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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BEE 21503
PROGRAMME	:	BEV / BEJ
EXAMINATION DATE	:	DECEMBER 2019/JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) If $w = \sqrt{x^2 + y^2 + z^2}$, $x = 9 \ln t$, $y = 6 \cos 3t$, $z = 2e^t \sin t$, use Chain rule to find $\frac{dw}{dt}$.

(8 marks)

(b) Find the rate of change of volume of a cylinder with radius 6 cm and height 14 cm if the increasing rate of radius is 0.3 cms^{-1} and the decreasing rate of height is 0.4 cms^{-1}.

(4 marks)

(c) Show that the function $z = e^x \sin y + e^y \cos x$ satisfies Laplace equation as the following:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

(6 marks)

(d) Find $\frac{\partial z}{\partial x}$, if $z = f(x, y)$ is implicitly defined as a function of x and y for equation $2x^2z^4 - 5\ln(xyz^2) = 4z$.

(7 marks)

Q2 (a) Given the functions of

$$y = (x + 2)^2 - 9 \text{ and } y = 4 - (x + 1)^2$$

(i) In the same figure, sketch the graphs of the functions above.

(6 marks)

(ii) Determine the domain and range of the functions based on **Q1(a)(i)**.

(4 marks)

(iii) Determine the area of the enclosed regions by using double integrals method.

(5 marks)

(b) Evaluate the volume of the solid bounded by $x = y^2 + z^2$ and plane $x = 9$ using triple integrals of the cylindrical coordinate.

(10 marks)

Q3 (a) A quantity can be either a scalar or a vector, while a field is a function that specifies a particular quantity everywhere in a region. If the quantity is scalar (or vector), the field is said to be a scalar (or vector) field. With the aid of diagram, define the scalar and vector fields and give examples of each field.

(6 marks)

(b) Calculate the line integral $\int_C x^2zdx - yx^2dy + 3dz$ if C is the path given as shown in **Figure Q3(b).**

(7 marks)

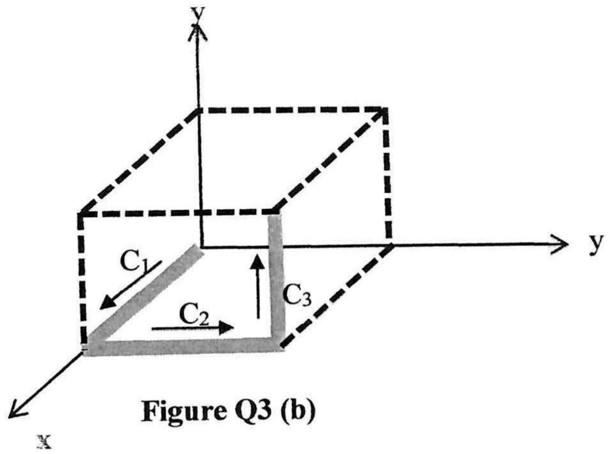

Figure Q3 (b)

(c) (i) Compute the surface integral $\iint_\sigma dS$, where σ is the first octant portion of the plane $2x + y + 2z = 6$.

(6 marks)

(ii) Hence, find the total mass of thin sheet (lamina) that has the same shape as in **Q3(c)(i)** if the density function, $\rho(x, y, z)$ at the point (x, y, z) is $2xz$.

(6 marks)

Q4 (a) Differentiate between the Gauss's theorem and Stokes' theorem.

(2 marks)

(b) Given that σ is the surface of the solid G enclosed by cone $z = 4 - \sqrt{x^2 + y^2}$ and plane $z = 0$.

(i) Compute the flux of water flowing through the surface of the cone if the velocity vector, $\vec{F} = 3x\hat{\imath} + 3y\hat{\jmath} + 6\hat{k}$. Assume that the unit normal vector is oriented outward.

(6 marks)

(ii) Evaluate $\iint_{\sigma} \vec{F} \cdot \hat{n} ds$ by using Gauss's theorem.

(5 marks)

(c) Let σ be the portion of paraboloid $z = 4 - x^2 - y^2$, $z \geq 0$, and oriented outward. Suppose that the curve C is the boundary of σ in the xy-plane and the force field is given by $\vec{F} = 2z\hat{\imath} + 3x\hat{\jmath} + 5y\hat{k}$.

(i) Find the work done by the force field along the curve C.

(5 marks)

(ii) Verify Stokes' theorem.

(7 marks)

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FORMULAS

Polar coordinate

$x = r\cos\theta,\ y = r\sin\theta,\ \theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)\,dA = \iint_R f(r,\theta)\,r\,dr\,d\theta$

Cylindrical coordinate

$x = r\cos\theta,\ y = r\sin\theta,\ z = z$ and $\iiint_G f(x,y,z)\,dV = \iiint_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$

Spherical coordinate

$x = \rho\sin\phi\cos\theta,\ y = \rho\sin\phi\sin\theta,\ z = \rho\cos\phi$, then $x^2 + y^2 + z^2 = \rho^2$, for $0 \le \theta \le 2\pi,\ 0 \le \phi \le \pi$,

and $\iiint_G f(x,y,z)\,dV = \iiint_G f(\rho,\phi,\theta)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$$A = \iint_R dA$$

$m = \iint_R \delta(x,y)\,dA$, where $\delta(x,y)$ is a density of lamina

$$V = \iint_R f(x,y)\,dA$$

$$V = \iiint_G dV$$

$$m = \iiint_G \delta(x,y,z)\,dV$$

If f is a differentiable function of x, y and z, then the

Gradient of f, grad $f(x,y,z) = \nabla f(x,y,z) = \dfrac{\partial f}{\partial x}\mathbf{i} + \dfrac{\partial f}{\partial y}\mathbf{j} + \dfrac{\partial f}{\partial z}\mathbf{k}$

If $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is a vector field in Cartesian coordinate, then the

Divergence of $\mathbf{F}(x,y,z)$, div $\mathbf{F} = \nabla\cdot\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$ **Curl** of $\mathbf{F}(x,y,z)$,

$$\text{curl}\,\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial Z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial Z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

$\mathbf{F}$ is conservative vector field if Curl of $\mathbf{F} = 0$.

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Surface Integral

Let S be a surface with equation $z = g(x, y)$ and let R be its projection on the xy-plane.

$$\iint_S f(x,y,z)\,dS = \iint_R f(x,y,g(x,y))\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA$$

Gauss's Theorem

$$\iint_S \mathbf{F}\bullet\mathbf{n}dS = \iiint_G \nabla\bullet\mathbf{F}dV$$

Stokes' Theorem

$$\iint_S (\nabla\times\mathbf{F})\bullet\mathbf{n}dS = \oint_C \mathbf{F}\bullet dr$$

Identities of Trigonometry and Hyperbolic

Trigonometric Functions

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$

$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$

$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$

$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$

$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$

$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$

Hyperbolic Functions

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x\cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 2\cosh^2 x - 1$

$= 1 + 2\sinh^2 x$

$1 - \tanh^2 x = \text{sech}^2 x$

$\coth^2 x - 1 = \text{csch}^2 x$

$\tanh 2x = \dfrac{2\tanh x}{1+\tanh^2 x}$

$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$

$\sinh(x \pm y) = \sinh x\cosh y \pm \cosh x\sinh y$

$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$

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The derivative of f(x) with respect to x

$$f_x(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Indefinite Integrals and Integration of Inverse Functions

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1}+C, \qquad n\neq -1$$

$$\int \frac{1}{x}dx = \ln|x|+C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x dx = \sinh x + C$$

$$\int \sinh x dx = \cosh x + C$$

$$\int \operatorname{sech}^2 x dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x dx = -\coth x + C$$

$$\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+C, \quad x^2<a^2$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right)+C, \quad x^2<a^2$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{-1}{a^2+x^2}dx = \cot^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{1}{|x|\sqrt{x^2-a^2}}dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C, \quad x^2>a^2$$

$$\int \frac{-1}{|x|\sqrt{x^2-a^2}}dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right)+C, \quad x^2>a^2$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \sinh^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \cosh^{-1}\left(\frac{x}{a}\right)+C, \quad x>a>0$$

$$\int \frac{-1}{|x|\sqrt{a^2-x^2}}dx = \frac{1}{a}\operatorname{sech}^{-1}\left|\frac{x}{a}\right|+C, \quad 0<x<a$$

$$\int \frac{-1}{|x|\sqrt{a^2+x^2}}dx = \frac{1}{a}\operatorname{csch}^{-1}\left|\frac{x}{a}\right|+C, \quad x\neq 0$$

$$\int \frac{1}{a^2-x^2}dx = \begin{cases}\frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+C, & x^2<a^2\\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+C, & x^2>a^2\end{cases}$$