CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ENGINEERING MATHEMATICS V
COURSE CODE	:	BEE 31702
PROGRAMME CODE	:	BEJ/BEV
EXAMINATION DATE	:	DECEMBER 2019/ JANUARY 2020
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS.
		2. WRITE YOUR ANSWER IN THIS BOOKLET.
		3. PROVIDE YOUR ANSWER IN 4 DECIMAL NUMBER.

THIS QUESTION PAPER CONSISTS OF **THIRTEEN (13)** PAGES

CONFIDENTIAL

Q1 A prototype for measuring heart rate has been developed by FKEE student for her FYP project. The prototype is developed using the photoplethysmography (PPG) sensor, which collects data by shining a light on patient's fingertip. The accuracy of the prototype measurement is compared to those collected by using the electrocardiogram (ECG) device as reference values. The ECG device is commonly used by medical practitioners for heart rate reading. Normal heart rate of a person (age 10 years and above) should be between 60-100 beats per second while resting. The collected data is tabulated in **Table Q1**.

Table Q1

Test Sample	Independent Variable Heart beat obtained by ECG device, x	Dependent Variable Heart beat obtained by PPG prototype, y
1	90	95
2	92	91
3	85	87
4	73	70
5	64	66
6	95	80
7	88	85
8	70	79
9	65	70
10	90	85

(a) Determine the regression line equation from the given data.

(12.5 marks)

(b) Calculate the coefficient of Pearson correlation from the given data.

(10.5 marks)

(c) Conclude the correlation between heart rate data collected using PPG and ECG devices.
(2 marks)

Q2 (a) Referring to the central limit theorem, define how the standard deviation of the sampling distribution of the sample mean is calculated?

(2 marks)

(b) Suppose that the population of the gripping strength of high-speed packaging machine is known has mean 72 and standard deviation, 6. For a random sample of 45 packages, what is the probability that the sample mean gripping strength will differ at least 2 units from the population mean (i.e. less than 74 and more than 70)?

(10 marks)

(c) A new low-noise transistor is being developed by a company, which will be used in electrical products. Suppose that the noise level for the current transistor is normally distributed with a mean on 2.5 decibels, while the noise level of all new transistor is normally distributed with a mean of 2.35 decibels. Given the standard deviation of the old transistor is 0.1 decibels and new transistor is 0.2 decibels, respectively. 16 random samples of both low-noise transistors were tested.

(i) Find the probability of sample of new transistor is better than the old one.

(11 marks)

(ii) How did the variance of sampling distribution of $\overline{x}$ change when the number of transistor samples manufactured increased?

(2 marks)

Q3 (a) Compute a number of samples needed to estimate the true mean score within 8 points with 95% confidence if the variance of a BEE31702 Engineering Mathematics V examination score is 440.

(7 marks)

(b) A computer must be connected to an Arduino microcontroller using a USB cable to upload source codes. A female USB connector mounts on the microcontroller. Assuming a standard deviation of 0.0013 and an approximate normal distribution. A random sample of 100 female USB connectors has an average of 0.250 inch. Solve a 99% confidence interval for the mean of all USB cables made by a certain manufacturing company.

(7 marks)

(c) In a semiconductor manufacturing, wet chemical etching is used to remove silicon from the backs of wafers prior to metallization. Two different etching solutions have been compared using two random samples of 8 wafers. The observed etching rates are given in **Table Q3**.

Table Q3

Observation Number	Solution 1	Solution 2
1	91.50	89.19
2	94.18	90.95
3	92.18	90.46
4	95.39	93.21
5	91.79	97.19
6	89.07	97.04
7	94.72	91.07
8	89.21	92.75

Suppose that the two samples are independent and come from population which are normally distributed. Also, assume that their population variances are equal. Calculate a 95% confidence interval for the difference between the two mean etching rates of the two solutions.

(11 marks)

Q4 (a) The time spent for engineering students in their study has a normal distribution with the mean number of hours spent on study per week is 23 hours with a standard deviation of 9 hours per week. A lecturer claims that the engineering students put in more hours studying as compared to other students. A sample of 25 engineering students was selected at random and the mean number of hours spent on study per week is 19 hours.

(i) Explain the type I and type II error for this situation.

(2 marks)

(ii) State the suitable null and alternative hypothesis.

(2 marks)

(iii) Analyze the lecturer's claim at 1% significant level.

(6 marks)

(b) Two types of battery were used on 5 and 7 new car model in Tesla Motors Company and the results are tabulated in **Table Q4**. Compute an appropriate hypothesis testing if there are significant differences in Battery A and Battery B by using 0.001 of significance level. Assume that the variances of population are unknown but equal.

Table Q4

Battery A	Battery B
10	8
12	9
13	12
11	14
14	15
	10
	9

(9 marks)

(c) A manufacturer wishes to determine whether there is less variability in the silicon rubber coating produced by Company A and Company B. Given that the sample size and sample standard deviation of Company A is 13 and 0.035, respectively, while Company B is 11 and 0.062. Calculate if the populations have different variances at 5% significant level.

(6 marks)

-END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER / SESSION	: SEM I / 2019/2020	PROGRAMME CODE	: BEV / BEJ
COURSE NAME	: ENGINEERING MATHEMATICS V	COURSE CODE	: BEE 31702

List of Formulas

Random Variables:

$$\sum_{i=-\infty}^{\infty} P(x_i) = 1, \quad E(X) = \sum_{\forall x} x \cdot P(x), \quad E(X^2) = \sum_{\forall x} x^2 \cdot P(x), \quad \int_{-\infty}^{\infty} f(x)\,dx = 1,$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot P(x)\,dx, \quad E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot P(x)\,dx, \quad Var(X) = E(X^2) - [E(X)]^2.$$

Special Probability Distributions:

$$P(x=r) = {}^nC_r \cdot p^r \cdot q^{n-r},\ r = 0, 1, \ldots, n,\ X \sim B(n, p),\ P(X=r) = \frac{e^{-\mu} \cdot \mu^r}{r!},\ r = 0, 1, \ldots, \infty,$$

$$X \sim P_0(\mu),\ Z = \frac{X-\mu}{\sigma},\ Z \sim N(0,1),\ X \sim N(\mu, \sigma^2).$$

Sampling Distributions:

$$\bar{X} \sim N(\mu, \sigma^2/n),\ Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1),\ T = \frac{\bar{x}-\mu}{s/\sqrt{n}},\ \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right).$$

Estimations:

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2,\ \left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}},$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}},$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,v} \cdot S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,v} \cdot S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where Pooled estimate of variance, $S_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$ with $v = n_1 + n_2 - 2$,

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,v}\sqrt{\frac{1}{n}\left(s_1^2 + s_2^2\right)} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,v}\sqrt{\frac{1}{n}\left(s_1^2 + s_2^2\right)} \text{ with } v = 2(n-1),$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2019/2020	PROGRAMME CODE : BEV / BEJ
COURSE NAME : ENGINEERING MATHEMATICS V	COURSE CODE : BEE 31702

$$\left(\bar{x}_1-\bar{x}_2\right)-t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}<\mu_1-\mu_2<\left(\bar{x}_1-\bar{x}_2\right)+t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \text{ with } v=\frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}},$$

$$\frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2,v}}<\sigma^2<\frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2,v}} \text{ with } v=n-1,$$

$$\frac{s_1^2}{s_2^2}\cdot\frac{1}{f_{\alpha/2,v_1,v_2}}<\frac{\sigma_1^2}{\sigma_2^2}<\frac{s_1^2}{s_2^2}\cdot f_{\alpha/2,v_2,v_1} \text{ with } v_1=n_1-1 \text{ and } v_2=n_2-1.$$

Hypothesis Testing:

$$Z=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}},\ T=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{S_p\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \text{ with } v=n_1+n_2-2,$$

$$Z=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}},\ T=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{1}{n}\left(s_1^2+s_2^2\right)}},\ T=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} \text{ with}$$

$$v=\frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}.\ ;\ S_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}\ ;\ \chi^2=\frac{(n-1)s^2}{\sigma^2}$$

$$S_{xy}=\sum x_iy_i-\frac{\sum x_i\cdot\sum y_i}{n},\ S_{xx}=\sum x_i^2-\frac{\left(\sum x_i\right)^2}{n},\ S_{yy}=\sum y_i^2-\frac{\left(\sum y_i\right)^2}{n},\ \bar{x}=\frac{\sum x}{n},\ \bar{y}=\frac{\sum y}{n},$$

$$\hat{\beta}_1=\frac{S_{xy}}{S_{xx}},\ \hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x},\ \hat{y}=\hat{\beta}_0+\hat{\beta}_1x,\ R^2=\frac{S_{yy}-SSE}{S_{yy}}=1-\frac{SSE}{S_{yy}},\ r=\frac{S_{xy}}{\sqrt{S_{xx}\cdot S_{yy}}},$$

$$SSE=S_{yy}-\hat{\beta}_1S_{xy},\ MSE=\frac{SSE}{n-2},\ T=\frac{\hat{\beta}_1-\beta_1^*}{\sqrt{\frac{MSE}{S_{xx}}}}\sim t_{n-2},\ T=\frac{\hat{\beta}_0-\beta_0^*}{\sqrt{MSE\left(\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right)}}\sim t_{n-2}.$$