Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	SIGNALS AND SYSTEMS
COURSE CODE	:	BEJ 20203
PROGRAMME	:	BEJ
EXAMINATION DATE	:	DECEMBER 2019/JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

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THIS QUESTION PAPER CONSISTS OF **FOURTEEN (14)** PAGES

Q1 (a) Given an arbitrary signal, $x(t)$ as shown in **Figure Q1(a)**. Plot the following functions:

(i) $x(t)\delta(t+1) + x(3t)$,

(2 marks)

(ii) $0.5x(2t+2)u(t+1)$,

(3 marks)

(iii) odd and even part of $x(t)$.

(5 marks)

(b) Based on the system in **Figure Q1(b)**, given that

$$x(t) = 2u(t+2) - u(t) - u(t-2)$$

and

$$y(t) = 2u(t+1.5) + (2t-2)u(t) - 2tu(t-2).$$

Express the signal $g(t)$ using a single analytical expression with the aid of unit step function, $u(t)$.

(10 marks)

Q2 (a) A system can be classified into several types of system. State **FOUR (4)** types of system classification and its pair.

(4 marks)

(b) A continuous time input signal $x(t) = u(t) - u(t-1)$ is passed to a parallel connection shown in **Figure Q2(b)**, which the impulse response $h_1(t)$ and $h_2(t)$ are given by

$$h_1(t) = r(t)$$

$$h_2(t) = u(t) - u(t-2).$$

(i) Show the overall impulse response, $h(t)$ as in **Figure Q2(b)(i)**.

(2 marks)

(ii) Compute the output, $y(t)$ of the system.

(12 marks)

(iii) Deduce the stability of the system and provide your justification.

(2 marks)

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Q3 Given a periodic signal $x_1(t)$ as follows:

$$x_1(t) = \begin{cases} -\frac{4}{T}t & ; -\frac{T}{2} < t < 0 \\ \frac{4}{T}t & ; 0 < t < \frac{T}{2} \end{cases}$$

(a) (i) Determine the symmetry type of the signal $x_1(t)$.

(1 mark)

(ii) State the trigonometric Fourier series coefficients (a_0, a_n and b_n) of the signal $x_1(t)$ based on the symmetry properties selected in **Q3(a)(i)**.

(2 marks)

(iii) Proof the value in **Q3(a)(ii)** by computing the trigonometric Fourier series coefficients (a_0, a_n and b_n) of the signal $x_1(t)$.

(12 marks)

(b) The signal $x_2(t)$ with the period $T = 10^{-5}s$ is passed through Linear Time Invariant (LTI) system with frequency response, $H(f)$ given in **Figure Q3(b)**. Calculate the output $y(t)$ of the system.

$$x_2(t) = 4 - \frac{2}{\pi}\left(e^{j\pi t} + e^{-j\pi t}\right) - \frac{1}{\pi}\left(e^{j2\pi t} + e^{-j2\pi t}\right) - \frac{2}{3\pi}\left(e^{j3\pi t} + e^{-j3\pi t}\right)$$
$$- \frac{1}{2\pi}\left(e^{j4\pi t} + e^{-j4\pi t}\right) - \frac{2}{5\pi}\left(e^{j5\pi t} + e^{-j5\pi t}\right)$$

(5 marks)

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Q4. (a) Explain how Fourier transform has been developed from Fourier series and applicable for Fourier representation of non-periodic signals.

(5 marks)

(b) **Figure Q4(b)** shows an RC circuit with an input signal, $V_i(t) = 3e^{-5t}u(t)$. Determine;

(i) the differential equation of the RC circuit,

(2 marks)

(ii) the frequency response, $H(\omega)$ of the circuit. Given that $R = 3\Omega$ and $C = 2F$,

(3 marks)

(iii) the output voltage, $v_0(t)$.

(5 marks)

(c) Given two signals

$$x_1(t) = 4 \text{ rect}\left(\frac{t-3}{4}\right)$$

and

$$x_2(t) = 2 \text{ rect}\left(\frac{t}{2}\right).$$

Prove that the Fourier transform of signal $y(t) = x_1(t) + x_2(t)$ is

$$Y(\omega) = \frac{4}{j\omega}\left(e^{-j5\omega} + \cos\omega\right).$$

(5 marks)

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Q5. (a) Define region of convergence (ROC).

(2 marks)

(b) Write THREE (3) properties of ROC of Laplace transform.

(3 marks)

(c) The output of an LTI system can be easily determined in s-domain using the convolution property of Laplace transform. If a signal

$$x(t) = e^{-2t}\big(u(t) - u(t-3)\big)$$

is an input to a system with the impulse response given by

$$h_1(t) = 3e^{-3t}u(t).$$

(i) Determine the output $y(t)$ using the Laplace transform convolution property.

(10 marks)

(ii) The system $h_1(t)$ is connected in series to another system $h_2(t)$ with its transfer function given by

$$H(s) = \frac{s-1}{s-2}$$

forming a new system $h(t)$ as shown in **Figure Q5(c)(ii)**. Determine the total response of this system, $h(t)$ if the system is stable.

(5 marks)

- END OF QUESTIONS -

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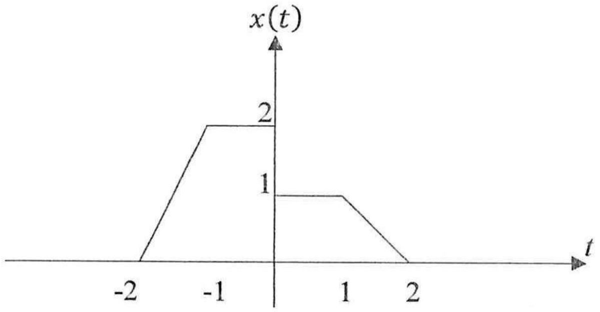

Figure Q1(a)

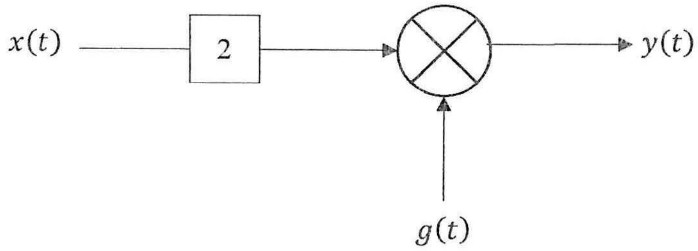

Figure Q1(b)

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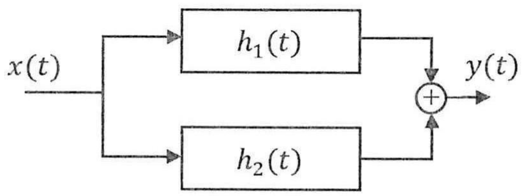

Figure Q2(b)

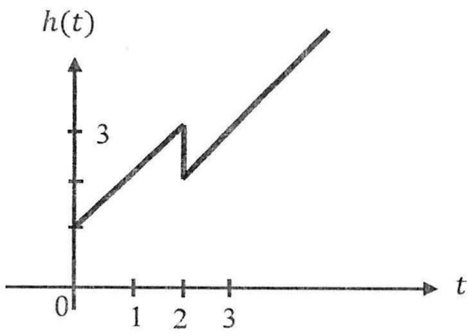

Figure Q2(b)(i)

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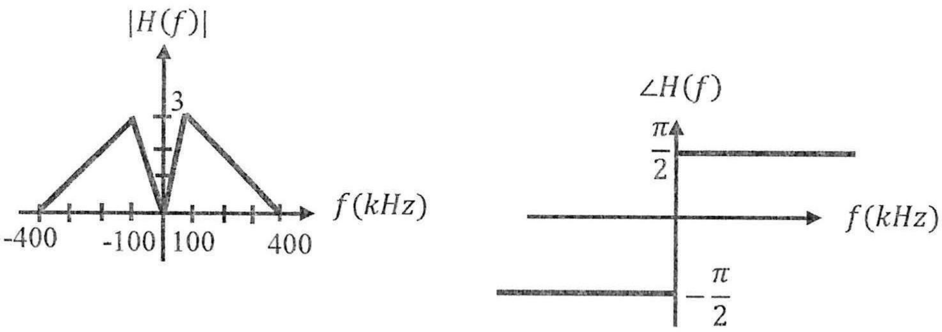

Figure Q3(b)

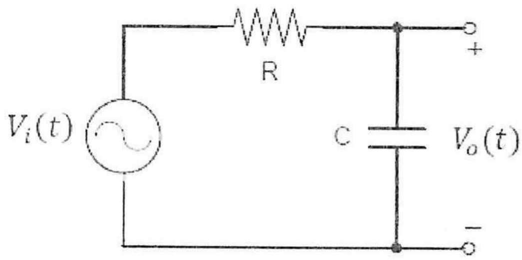

Figure Q4(b)

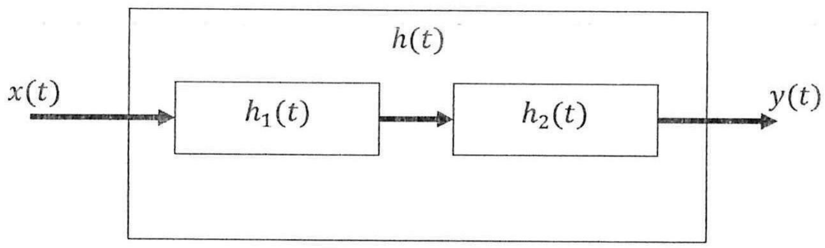

Figure Q5(c)(ii)

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TABLE 1: INDEFINITE INTEGRALS

$\int \cos at\,dt = \frac{1}{a}\sin at$	$\int \sin at\,dt = -\frac{1}{a}\cos at$
$\int t\cos at\,dt = \frac{1}{a^2}\cos at + \frac{1}{a}t\sin at$	$\int t\sin at\,dt = \frac{1}{a^2}\sin at - \frac{1}{a}t\cos at$
$\int te^{at}dt = \frac{1}{a^2}e^{at}(at-1)$	$\int \frac{1}{(a^2+t^2)}dt = \frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)$

TABLE 2: EULER'S IDENTITY

$e^{\pm j\pi/2} = \pm j$	$A\angle \pm \theta = Ae^{\pm j\theta}$
$e^{\pm jk\pi} = \cos k\pi$	$e^{\pm}j\theta = \cos\theta \pm j\sin\theta$
$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$	$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$

TABLE 3: COMPLEX NUMBER

$s = a + jb = \lvert s\rvert\angle \pm \theta = \lvert s\rvert e^{\pm j\theta}$	$\lvert s\rvert = \sqrt[2]{a^2+b^2}$	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

TABLE 4: TRIGONOMETRIC IDENTITIES

$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right))$	$\cos\theta = \sin(\theta + \frac{\pi}{2})$
$\sin(\propto \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin^2\alpha + \cos^2\beta = 1$	
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos\ 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 5: VALUES OF COSINE, SINE AND EXPONENTIAL FUNCTIONS FOR INTEGRAL MULTIPLE OF π.

Function	Value	Function	Value
$\cos 2n\pi$	1	$e^{j2n\pi}$	1
$\sin 2n\pi$	0	$e^{jn\pi}$	$(-1)^n$
$\cos n\pi$	$(-1)^n$	$e^{\frac{jn\pi}{2}}$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = \text{even} \\ j(-1)^{\frac{n-1}{2}} & ,n = \text{odd} \end{cases}$
$\sin n\pi$	0		
$\cos\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = \text{even} \\ 0 & ,n = \text{odd} \end{cases}$	$\sin\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n-1}{2}} & ,n = \text{odd} \\ 0 & ,n = \text{even} \end{cases}$

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TABLE 6: PARTIAL FRACTION FORMULA

Type of proper rational function	Partial Fraction
$\dfrac{px+q}{(x-a)(x-b)}, a \neq b$	$\dfrac{A}{x-a}+\dfrac{B}{x-b}$
$\dfrac{px^2+qx+r}{(x-a)(x-b)(x-c)}, a \neq b \neq c$	$\dfrac{A}{x-a}+\dfrac{B}{x-b}+\dfrac{C}{x-c}$
$\dfrac{px+q}{(x-a)^3}$	$\dfrac{A}{x-a}+\dfrac{B}{(x-a)^2}+\dfrac{C}{(x-a)^3}$
$\dfrac{px^2+qx+r}{(x-a)^2(x-b)}$	$\dfrac{A}{x-a}+\dfrac{B}{(x-a)^2}+\dfrac{C}{x-b}$
$\dfrac{px^2+qx+r}{(x-a)(x^2+bx+c)}$, where x^2+bx+c cannot be factorised.	$\dfrac{A}{x-a}+\dfrac{Bx+C}{x^2+bx+c}$
$\dfrac{px^3+qx^2+rx+s}{(x^2+ax+b)(x^2+cx+d)}$, where (x^2+ax+b) and (x^2+cx+d) cannot be factorised.	$\dfrac{Ax+B}{x^2+ax+b}+\dfrac{Cx+D}{x^2+cx+d}$

TABLE 7: FOURIER SERIES

Exponential	$x(t)=\sum_{n=-\infty}^{\infty} x_n e^{jn\frac{2\pi}{T}t}$ $x_n=\frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\, e^{-jn\frac{2\pi}{T}t}\, dt$
Trigonometric	$x(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n \cos n\frac{2\pi}{T}t + b_n \sin n\frac{2\pi}{T}t\right)$ $a_n=\frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\cos n\frac{2\pi}{T}t\, dt, \quad n=0,1,2,3\ldots$ $b_n=\frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\sin n\frac{2\pi}{T}t\, dt, \quad n=1,2,3\ldots$
Amplitude-phase	$x(t)=X_0+\sum_{n=1}^{\infty} A_n \cos(n\frac{2\pi}{T}t+\theta_n)$ $A_n=2\lvert X_n\rvert=\sqrt[2]{a_n^2+b_n^2} \quad , \quad \theta_n=\angle X_n=-\tan^{-1}\left(\frac{b_n}{a_n}\right)$
Average Power	$P=V_{dc}I_{dc}+\frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_{V_n}-\theta_{I_n})$

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TABLE 8: DEFINITION OF FOURIER AND LAPLACE TRANSFORM

FOURIER TRANSFORM	INVERSE FOURIER TRANSFORM
$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ $\mathcal{F}[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$	$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$ $x(t) = \mathcal{F}^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df$
LAPLACE TRANSFORM	**INVERSE LAPLACE TRANSFORM**
Bilateral $\mathrm{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$ **Unilateral** $\mathrm{L}[x(t)] = X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$ $s = \sigma + j\omega$	$x(t) = \mathrm{L}^{-1}[X(s)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$

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TABLE 9: FOURIER TRANSFORM PAIRS

Time domain, $x(t)$	**Frequency domain,** $X(\omega)$	**Frequency domain,** $X(f)$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	$\frac{\delta(f)}{2}+\frac{1}{j2\pi f}$
$u(t+\tau)-u(t-\tau)$	$\frac{2\sin(\omega\tau)}{\omega}=2\tau\,\mathrm{sinc}(\omega\tau)$	$2\tau\,\mathrm{sinc}\,2f\tau$
$\mathrm{rect}(t)$	$\mathrm{sinc}(\frac{\omega}{2})$	$\mathrm{sinc}(f)$
$\|t\|$	$-\frac{2}{\omega^2}$	$-\frac{2}{(2\pi f)^2}$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$\frac{1}{\alpha+j2\pi f}$
$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$\frac{1}{\alpha-j2\pi f}$
$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$\frac{2a}{a^2+4\pi^2f^2}$
$e^{j\omega_o t}$	$2\pi\delta(\omega-\omega_0)$	$\delta(f-f_0)$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$\frac{n!}{(a+j2\pi f)^{n+1}}$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$\frac{\delta(f-f_o)-\delta(f+f_o)}{2j}$
$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$	$\frac{\delta(f-f_o)+\delta(f+f_o)}{2}$
$e^{-at}\sin\omega_0 t\,u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$\frac{2\pi f_0}{(a+j2\pi f)^2+(2\pi f_0)^2}$
$e^{-at}\cos\omega_0 t\,u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$\frac{a+2\pi f}{(a+j2\pi f)^2+(2\pi f_0)^2}$

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TABLE 10: FOURIER TRANSFORM PROPERTIES

Property	Time domain, $x(t)$	Frequency domain, $X(\omega)$	Frequency domain, $X(f)$
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(\omega)+a_2X_2(\omega)$	$a_1X_1(f)+a_2X_2(f)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
Time shifting	$x(t-t_0)u(t-t_0)$	$e^{-j\omega t_0}X(\omega)$	$e^{-j2\pi f t_0}X(f)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$	$X(f-f_o)$
Modulation	$\cos(\omega_0 t)\,x(t)$ $\sin(\omega_0 t)\,x(t)$	$\frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$ $\frac{1}{2j}[X(\omega-\omega_0)-X(\omega+\omega_0)]$	$\frac{1}{2}[X(f-f_o)+X(f+f_o)]$ $\frac{1}{2j}[X(f-f_o)-X(f+f_o)]$
Time differentiation	$\frac{d}{dt}(x(t))$ $\frac{d^n}{dt^n}(x(t))$	$j\omega X(\omega)$ $(j\omega)^n X(\omega)$	$j2\pi f X(f)$ $(j2\pi f)^n X(f)$
Time integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{X(\omega)}{j\omega}+\pi X(\omega)\,\delta(\omega)$	$\frac{X(f)}{j2\pi f}+\frac{1}{2}X(0)\delta(f)$
Frequency differentiation	$t^n x(t)$	$(j)^n\frac{d^n}{d\omega^n}X(\omega)$	$\left(\frac{j}{2\pi}\right)^n\frac{d^n}{df^n}X(f)$
Time Reversal	$x(-t)$	$X(-\omega)$ or $X^*(\omega)$	$X(-f)$
Duality	$X(t)$	$2\pi x(-\omega)$	$X(-f)$
Convolution in t	$x_1(t)*x_2(t)$	$X_1(\omega)\cdot X_2(\omega)$	$X(f)\cdot Y(f)$
Multiplication	$x_1(t)\cdot x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$	$X(f)*Y(f)$
Parseval's Theorem	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2d\omega$	$\int_{-\infty}^{\infty}\lvert X(f)\rvert^2df$

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TABLE 11: LAPLACE TRANSFORM PAIR

Time domain, $x(t)$, $t > 0$	s-domain, $X(s)$	ROC	Time domain, $x(t)$, $t > 0$	s-domain, $X(s)$	ROC
$\delta(t)$	1	All s	$\cos bt$	$\frac{s}{s^2+b^2}$	$Re(s) > 0$
$u(t)$	$\frac{1}{s}$	$Re(s) > 0$	$\sin bt$	$\frac{b}{s^2+b^2}$	$Re(s) > 0$
t	$\frac{1}{s^2}$	$Re(s) > 0$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$Re(s) > -a$
t^n	$\frac{n!}{s^{n+1}}$	$Re(s) > 0$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$	$Re(s) > -a$
e^{-at}	$\frac{1}{s+a}$	$Re(s) > -a$	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$	$Re(s) > 0$
te^{-at}	$\frac{1}{(s+a)^2}$	$Re(s) > -a$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s) > 0$

TABLE 12: LAPLACE TRANSFORM PROPERTIES

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t), x_2(t)$	$X_1(s), X_2(s)$	R_1, R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s-s_0)$	Shifted version of R (i.e., s is in the ROC if $s-s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s) \cdot X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
		$sX(s) - x(0^+)$ (Unilateral)	R right hand plane
	$\frac{d^n}{dt^n}x(t)$	$s^nX(s) - s^{n-1}x(0^+) - \cdots - sx^{n-2}(0^+) - x^{n-1}(0^+)$	
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{Re(s) > 0\}$

Initial- and Final- Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher order singularities at $t = 0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and has a finite limit as $t \to \infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to\infty} sX(s)$$

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