Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	: ELECTROMAGNETIC TECHNOLOGY
COURSE CODE	: BNR 20603
PROGRAMME	: BND
EXAMINATION DATE	: DEC 2013 / JAN 2014
DURATION	: 2 HOURS 30 MINUTES
INSTRUCTION	: A) ANSWER ONE (1) **QUESTION**
	B) **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

SECTION A (ANSWER ONE QUESTION ONLY)

Q1 A feed through capacitor (shown in **Figure Q1 (a)**) is a type of capacitor for bypassing RF voltage frequencies to ground of a transmitter's chassis. It is built by putting a dielectric around a conductor and encasing it in a metallic tube, so that the conductor is one plate and the outer tube is the second plate.

An engineer decided to design a 4 cm long feed through capacitor with a separation distance d = 1 cm. The inner conductor is made of solid copper (radius, a = 0.5 cm) and the outer conductor is made of copper tube with inner side coated with 0.2 cm glass (ε_r = 6.5) as shown in **Figure Q1 (b).** The leftover space between the conductors is air with dielectric strength 30 kV/cm. Assuming that the copper is a perfect electric conductor,

(a) Name the Maxwell equation (and also in point form and integral form) that can be used to determine **E** between the conductors of the capacitor.

(9 marks)

(b) Explain your answer in **Q1(a)** in one sentence.

(5 marks)

(b) Develop expressions for the electric field **E**, potential different V, and capacitance C for all regions.

(10 marks)

(c) Solve the capacitance of the air and the glass coating.

(7 marks)

(d) Analyse why when being put under a load of 30kV, suddenly the air between the conductors breaks down.

(Hints: $Q=C_1V_1=C_2V_2$, $V_{total}=V_1+V_2$, and Dielectric strength = *Voltage / distance*)

(9 marks)

(Total 40 marks)

Q2 (a) States the difference between Ampere's law and Biot Savart's law.

(5 marks)

(b) A communication engineer decided to use a coaxial cable as the transmission line for a mission critical communication link. The coaxial cable has an inner radius a and an outer conductor has inner radius b with thickness t. If we assume that the current $+I$ is along the positive z-direction and is uniformly distributed in both conductors,

(i) Provides **ONE (1)** reason why a coaxial cable is chosen.

(6 marks)

(ii) Calculate **H** everywhere.

(10 marks)

(iii) Calculate **B** at the location between the two conductors (assuming free space).

(5 marks)

(iv) Sketch a graph depicting the relationship between the magnitude of **H** and the radial distance from the center or the coaxial cable.

(7 marks)

(v) If a conductor carrying a current is brought near to the coaxial cable, explain what will happen to the conductor and why.

(7 marks)

(Total 40 marks)

SECTION B (ANSWER ALL QUESTIONS)

Q3 The AC electrical supply that we enjoyed today is as a consequence of Faraday's Law where the electromagnetic generator converts mechanical motion to an AC electrical supply. As illustrated in **Figure Q3**, a conductive loop is turned in the presence of the magnetic field from a permanent magnet. The loop rotates with an angular velocity ω radians per second.

(a) Explain mathematically why side a, and c are not contributing to the total V_{emf}, thus can be ignored.

(6 marks)

(b) Produce an equation for velocity **u** (angular velocity).

(8 marks)

(c) Calculate the total V_{emf} generated when the loop is turning.

(8 marks)

(d) An engineer decided to generate some electricity by turning the loop at the rate of 120 revolutions per minute in the presence of 60 mT field. Plot V_{emf} versus time t. Assume $\varphi = 0^{o}$ at $t = 0$.

(8 marks)

(Total 30 marks)

Q4 "On 31 October 2013, airline passengers will soon be able to use certain electronic devices throughout their entire flight after the U.S. Federal Aviation Administration ended a long-standing ban".

As a consequence, some parts of an aircraft require some lossy material to absorb some electromagnetic fields generated by certain electronic devices.

(a) Explain the term lossy material in terms of its conductivity, permittivity and permeability.

(4 marks)

(b) A certain mobile phone produces a plane wave propagating through the lossy material, at a particular radian frequency ω with the **H** component of

$$5e^{-\alpha x}\cos\left(\varpi t - \frac{1}{4}x\right)a_y \ {}^{A}\!/_{m}$$

(i) Solve **E** if the lossy dielectric of his choice has an intrinsic impedance of $100e^{j\pi/6}$ at that particular radian frequency.

(8 marks)

(ii) Explain the term "skin depth of a material". Calculate α and then the minimum depth of this material for it to be effective.

(7 marks)

(iii) Define and calculate the lost tangent of the material.

(5 marks)

(iv) Based on the calculated skin depth and loss tangent in **Q4** b (ii) and **Q4** b (iii) respectively, analyse if the material can be used in an airplane realistically.

(6 marks)

(Total 30 marks)

- END OF QUESTION -

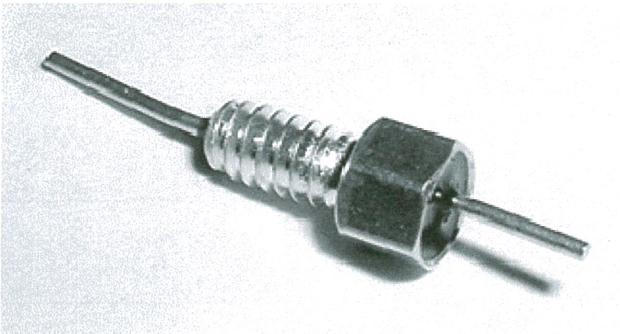

FIGURE Q1 (a)

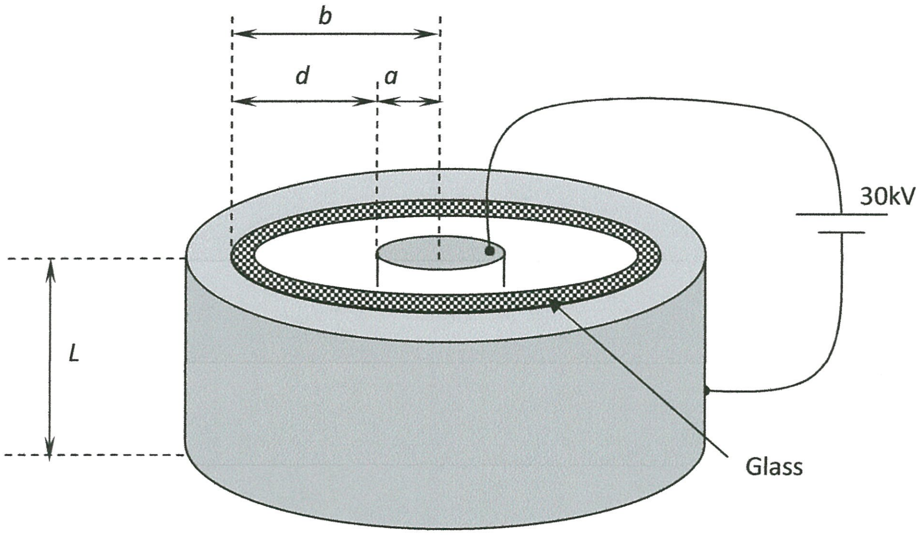

FIGURE Q1 (b)

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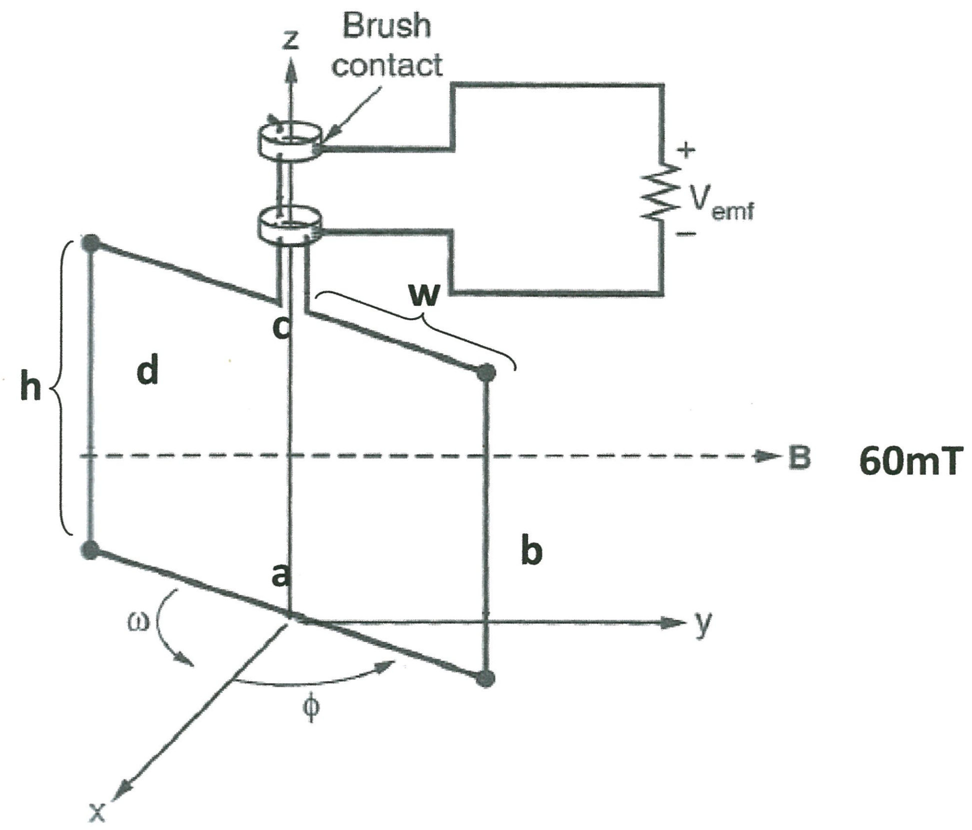

h= 10 cm

w = 8 cm

FIGURE Q3

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Formula

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}}$$

Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{r}\left[\frac{\partial (rA_r)}{\partial r}\right] + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{R^2}\frac{\partial (R^2 A_R)}{\partial R} + \frac{1}{R\sin\theta}\left[\frac{\partial (A_\theta \sin\theta)}{\partial \theta}\right] + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{r}\left(\frac{\partial (rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \frac{1}{R\sin\theta}\left[\frac{\partial (\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{R}} + \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial (RA_\phi)}{\partial R}\right]\hat{\boldsymbol{\theta}} + \frac{1}{R}\left[\frac{\partial (RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta}\right]\hat{\boldsymbol{\varphi}}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

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	Cartesian	**Cylindrical**	**Spherical**
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\phi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\varphi}}$
Magnitude $\vec{A}$	$\sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$	$\sqrt{A_r{}^2 + A_\phi{}^2 + A_z{}^2}$	$\sqrt{A_R{}^2 + A_\theta{}^2 + A_\phi{}^2}$
Position vector, $\overrightarrow{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1, y_1, z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1, \phi_1, z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1, \theta_1, \phi_1)$
Unit vector product	$\hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\bullet\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}$	$\hat{\mathbf{R}}\bullet\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = 1$ $\hat{\mathbf{R}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A}\bullet\vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A}\times\vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\varphi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\varphi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\overrightarrow{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\hat{\boldsymbol{\varphi}}$
Differential surface, $\overrightarrow{ds}$	$\overrightarrow{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\overrightarrow{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\overrightarrow{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\overrightarrow{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\varphi}}$ $\overrightarrow{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\overrightarrow{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\overrightarrow{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\varphi}}$
Differential volume, $\overrightarrow{dv}$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Transformation	**Coordinate Variables**	**Unit Vectors**	**Vector Components**
Cartesian to Cylindrical	$r=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{\mathbf{r}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
Cylindrical to Cartesian	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{\mathbf{x}}=\hat{\mathbf{r}}\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{r}}\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_x=A_r\cos\phi-A_\phi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Cartesian to Spherical	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{\mathbf{R}}=\hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
Spherical to Cartesian	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{\mathbf{x}}=\hat{\mathbf{R}}\sin\theta\cos\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{R}}\sin\theta\sin\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical to Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{\mathbf{R}}=\hat{\mathbf{r}}\sin\theta+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{r}}\cos\theta-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
Spherical to Cylindrical	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{\mathbf{r}}=\hat{\mathbf{R}}\sin\theta+\hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

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$$Q = \int \rho_\ell d\ell,$$

$$Q = \int \rho_s dS,$$

$$Q = \int \rho_v dv$$

$$\bar{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$$

$$\bar{E} = \frac{\bar{F}}{Q},$$

$$\bar{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\bar{E} = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\bar{E} = \int \frac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\bar{E} = \int \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\bar{D} = \varepsilon\bar{E}$$

$$\psi_e = \int \bar{D} \bullet d\bar{S}$$

$$Q_{enc} = \oint_S \bar{D} \bullet d\bar{S}$$

$$\rho_v = \nabla \bullet \bar{D}$$

$$V_{AB} = -\int_A^B \bar{E} \bullet d\bar{\ell} = \frac{W}{Q}$$

$$V = \frac{Q}{4\pi\varepsilon r}$$

$$V = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon r}$$

$$\oint \bar{E} \bullet d\bar{\ell} = 0$$

$$\nabla \times \bar{E} = 0$$

$$\bar{E} = -\nabla V$$

$$\nabla^2 V = 0$$

$$R = \frac{\ell}{\sigma S}$$

$$I = \int \bar{J} \bullet dS$$

$$d\bar{H} = \frac{Id\bar{\ell} \times \bar{R}}{4\pi R^3}$$

$$Id\bar{\ell} \equiv \bar{J}_s dS \equiv \bar{J}dv$$

$$\oint \bar{H} \bullet d\bar{\ell} = I_{enc} = \int \bar{J}_s dS$$

$$\nabla \times \bar{H} = \bar{J}$$

$$\psi_m = \int_s \bar{B} \bullet d\bar{S}$$

$$\psi_m = \oint \bar{B} \bullet d\bar{S} = 0$$

$$\psi_m = \oint \bar{A} \bullet d\bar{\ell}$$

$$\nabla \bullet \bar{B} = 0$$

$$\bar{B} = \mu\bar{H}$$

$$\bar{B} = \nabla \times \bar{A}$$

$$\bar{A} = \int \frac{\mu_0 Id\bar{\ell}}{4\pi R}$$

$$\nabla^2 \bar{A} = -\mu_0 \bar{J}$$

$$\bar{F} = Q(\bar{E} + \bar{u} \times \bar{B}) = m\frac{d\bar{u}}{dt}$$

$$d\bar{F} = Id\bar{\ell} \times \bar{B}$$

$$\bar{T} = \bar{r} \times \bar{F} = \bar{m} \times \bar{B}$$

$$\bar{m} = IS\hat{a}_n$$

$$V_{emf} = -\frac{\partial\psi}{\partial t}$$

$$V_{emf} = -\int \frac{\partial\bar{B}}{\partial t} \bullet d\bar{S}$$

$$V_{emf} = \int (\bar{u} \times \bar{B}) \bullet d\bar{\ell}$$

$$I_d = \int J_d . d\bar{S}, J_d = \frac{\partial\bar{D}}{\partial t}$$

$$\gamma = \alpha + j\beta$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\bar{F}_1 = \frac{\mu I_1 I_2}{4\pi}\oint_{L1}\oint_{L2} \frac{d\bar{\ell}_1 \times (d\bar{\ell}_2 \times \hat{a}_{R_{21}})}{R_{21}^{\ 2}}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$$

$$tan\, 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$$

$$tan\,\theta = \frac{\sigma}{\omega\varepsilon} = \frac{\bar{J}_s}{\bar{J}_{ds}}$$

$$\delta = \frac{1}{\alpha}$$

$$\varepsilon_0 = 8.854 \times 10^{-12}\ \text{Fm}^{-1}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$$

$$\int \frac{dx}{(x^2 + c^2)^{3/2}} = \frac{x}{c^2(x^2 + c^2)^{1/2}}$$

$$\int \frac{xdx}{(x^2 + c^2)^{3/2}} = \frac{-1}{(x^2 + c^2)^{1/2}}$$

$$\int \frac{dx}{(x^2 \pm c^2)^{1/2}} = ln\left(x + \sqrt{x^2 \pm c^2}\right)$$

$$\int \frac{dx}{(x^2 + c^2)} = \frac{1}{c}tan^{-1}\left(\frac{x}{c}\right)$$

$$\int \frac{xdx}{(x^2 + c^2)} = \frac{1}{2}ln(x^2 + c^2)$$

$$\int \frac{xdx}{(x^2 + c^2)^{1/2}} = \sqrt{x^2 + c^2}$$