UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2010/2011

COURSE NAME	:	ALGEBRA
COURSE CODE	:	DAS 10103
PROGRAMME	:	1 DAA / DAC / DAE 1 DAI / DAL / DAM / DFT
EXAMINATION DATE	:	APRIL/MAY 2011
DURATION	:	3 HOURS
INSTRUCTIONS	:	ANSWER **ALL** QUESTIONS IN PART **A** AND **THREE (3)** QUESTIONS IN PART **B**

THIS QUESTION PAPER CONSISTS OF EIGHT (8) PAGES

PART A

Q1 (a) Let $\mathbf{u} = 4\mathbf{i} + \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{w} = a\mathbf{i} - \mathbf{j} - 4b\mathbf{k}$. Find

(i) $4\mathbf{u} - 3\mathbf{v} + \mathbf{w}$

(ii) $\mathbf{u} \times \mathbf{v}$

(iii) the value of a and b if $4\mathbf{u} - 3\mathbf{v} + \mathbf{w} = \mathbf{u} \times \mathbf{v}$

(10 marks)

(b) Find the parametric equation of the line that passes through points $S(2,-1,3)$ and $T(-1,1,-2)$

(4 marks)

(c) Given three points $K_1(1,2,-1)$, $K_2(2,3,1)$ and $K_3(3,-1,2)$. Find

(i) the normal vector, **N** where $\mathbf{N} = K_1K_2 \times K_1K_3$

(ii) the equation of the plane with points K_1, K_2 and K_3 on it. Let $K_1 = K_0$

(6 marks)

Q2 (a) Given $z_1 = 3 + 4i$ and $z_2 = 1 - \sqrt{3}\,i$, find

(i) the modulus and argument for z_1

(ii) the modulus and argument for z_2

(iii) find the expression of $z_1 z_2$ in polar form

(iv) find the expression of $\frac{z_1}{z_2}$ in polar form

(8 marks)

(b) Given $z = -\sqrt{3} + i$, by using De Moivre theorem

(i) Express z in polar form

(ii) Find all the root of three for z

(12 marks)

PART B

Q3 (a) Given $7(8^p)=9(5^q)$ and $7(16^{p+1})=12(5^q)$, show that $2^p=\dfrac{1}{12}$

(4 marks)

(b) (i) Find the x values for $\log_2(x^2+2)=1+\log_2(x+5)$

(iii) Find the smallest value for n integer so that $\left(\dfrac{1}{2}\right)^n<0.001$

(10 marks)

(c) Find the set of solutions for the following inequalities

$$x+2>\frac{30}{x+1}$$

(6 marks)

Q4 (a) Given $f(x)=x^3+3x-5$. If $f(x)=0$, by using secant method, find its root, x, between the interval of [1, 2]. Iterate until $|f(x_i)|<\varepsilon=0.005$. Do the calculation in 3 decimal places.

(5 marks)

(b) Determine whether the series converge. If converge find the summation,

$$\sum_{r=1}^{\infty}\left(-\frac{1}{2}\right)^r=-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots$$

(5 marks)

(c) By using Binomial series, expand the following until the term of x^3

(i) $\dfrac{1}{1-x}$

(ii) $\sqrt{4-x}$

(10 marks)

Q5 (a) If $h = \cos 10°$ and $k = \sin 40°$, express in terms of h and / or k for

(i) $\sin 50°$

(ii) $\sin 20°$

(iii) $\cos 5°$

(11 marks)

(b) Find the all angles between $0° \leq \theta \leq 360°$ that satisfy the equation of $\tan 2\theta = 10 \tan \theta$

(9 marks)

Q6 (a) Given $A = \begin{pmatrix} 1 & -2 & 2 \\ 2 & -3 & 6 \\ 1 & 1 & 7 \end{pmatrix}$. Use steps of row-operation method below to find A^{-1}

$$-2R_1 + R_2 \rightarrow R_2$$
$$-R_1 + R_3 \rightarrow R_3$$
$$2R_2 + R_1 \rightarrow R_1$$
$$-3R_2 + R_3 \rightarrow R_3$$
$$-R_3 \rightarrow R_3$$
$$-2R_3 + R_2 \rightarrow R_2$$
$$-6R_3 + R_1 \rightarrow R_1$$

(9 marks)

(b) Given $\begin{pmatrix} 1 & 1 & 7 \\ 3 & -2 & 1 \\ 1 & -5 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 11 \end{pmatrix}$

Solve the systems of linear equation using Gauss-Seidel iteration method with $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$. Iterate until max $\left| x^{(k+1)} - x^k < 0.005 \right|$. Do the calculation to 3 decimal places.

(11 marks)

BAHAGIAN A

S1 (a) Diberi $\mathbf{u} = 4\mathbf{i} + \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$ dan $\mathbf{w} = a\mathbf{i} - \mathbf{j} - 4b\mathbf{k}$. Tentukan

(i) $4\mathbf{u} - 3\mathbf{v} + \mathbf{w}$

(ii) $\mathbf{u} \times \mathbf{v}$

(iii) nilai a dan b jika $4\mathbf{u} - 3\mathbf{v} + \mathbf{w} = \mathbf{u} \times \mathbf{v}$

(10 markah)

(b) Tuliskan persamaan parameter bagi vektor garis yang melalui titik $S(2,-1,3)$ dan $T(-1,1,-2)$.

(4 markah)

(c) Diberi tiga titik $K_1\,(1,2,-1)$, $K_2\,(2,3,1)$ dan $K_3\,(3,-1,2)$. Tentukan

(i) vektor normal $\mathbf{N}$ di mana $\mathbf{N} = K_1K_2 \times K_1K_3$

(ii) persamaan bagi satah yang mengandungi titik K_1, K_2 and K_3. Anggap $K_1 = K_0$

(6 markah)

S2 (a) Diberi $z_1 = 3 + 4i$ dan $z_2 = 1 - \sqrt{3}\,i$, cari

(i) modulus dan hujah bagi z_1

(ii) modulus dan hujah bagi z_2

(iii) dapatkan sebutan $z_1 z_2$ dalam bentuk polar

(iv) dapatkan sebutan $\frac{z_1}{z_2}$ dalam bentuk polar

(8 markah)

(b) Diberi $z = -\sqrt{3} + i$, dengan menggunakan teorem De Moivre

(i) tulis z dalam bentuk polar

(iii) kira kesemua punca kuasa tiga bagi z

(12 marks)

BAHAGIAN B

S3 (a) Diberi $7(8^p) = 9(5^q)$ dan $7(16^{p+1}) = 12(5^q)$, tunjukkan $2^p = \frac{1}{12}$

(4 markah)

(b) (i) Cari nilai x bagi persamaan $\log_2(x^2 + 2) = 1 + \log_2(x + 5)$

(ii) Kira nilai terkecil bagi integer n supaya $\left(\frac{1}{2}\right)^n < 0.001$

(10 markah)

(c) Dapatkan set penyelesaian bagi ketaksamaan berikut

$$x + 2 > \frac{30}{x + 1}$$

(6 markah)

S4 (a) Diberi $f(x) = x^3 + 3x - 5$. Jika $f(x) = 0$, dengan menggunakan kaedah sekan, kira punca, x, antara selang [1, 2]. Lelarkan hingga $|f(x_i)| < \varepsilon = 0.005$. Buat pengiraan sehingga 3 titik perpuluhan.

(5 markah)

(b) Tentukan sama ada janjang berikut menumpu. Jika ya, kira hasil tambahnya,

$$\sum_{r=1}^{\infty}\left(-\frac{1}{2}\right)^r = -\frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$$

(5 markah)

(c) Dengan menggunakan janjang binomial, kembangkan janjang berikut hingga sebutan x^3

(i) $\frac{1}{1 - x}$

(ii) $\sqrt{4 - x}$

(10 markah)

S5 (a) Jika $h = \cos 10°$ dan $k = \sin 40°$, tuliskan dalam sebutan h dan/atau k bagi

(i) $\sin 50°$

(ii) $\sin 20°$

(iii) $\cos 5°$

(11 markah)

(b) Kira semua sudut antara $0° \le \theta \le 360°$ yang memenuhi persamaan $\tan 2\theta = 10 \tan \theta$

(9 markah)

S6 (a) Diberi $A = \begin{pmatrix} 1 & -2 & 2 \\ 2 & -3 & 6 \\ 1 & 1 & 7 \end{pmatrix}$. Gunakan langkah-langkah kaedah operasi baris berikut untuk dapatkan A^{-1}

$$-2R_1 + R_2 \to R_2$$
$$-R_1 + R_3 \to R_3$$
$$2R_2 + R_1 \to R_1$$
$$-3R_2 + R_3 \to R_3$$
$$-R_3 \to R_3$$
$$-2R_3 + R_2 \to R_2$$
$$-6R_3 + R_1 \to R_1$$

(9 markah)

(b) Diberi $\begin{pmatrix} 1 & 1 & 7 \\ 3 & -2 & 1 \\ 1 & -5 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 11 \end{pmatrix}$

Selesaikan sistem persamaan linear yang diberikan menggunakan kaedah lelaran Gauss-Seidel dengan $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$. Lelarkan sehingga maksimum $|x^{(k+1)} - x^k < 0.005|$. Buat pengiraan sehingga 3 titik perpuluhan.

(11 markah)

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Formulae

Arithmetic Sequences	**Geometric Sequences**	**Binomial Series**
(i) $u_n = a + (n-1)d$ (ii) $d = u_n - u_{n-1}$ (iii) $S_n = \frac{n}{2}(a + u_n)$ (iv) $S_n = \frac{n}{2}[2a + (n-1)d]$	(i) $u_n = ar^{n-1}$ (ii) $r = \frac{u_n}{u_{n-1}}$ (iii) $S_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$ (iv) $S_n = \frac{a(r^n - 1)}{r-1}$ if $r > 1$ (v) $S_\infty = \frac{a}{1-r}$	For any positive integer n $(1+x)^r = 1 + rx + \frac{r(r-1)}{2!}x^2 + \frac{r(r-1)(r-2)}{3!}x^3 + \ldots$ $\lvert x \rvert < 1$, r any real number

Trigonometry

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

De Moivre's Theorem

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{\theta + 360k}{n} + i\sin\frac{\theta + 360k}{n}\right), \qquad k = 0,1,2,3,\ldots$$