CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	SIGNALS AND SYSTEMS
COURSE CODE	:	BEB 20203
PROGRAMME CODE	:	BEJ
EXAMINATION DATE	:	DECEMBER 2019 /JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A: ANSWER **ALL** QUESTIONS SECTION B: ANSWER **THREE (3)** QUESTIONS ONLY

TERBUKA

THIS QUESTION PAPER CONSISTS OF **FOURTEEN (14)** PAGES

CONFIDENTIAL

SECTION A: ANSWER ALL QUESTION

Q1. A system in **Figure Q1** produces an output $y(t)$ which is represented as:

$$y(t) = 2[u(t+2) - u(t-2)] - [u(t+1) - u(t-1)].$$

By using graphical method, find the input signal $x(t)$ if $g(t)$ is given by

$$g(t) = u(t+1) - u(t-1).$$

(10 marks)

Q2 A periodic signal $x(t)$ is defined by

$$x(t) = 2 + 3\cos(2\omega_0 t) + 4\sin(4\omega_0 t + 45°)$$

(i) Determine the exponential Fourier series coefficients of the signal $x(t)$.

(6 marks)

(ii) Sketch the magnitude and phase spectrum of the signal $x(t)$.

(4 marks)

Q3 Given two signals,

$$x_1(t) = 0.5\,\text{rect}\left(\frac{t}{6}\right),$$

$$x_2(t) = \text{rect}\left(\frac{t}{2}\right).$$

(i) Sketch $y(t) = x_1(t) + x_2(t)$.

(4 marks)

(ii) Find the Fourier transform of $y(t)$ using the definition of Fourier transform.

(6 marks)

Q4. The signal in **Figure Q4** is given by

$$x(t) = 2e^{-2|t|}.$$

(i) Find the Laplace transform of $x(t)$ and plot its corresponding region of convergence (ROC).

(7 marks)

(ii) Determine the Laplace transform of $y(t)$, if the signals is transformed into
$$y(t) = x(2t-1).$$

(3 marks)

SECTION B: ANSWER THREE (3) QUESTIONS ONLY

Q5 (a) Test the stability of the following continuous-time systems.

(i) $h_1(t) = e^{2t}u(t)$

(2.5 marks)

(ii) $h_2(t) = e^{-4t}u(t)$

(2.5 marks)

(b) Using the graphical method of convolution integral, determine and sketch the output $y(t)$ for the input signal $x(t)$ and system impulse response $h(t)$ given as follow:

$$x(t) = u(t) - 2u(t-1) + u(t-2)$$

$$h(t) = \begin{cases} t & ,0 \leq t \leq 1 \\ 0 & ,elsewhere \end{cases}$$

(15 marks)

Q6 (a) A periodic signal $x(t)$ is given in **Figure Q6(a)**.

(i) Show that the exponential Fourier series of $x(t)$ is

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{4}\text{sinc}\left(\frac{n}{4}\right) e^{j\pi n 10^3 t}.$$

(6 marks)

(ii) Plot the magnitude of the Fourier series coefficient for $n = 0, \pm 1, \pm 2, \pm 3$.

(4 marks)

(b) The signal $x(t)$ in **Q6(a)** is an input to a system with frequency response given in **Figure Q6(b)**.

(i) Determine the output response, $y(t)$ of the system.

(5 marks)

(ii) Determine the percentage of output to input power.

(5 marks)

Q7 (a) Consider the system shown in **Figure Q7(a)(i)**, where the frequency response $H(\omega)$ has magnitude as shown in **Figure Q7(a)(ii)**. Given $x_1(t) = \cos(\pi t)$ and $x_2(t) = \cos(2\pi t)$.

(i) Determine the mathematical expression of $x(t)$ and its corresponding Fourier transform $X(\omega)$.

(4 marks)

(ii) Sketch the magnitude spectrum of $Y(\omega)$.

(6 marks)

(iii) Determine the output signal, $y(t)$.

(3 marks)

(b) By using the Fourier transform, calculate the voltage, $v(t)$ and the current, $i(t)$ in the circuit shown in **Figure Q7(b)** when the input , $e(t) = u(t) - u(t-2)\ V$.

(7 marks)

Q8 (a) Find the system function and impulse response for

$$\frac{dy(t)}{dt} + 3y(t) = x(t).$$

(5 marks)

(b) A cascaded system in **Figure Q8(b)** has the following transfer functions;

$$H_1(s) = \frac{1}{s+3}, \qquad H_2(s) = \frac{1}{s-2}$$

(i) Solve the transfer function $H(s)$.

(3 marks)

(ii) Determine all possible impulse responses of the system and its respective stability and causality.

(12 marks)

-END OF QUESTIONS-

TERBUKA

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

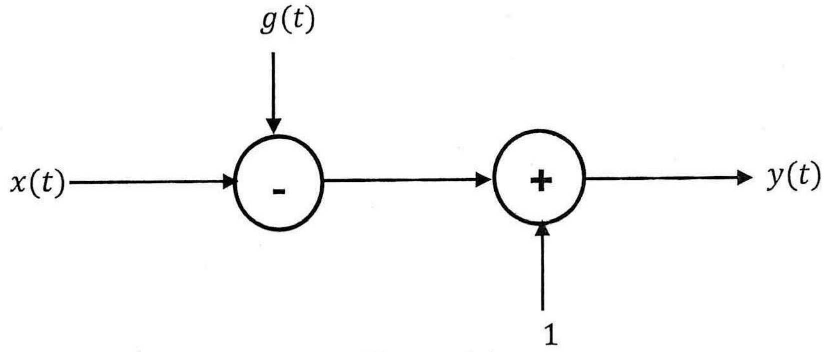

Figure Q1

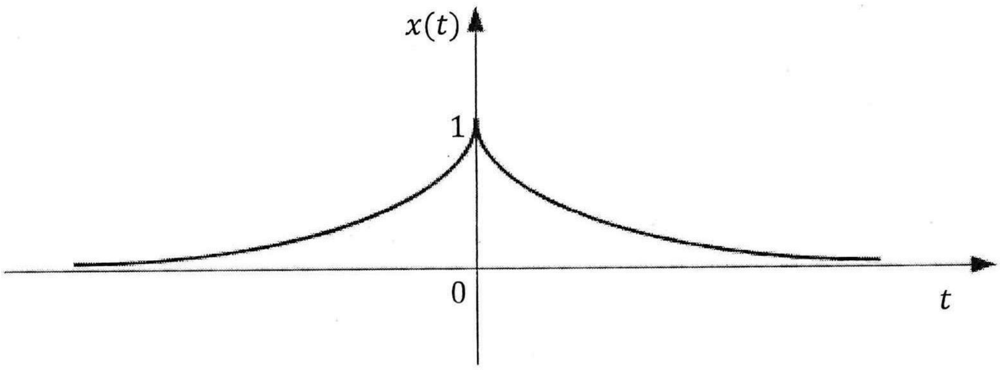

Figure Q4

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

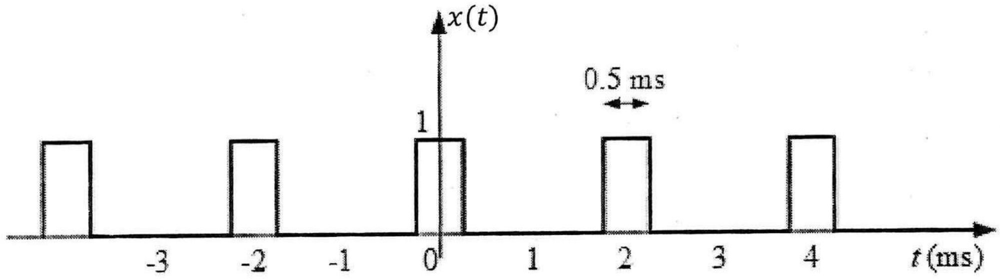

Figure Q6(a)

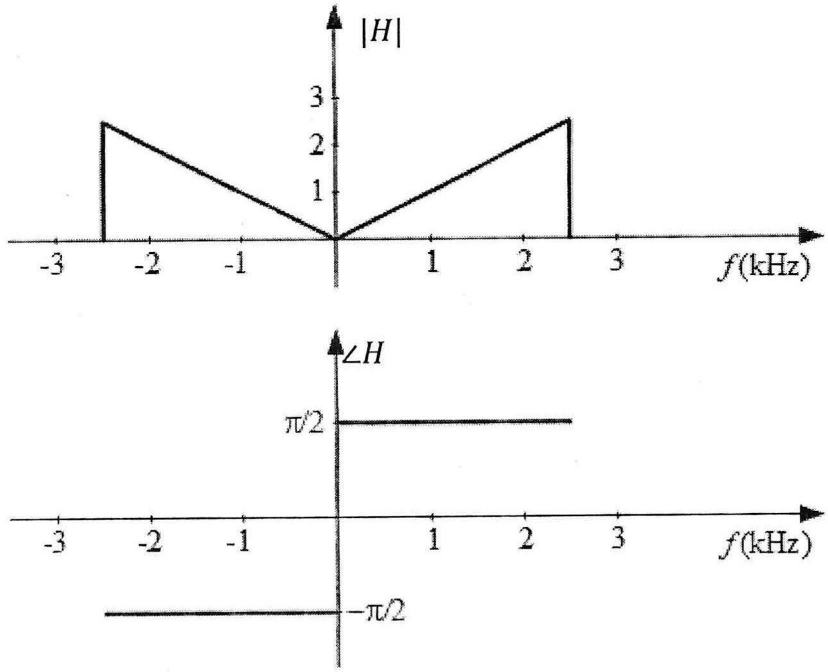

Figure Q6(b)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

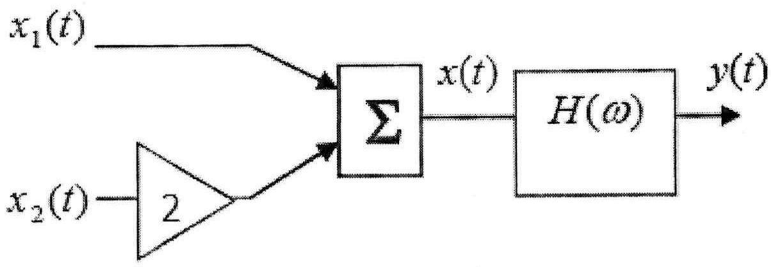

Figure Q7(a)(i)

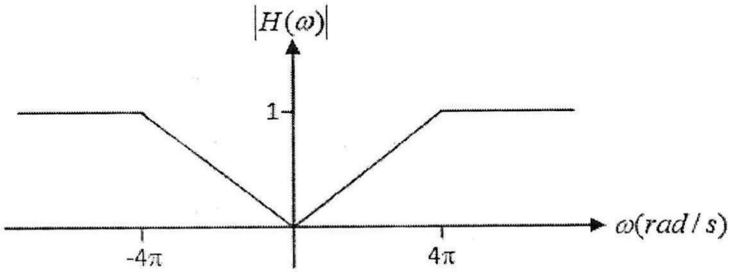

Figure Q7(a)(ii)

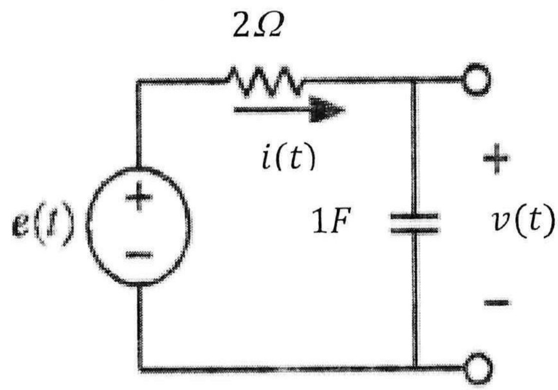

Figure Q7(b)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

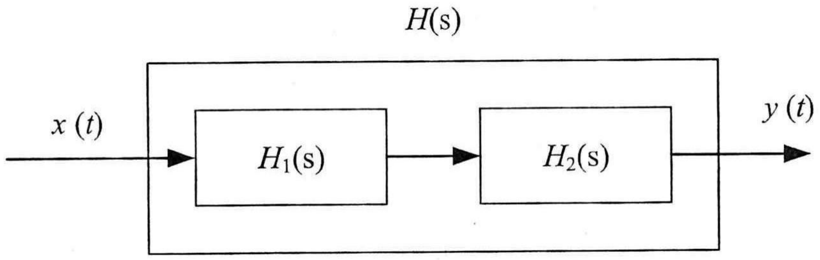

Figure Q8(b)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 1: INDEFINITE INTEGRALS

$\int \cos at\,dt = \frac{1}{a}\sin at$	$\int \sin at\,dt = -\frac{1}{a}\cos at$
$\int t\cos at\,dt = \frac{1}{a^2}\cos at + \frac{1}{a}t\sin at$	$\int t\sin at\,dt = \frac{1}{a^2}\sin at - \frac{1}{a}t\cos at$
$\int te^{at}dt = \frac{1}{a^2}e^{at}(at-1)$	$\int \frac{1}{(a^2+t^2)}dt = \frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)$

TABLE 2: EULER'S IDENTITY

$e^{\pm j\pi/2} = \pm j$	$A\angle \pm\theta = Ae^{\pm j\theta}$
$e^{\pm jk\pi} = \cos k\pi$	$e^{\pm}j\theta = \cos\theta \pm j\sin\theta$
$\cos\theta = \frac{1}{2}\left(e^{j\theta}+e^{-j\theta}\right)$	$\sin\theta = \frac{1}{2j}\left(e^{j\theta}-e^{-j\theta}\right)$

TABLE 3: COMPLEX NUMBER

$s = a + jb = \lvert s\rvert\angle \pm\theta = \lvert s\rvert e^{\pm j\theta}$	$\lvert s\rvert = \sqrt[2]{a^2+b^2}$	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

TABLE 4: TRIGONOMETRIC IDENTITIES

$\sin\theta = \cos\left(\left(\theta - \frac{\pi}{2}\right)\right)$	$\cos\theta = \sin(\theta + \frac{\pi}{2})$
$\sin(\propto \pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin^2\alpha + \cos^2\beta = 1$	
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos\ 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 5: VALUES OF COSINE, SINE AND EXPONENTIAL FUNCTIONS FOR INTEGRAL MULTIPLE OF π.

Function	Value	Function	Value
$\cos 2n\pi$	1	$e^{j2n\pi}$	1
$\sin 2n\pi$	0	$e^{jn\pi}$	$(-1)^n$
$\cos n\pi$	$(-1)^n$	$e^{\frac{jn\pi}{2}}$	$\begin{cases}(-1)^{\frac{n}{2}} & ,n = \text{even}\\ j(-1)^{\frac{n-1}{2}} & ,n = \text{odd}\end{cases}$
$\sin n\pi$	0		
$\cos\left(\frac{n\pi}{2}\right)$	$\begin{cases}(-1)^{\frac{n}{2}} & ,n = \text{even}\\ 0 & ,n = \text{odd}\end{cases}$	$\sin\left(\frac{n\pi}{2}\right)$	$\begin{cases}(-1)^{\frac{n-1}{2}} & ,n = \text{odd}\\ 0 & ,n = \text{even}\end{cases}$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 6: PARTIAL FRACTION FORMULA

Type of proper rational function	Partial Fraction
$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}, a \neq b \neq c$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
$\frac{px+q}{(x-a)^3}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{(x-a)^3}$
$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$, where x^2+bx+c cannot be factorised.	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$
$\frac{px^3+qx^2+rx+s}{(x^2+ax+b)(x^2+cx+d)}$, where (x^2+ax+b) and (x^2+cx+d) cannot be factorised.	$\frac{Ax+B}{x^2+ax+b}+\frac{Cx+D}{x^2+cx+d}$

TABLE 7: FOURIER SERIES

Exponential	$x(t)=\sum_{n=-\infty}^{\infty} x_n e^{jn\frac{2\pi}{T}t}$ $x_n=\frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\, e^{-jn\frac{2\pi}{T}t} dt$
Trigonometric	$x(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos n\frac{2\pi}{T}t + b_n \sin n\frac{2\pi}{T}t$ $a_n=\frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\cos n\frac{2\pi}{T}t\, dt, \quad n=0,1,2,3\ldots$ $b_n=\frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\sin n\frac{2\pi}{T}t\, dt, \quad n=1,2,3\ldots$
Amplitude-phase	$x(t)=X_0+\sum_{n=1}^{\infty} A_n \cos(n\frac{2\pi}{T}t+\theta_n)$ $A_n=2\lvert X_n\rvert=\sqrt[2]{a_n^2+b_n^2} \quad , \quad \theta_n=\angle X_n=-\tan^{-1}\left(\frac{b_n}{a_n}\right)$
Average Power	$P=V_{dc}I_{dc}+\frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_{V_n}-\theta_{I_n})$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 8: DEFINITION OF FOURIER AND LAPLACE TRANSFORM

FOURIER TRANSFORM	**INVERSE FOURIER TRANSFORM**
$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ $\mathcal{F}[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$	$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$ $x(t) = \mathcal{F}^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df$
LAPLACE TRANSFORM **Bilateral** $\mathcal{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$ **Unilateral** $\mathcal{L}[x(t)] = X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$ $s = \sigma + j\omega$	**INVERSE LAPLACE TRANSFORM** $x(t) = \mathcal{L}^{-1}[X(s)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 9: FOURIER TRANSFORM PAIRS

Time domain, $x(t)$	**Frequency domain,** $X(\omega)$	**Frequency domain,** $X(f)$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\frac{\delta(f)}{2} + \frac{1}{j2\pi f}$
$u(t+\tau) - u(t-\tau)$	$\frac{2\sin(\omega\tau)}{\omega} = 2\tau\,\mathrm{sinc}(\omega\tau)$	$2\tau\,\mathrm{sinc}\,2f\tau$
$\mathrm{rect}(t)$	$\mathrm{sinc}(\frac{\omega}{2})$	$\mathrm{sinc}(f)$
$\|t\|$	$-\frac{2}{\omega^2}$	$-\frac{2}{(2\pi f)^2}$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$\frac{1}{\alpha+j2\pi f}$
$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$\frac{1}{\alpha-j2\pi f}$
$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$\frac{2a}{a^2+4\pi^2 f^2}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$\delta(f-f_0)$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$\frac{n!}{(a+j2\pi f)^{n+1}}$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$	$\frac{\delta(f-f_o) - \delta(f+f_o)}{2j}$
$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$	$\frac{\delta(f-f_o) + \delta(f+f_o)}{2}$
$e^{-at}\sin\omega_0 t\,u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$\frac{2\pi f_0}{(a+j2\pi f)^2+(2\pi f_0)^2}$
$e^{-at}\cos\omega_0 t\,u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$\frac{a+2\pi f}{(a+j2\pi f)^2+(2\pi f_0)^2}$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 10: FOURIER TRANSFORM PROPERTIES

Property	Time domain, $x(t)$	Frequency domain, $X(\omega)$	Frequency domain, $X(f)$
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$	$a_1X_1(f) + a_2X_2(f)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
Time shifting	$x(t-t_0)u(t-t_0)$	$e^{-j\omega t_0}X(\omega)$	$e^{-j2\pi f t_0}X(f)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$	$X(f-f_o)$
Modulation	$\cos(\omega_0 t)\,x(t)$ $\sin(\omega_0 t)\,x(t)$	$\frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$ $\frac{1}{2j}[X(\omega-\omega_0)-X(\omega+\omega_0)]$	$\frac{1}{2}[X(f-f_o)+X(f+f_o)]$ $\frac{j}{2}[X(f+f_o)-X(f-f_o)]$
Time differentiation	$\frac{d}{dt}(x(t))$ $\frac{d^n}{dt^n}(x(t))$	$j\omega X(\omega)$ $(j\omega)^n X(\omega)$	$j2\pi f X(f)$ $(j2\pi f)^n X(f)$
Time integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{X(\omega)}{j\omega} + \pi X(\omega)\,\delta(\omega)$	$\frac{X(f)}{j2\pi f} + \frac{1}{2}X(0)\delta(f)$
Frequency differentiation	$t^n x(t)$	$(j)^n \frac{d^n}{d\omega^n}X(\omega)$	$\left(\frac{j}{2\pi}\right)^n \frac{d^n}{df^n}X(f)$
Time Reversal	$x(-t)$	$X(-\omega)$ or $X^*(\omega)$	$X(-f)$
Duality	$X(t)$	$2\pi x(-\omega)$	$X(-f)$
Convolution in t	$x_1(t) * x_2(t)$	$X_1(\omega)\cdot X_2(\omega)$	$X(f)\cdot Y(f)$
Multiplication	$x_1(t)\cdot x_2(t)$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$	$X(f) * Y(f)$
Parseval's Theorem	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$	$\int_{-\infty}^{\infty} \lvert X(f)\rvert^2 df$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2019/2020 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 11: LAPLACE TRANSFORM PAIR

Time domain, $x(t)$, $t>0$	s-domain, $X(s)$	ROC	Time domain, $x(t)$, $t>0$	s-domain, $X(s)$	ROC
$\delta(t)$	1	All s	$\cos bt$	$\frac{s}{s^2+b^2}$	$Re(s)>0$
$u(t)$	$\frac{1}{s}$	$Re(s)>0$	$\sin bt$	$\frac{b}{s^2+b^2}$	$Re(s)>0$
t	$\frac{1}{s^2}$	$Re(s)>0$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$Re(s)>-a$
t^n	$\frac{n!}{s^{n+1}}$	$Re(s)>0$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$	$Re(s)>-a$
e^{-at}	$\frac{1}{s+a}$	$Re(s)>-a$	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$	$Re(s)>0$
te^{-at}	$\frac{1}{(s+a)^2}$	$Re(s)>-a$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s)>0$

TABLE 12: LAPLACE TRANSFORM PROPERTIES

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t), x_2(t)$	$X_1(s), X_2(s)$	R_1, R_2
Linearity	$ax_1(t)+bx_2(t)$	$aX_1(s)+bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s-s_0)$	Shifted version of R (i.e., s is in the ROC if $s-s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t)*x_2(t)$	$X_1(s)\cdot X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
		$sX(s)-x(0^+)$ (Unilateral)	R right hand plane
	$\frac{d^n}{dt^n}x(t)$	$s^nX(s)-s^{n-1}x(0^+)-\cdots-sx^{n-2}(0^+)-x^{n-1}(0^+)$	
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R\cap\{Re(s)>0\}$

Initial- and Final- Value Theorems

If $x(t)=0$ for $t<0$ and $x(t)$ contains no impulses or higher order singularities at $t=0$, then

$$x(0^+)=\lim_{s\to\infty} sX(s)$$

If $x(t)=0$ for $t<0$ and has a finite limit as $t\to\infty$, then

$$\lim_{t\to\infty} x(t)=\lim_{s\to\infty} sX(s)$$