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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BEE 11403
PROGRAMME CODE	:	BEV / BEJ
EXAMINATION DATE	:	DECEMBER 2019/JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	**ANSWER ALL QUESTIONS**

THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

Q1 (a) By Newton's law of cooling, the rate of change of temperature T of an object is stated by the first-order differential equation as

$$\frac{dT}{dt} = k\left(T - T_s\right),$$

where k is a constant and T_s is temperature of surrounding environment.

(i) Find the general solution of the above first-order differential equation if $T_s = 20$.

(5 marks)

(ii) Given the initial $T(t)$ of the object is 100°C, find the particular solution of the above first-order differential equation.

(3marks)

(iii) After **FIVE (5)** minutes, the object is cooled to 60°C, find the value of k and hence temperature $T(t)$.

(5 marks)

(b) By using power series $i = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + \cdots$ and $\sin t = \sum_{m=0}^{\infty} \frac{(-1)^m t^{2m+1}}{(2m+1)!}$,

show that the particular solution of the first-order differential equation of

$$\frac{di}{dt} + i = \sin 2t,\ i(0) = 1$$

is

$$i = 1 + t + \frac{1}{2}t^2 - \frac{1}{6}t^3 - \frac{7}{24}t^4 \cdots.$$

(12 marks)

Q2 (a) **Figure Q2** shows a *RLC* parallel circuit which is governed by the second-order ODE as

$$\frac{d^2V}{dt^2} + 4\frac{dV}{dt} + 400V = 0.$$

(i) What type of second-order ODE is this?

(1 mark)

(ii) How do you find the general solution of **Q2(a)(i)**?

(1 mark)

(iii) Based on your solution on **Q2(a)(ii)**, state cases of solution of **Q2(a)(ii).**

(2 marks)

(iv) Find the particular solution of $V(t)$ when $V(0) = 5$ and $V(1) = 0$.

(7 marks)

(b) Given a second-order ODE as

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + i = 5\cos 2t$$

(i) Find the general solution, i_c for the second-order homogeneous equation.

(3 marks)

(ii) Evaluate the particular integral, i_p for the second-order non-homogeneous equation using undetermined coefficient method.

(10 marks)

(iii) Formulate the general solution of i.

(1 mark)

Q3 By applying Kirchhoff's voltage law, the RL-network given in **Figure Q3** can be modeled as below:

$$\begin{pmatrix} i_1' \\ i_2' \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} 10 \\ 0 \end{pmatrix}$$

(a) State what type of differential equation is this?

(1 mark)

(b) Explain how to find the solution of differential equation in **Q3**?

(2 marks)

(c) Find eigenvalues λ_1 and λ_2 of the homogeneous system.

(3 marks)

(d) Determine eigenvectors V_1 and V_2 of the homogeneous system.

(8 marks)

(e) Generate the general solution for the homogeneous system.

(1 mark)

(f) Propose the particular integral for the non-homogeneous system.

(6 marks)

(g) Formulate the general solution of the non-homogeneous system.

(1 mark)

(h) Compose the particular solution for the current i_1, and i_2 given there is no currents flow through at initial state.

(3 marks)

Q4 (a) A rectangular pulses, $E(t)$ is applied to the RC circuit as shown in **Figure Q4(a)**.

(i) Show that the circuit can be modelled as

$$RC\frac{dV}{dt} + V = u(t) - u(t-2).$$

(4 marks)

(ii) Calculate the response, $v(t)$ with $v(0) = 0$ using Laplace transform.

(7 marks)

(b) Consider the network circuit as shown in **Figure Q4 (b)** with initial current and their derivatives are zero for $t = 0$.

(i) Show the loop current, $i_2(t)$ can be formulated as

$$\frac{di_2}{dt} + 250i_2 = 20$$

(6 marks)

(ii) Compute the loop current, $i_2(t)$ using Laplace transform.

(8 marks)

- END OF QUESTIONS -

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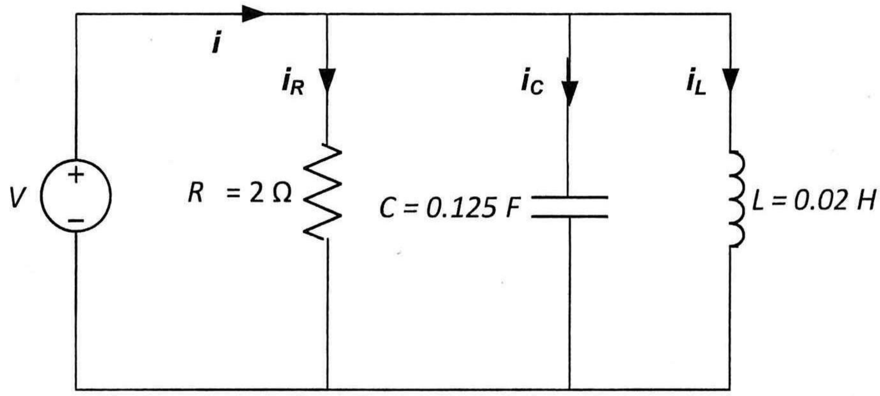


Figure Q2(a)

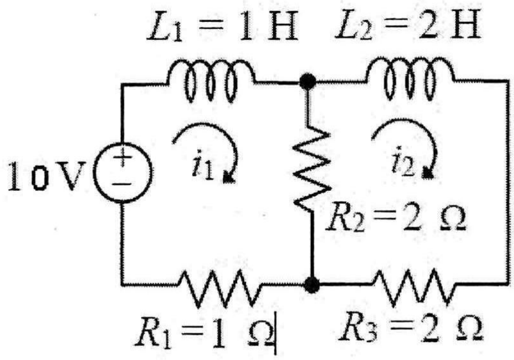
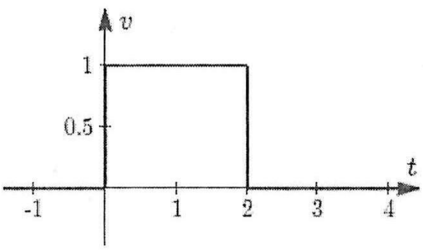


Figure Q3



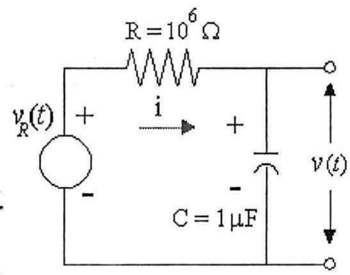


Figure Q4(a)

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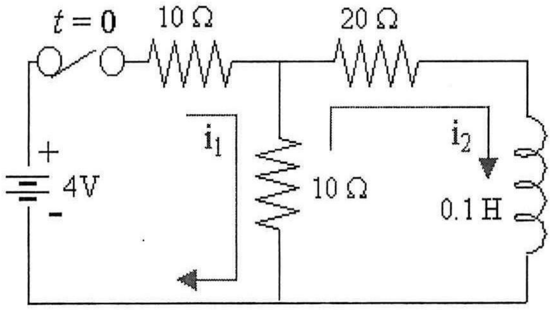

Figure Q4(b)

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FORMULAS

Electrical Formula

1. Voltage drop across resistor, R (Ohm's Law): $v_R = iR$

2. Voltage drop across inductor, L (Faraday's Law): $v_L = L\dfrac{di}{dt}$

3. Voltage drop across capacitor, C (Coulomb's Law): $v_c = \dfrac{q}{C}$ or $i = C\dfrac{dv_c}{dt}$ or

   $V_C = \dfrac{1}{C}\int i\,dt$

4. The relation between current, i and charge, q: $i = \dfrac{dq}{dt}$

5. Linear equation: $\dfrac{dy}{dt} + p(t)y = q(t)$

$$I = e^{\int p(t)dt}$$

$$I\,y = \int I\,q(t)dt + C$$

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FORMULAS

Second-order homogeneous Differential Equation

The roots of characteristic equation and the general solution for second-order differential equation $ay''(t)+by'(t)+cy(t)=0$.

Characteristic equation: $am^2+bm+c=0$.		
Case	**The roots of characteristic equation**	**General solution**
1.	Real and different roots: m_1 and m_2	$y=Ae^{m_1 t}+Be^{m_2 t}$
2.	Real and equal roots: $m=m_1=m_2$	$y=(A+Bt)e^{mt}$
3.	Complex roots: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y=e^{\alpha t}(A\cos\beta t+B\sin\beta t)$

The method of undetermined coefficients for second order non-homogeneous differential equations

For second order non-homogeneous differential equations, $ay''(t)+by'(t)+cy(t)=f(t)$, the particular integral, y_p is given by:

$f(t)$	Example of $f(t)$	Assume
Exponent	ke^{nt}	$y_p=Ce^{nt}$
Polynomial	k	$y_p=C$
	kt	$y_p=Ct+D$
	kt^2	$y_p=Ct^2+Dt+E$
Trigonometry	$k\sin nt$ or $k\cos nt$	$y_p=C\cos nt+D\sin nt$

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Homogeneous System of First-Order Differential Equation

$Y' = AY$

Eigenvalues

$|A-\lambda I| = 0$

Eigenvectors

$(A-\lambda I)V = 0$

General solution of Homogeneous System

$$Y = AV_1e^{\lambda_1 t} + BV_2e^{\lambda_2 t}$$
$$= A\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}e^{\lambda_1 t} + B\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}e^{\lambda_2 t}$$

Non-Homogeneous System of First-Order Linear Differential Equation

Assume Y_p based on G	
G	$\boldsymbol{Y_p}$
Case I: Polynomial $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$; $\begin{pmatrix} a_1t+b_1 \\ a_2t+b_2 \end{pmatrix}$; $\begin{pmatrix} a_1t^2+b_1t+c_1 \\ a_2t^2+b_2t+c_2 \end{pmatrix}$	$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$; $\begin{pmatrix} u_1t+v_1 \\ u_2t+v_2 \end{pmatrix}$; $\begin{pmatrix} u_1t^2+v_1t+w_1 \\ u_2t^2+v_2t+w_2 \end{pmatrix}$
Case II: Exponent $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}e^{\lambda t}$	$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}e^{\lambda t}$ if $Y_p = Y_c$, then $\begin{pmatrix} u_1t+v_1 \\ u_2t+v_2 \end{pmatrix}e^{\lambda t}$
Case III: Trigonometric $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\sin t$ or $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\cos t$	$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\sin t + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\cos t$

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Power Series Method

$$\sum_{m=0}^{\infty} c_m t^m = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + \cdots$$

Where $c_0,\ c_1,\ c_2$ ... are constants

Representation of Functions in Power Series

$e^t = 1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots \sum_{m=0}^{\infty} \frac{t^m}{m!}, -\infty < t < \infty$
$\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots \sum_{m=0}^{\infty} (-1)^m \frac{t^{2m+1}}{(2m+1)!}, -\infty < t < \infty$
$\cos t = 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \cdots \sum_{m=0}^{\infty} (-1)^m \frac{t^{2m}}{(2m)!}, -\infty < t < \infty$
$\ln(1+t) = t - \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots \sum_{m=1}^{\infty} (-1)^{m+1} \frac{t^m}{m!}$
$\frac{1}{1-t} = 1 + t + t^2 + t^3 + \cdots \sum_{m=0}^{\infty} t^m$
$(1+t)^{\alpha} = 1 + \alpha t + \frac{\alpha(\alpha-1)}{2!} t^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} t^3 + \cdots$

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Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
e^{at}	$\frac{1}{s-a}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\sinh at$	$\frac{a}{s^2-a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y(t)$	$Y(s)$
$e^{at}f(t)$	$F(s-a)$	$y'(t)$	$sY(s)-y(0)$
$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$