E_n = \rho_s$ |

**Electrostatic Boundary-Value Problems (Resistance and Capacitance)**

| | | |
|---|---|---|
| $R = \frac{V}{I} = \frac{\int \tilde{E}.d\tilde{l}}{\oint \sigma\tilde{E}.d\tilde{S}}$ | $C = \frac{Q}{V} = \frac{\varepsilon\oint \sigma\tilde{E}.d\tilde{S}}{\int \tilde{E}.d\tilde{l}}$ | Parallel-plate Capacitor <br> $C = \frac{Q}{V} = \frac{\varepsilon S}{d} \qquad R = \frac{d}{\sigma S}$ |
| Parallel-plate Capacitor <br> $W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$ | Coaxial/Cylindrical Capacitor <br> $C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}} \qquad R = \frac{\ln\frac{b}{a}}{2\pi\sigma L}$ | Spherical Capacitor <br> $C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}} \qquad R = \frac{\frac{1}{a}-\frac{1}{b}}{4\pi\sigma}$ |
| Spherical Capacitor <br> $RC = \frac{\varepsilon}{\sigma}$ | | |

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**Magnetostatics**

| | | |
|---|---|---|
| $d\overline{H} = \dfrac{Id\ell \times \overline{R}}{4\pi R^3}$ | $Id\overline{\ell} \equiv \overline{J}_s dS \equiv \overline{J}dv$ | $\nabla \times \overline{H} = \overline{J}$ |
| $H = \dfrac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\underset{\sim}{a}_\phi$ | $I_{enc} = \oint \tilde{H}.d\tilde{l} = \int \tilde{J}_s d\tilde{S}$ | $\psi_m = \int \overline{B}.d\overline{S} = BS = \int_{enc} \overline{B}.d\overline{S}$ |
| $\psi_m = \oint \bar{B}.d\bar{S} = 0$ | $\psi_m = \oint \bar{A}.d\bar{l}$ | $\nabla.\overline{B} = 0$ |
| $\overline{B} = \mu\overline{H}$ <br> $\lvert\overline{B}\rvert = \dfrac{\mu_o NI}{\ell}$ | $\overline{B} = \nabla \times \overline{A}$ | $\overline{A} = \int \dfrac{\mu_o Id\ell}{4\pi R}$ |
| $\nabla^2\overline{A} = -\mu_o\overline{J}$ | $\overline{F} = Q(\overline{E} + \overline{u} \times \overline{B}) = m\dfrac{d\overline{u}}{dt}$ | $d\overline{F} = Id\overline{\ell} \times \overline{B}$ |
| $\overline{T} = \overline{r} \times \overline{F} = \overline{m} \times \overline{B}$ | $\overline{m} = IS\underset{\sim}{a}_n$ | $H = \dfrac{I}{2\pi\rho}\underset{\sim}{a}_\phi \;\; (l = \infty)$ |

**Time Varying & Faraday Law**

| | | |
|---|---|---|
| $V_{emf} = -\dfrac{\partial\psi}{\partial t} = \dfrac{-\partial B}{\partial t}S = IR$ | $V_{emf} = -\int \dfrac{-\partial B}{\partial t} \bullet dS$ | $V_{emf} = \int (\overline{u} \times \overline{B}) \bullet d\overline{\ell}$ |
| $I_d = \int J_d \bullet d\overline{S}$ | $J_d = \dfrac{\partial\overline{D}}{\partial t}$ | $\oint \bar{E}.d\bar{l} = -\dfrac{d}{dt}\int \bar{B}.d\bar{S} = V_1 - V_2$ |
| | | |

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**Wave Propagation**

| | | |
|---|---|---|
| $\gamma = \alpha + j\beta$ | $\alpha = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\dfrac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$ | $\beta = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\dfrac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$ |
| Tangent Loss <br> $u = \tan 2\theta_\eta = \dfrac{\sigma}{\omega\varepsilon}$ | $\tan\theta = \dfrac{\sigma}{\omega\varepsilon} = \dfrac{\overline{J}_s}{\overline{J}_{ds}}$ | $\lvert\eta\rvert = \dfrac{\sqrt{\dfrac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left[\dfrac{\sigma}{\omega\varepsilon}\right]^2}}$ |
| Complex Permittivity <br> $\varepsilon_c = \varepsilon' - j\varepsilon'' = \varepsilon - j\dfrac{\sigma}{\omega}$ | | |

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### Derivatives

| | | |
|---|---|---|
| $\frac{d}{dx}\log_a U = \frac{\log_a e}{U}\frac{dU}{dx}$ | $\frac{d}{dx}\ln U = \frac{1}{U}\frac{dU}{dx}$ | $\frac{d}{dx}a^u = a^u \ln a \frac{dU}{dx}$ |
| $\frac{d}{dx}e^u = e^u \frac{dU}{dx}$ | $\frac{d}{dx}\sin U = \cos U \frac{dU}{dx}$ | $\frac{d}{dx}\cos U = -\sin U \frac{dU}{dx}$ |
| $\frac{d}{dx}\tan U = \sec^2 U \frac{dU}{dx}$ | $\frac{d}{dx}\sinh U = \cosh U \frac{dU}{dx}$ | $\frac{d}{dx}\cosh U = \sinh U \frac{dU}{dx}$ |
| $\frac{d}{dx}\tanh U = \text{sech}^2 U \frac{dU}{dx}$ | | |

### Indefinite Integrals

| | | |
|---|---|---|
| $\int e^U dU = e^U + C$ | $\int e^{ax} dU = \frac{1}{a}e^{ax}$ | $\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$ |
| $\int x^2e^{ax}dx = \frac{e^{ax}}{a^3}\left(a^2x^2-2ax+2\right)$ | $\int \ln x dx = x\ln x - x$ | $\int \sin ax dx = -\frac{1}{a}\cos ax$ |

| | |
|---|---|
| $\int \cos ax dx = \frac{1}{a}\sin ax + c$ | $\int \tan ax dx = \frac{1}{a}\ln \sec ax = -\frac{1}{a}\ln \cos ax$ |
| $\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + c$ | $\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + c$ |
| $\int x\cos ax dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$ | $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$ |
| $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$ | $\int x\sin ax dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$ |
| $\int \sec ax dx = \frac{1}{a}\ln(\sec ax + \tan ax)$ | $\int \sin ax \sin bx dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}$ |
| $\int \sinh ax dx = \frac{1}{a}\cosh ax$ | $\int \sin ax \cos bx dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$ |
| $\int \cosh ax dx = \frac{1}{a}\sinh ax + c$ | $\int \cos ax \cos bx dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}$ |
| $\int \tanh ax dx = \frac{1}{a}\ln \cosh ax$ | $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ |
| $\int \frac{x^2dx}{x^2+a^2} = x - a\tan^{-1}\frac{x}{a} + C$ | $\int \frac{xdx}{x^2+a^2} = \frac{1}{2}\ln\left(x^2+a^2\right)C$ |
| $\int \frac{xdx}{\left(x^2+a^2\right)^{\frac{3}{2}}} = -\frac{1}{\sqrt{x^2+a^2}} + C$ | $\int \frac{x^2dx}{\left(x^2+a^2\right)^{\frac{3}{2}}} = \ln\left(\frac{\sqrt{x^2+a^2}}{a} + \frac{x}{a}\right) - \frac{x}{\sqrt{x^2+a^2}} + C$ |
| $\int \frac{dx}{\left(x^2+a^2\right)^{\frac{3}{2}}} = \frac{x/a^2}{\sqrt{x^2+a^2}} + C$ | $\int \frac{dx}{\left(x^2+a^2\right)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2+a^2} + \frac{1}{a}\tan^{-1}\frac{x}{a}\right) + C$ |