Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	: DIGITAL SIGNAL PROCESSING
COURSE CODE	: BEB 30503 / BEF 35603
PROGRAMME CODE	: BEJ / BEV
EXAMINATION DATE	: DECEMBER 2019 / JANUARY 2020
DURATION	: 3 HOURS
INSTRUCTION	: ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIFTEEN (15)** PAGES

Q1 (a) The operation of each subsystem in **Figure Q1(a)** are described as below:

- $T_1\{\cdot\}$ produces $y[n] = x\left[\frac{n}{2}\right]$ based on zero interpolation
- $T_2\{\cdot\}$ produces $z[n] = y\left[\frac{n}{3}\right]$ based on step interpolation

Let the input signal, $x[n] = \cos(2n\pi)$;

(i) Sketch the output signal z[n] for $0 \leq n \leq 11$

(4 marks)

(ii) Prove that the output signal can be represented by

$$z[n] = \sum_{p=-\infty}^{\infty} rect\left(\frac{n-(1+6p)}{2}\right)$$

(5 marks)

(b) Calculate the periodic convolution of $y_p[n] = x_p[n] \otimes h_p[n]$ for $x_p[n] = \{\overset{\Downarrow}{-1}, 2, 5\}$ and $h_p[n] = \{\overset{\Downarrow}{3}, 0, -4\}$ by using cyclic method.

(5 marks)

(c) Given the signal $x[n] = \{A, 2, \overset{\Downarrow}{3}, 2, A\}$. Analyze the possible value of A if autocorrelation of $x[n]$ gives $r_{xx}[0] = 19$. Use sum-by-column method for linear convolution process.

(6 marks)

Q2 (a) State **THREE (3)** cases of sampling according to the given condition of sampling rate, *S* and band-limited frequency, *B*

(i) $S = 2B$
(ii) $S > 2B$
(iii) $S < 2B$

(3 marks)

(b) An Analog to Digital Converter (ADC) is consists of 3 modules: (1) sampler, (2) quantizer and (3) coder modules. ADC is used to transform the analog signal into encoded digital signal before it can be transmitted to receiver station using a suitable transmission medium.

The analog signal input of ADC has two different sources which are:

Source A : $x_1(t) = 3\sin\left(2\pi t + \frac{\pi}{4}\right) - 3\cos\left(2\pi t + \frac{3\pi}{2}\right)$ V

Source B: $x_2(t) = 1.5\cos\left(3\pi t + \frac{3\pi}{4}\right) - \sin(3\pi t)$ V

Both analog signal is sampled at a sampling rate of 9 Hz with dynamic range of ± 2.5 V.

(i) Find the discrete signal of $x_1[n]$ and $x_2[n]$ for $0 \leq n \leq 3$.

(5 marks)

(ii) Given the encoded digital signal $x_c[n]$ for $0 \leq n \leq 3$ at the coder module in ADC is shown in **Figure Q2(b)**, determine the source of the encoded signals if the discrete signals are quantized by using up-truncation technique.

(12 marks)

Q3 (a) Compute the Discrete Fourier Transform (DFT) of $x[n] = (0.9)^n$ for $0 \leq n \leq 3$.
(5 marks)

(b) Fast Fourier Transform (FFT) is a collection of algorithms for fast computation of DFT. Given the Decimation in Time-FFT (DIT-FFT) of $a[n]$ and $b[n]$ are as follows:

$$A[k] = [-j, 1, -2j, 1]$$
$$B[k] = [0, 2, 0, 2]$$

(i) Sketch butterfly diagram of the DIT-FFT process for both $A[k]$ and $B[k]$. State the Twiddle factors in each diagram.
(5 marks)

(ii) Analyze the butterfly diagram in **Q3 (b)(i)** to compute $b[n]$.
(7 marks)

(iii) Plot the magnitude spectrum of $c[n] = a[n] \otimes b[n]$.
(3 marks)

Q4 (a) Convert the discrete signal below to z-domain, using the defining relation. Then identify the ROC for each of the signal.

(i) $x[n] = \delta[n+1] + 2\delta[n-2]$

(2 marks)

(ii) $y[n] = tri\left(\frac{n-3}{3}\right)$

(2 marks)

(b) The z-transform of $x[n]$ is $X(z) = \frac{4z}{(z+0.5)^2}, |z| > 0.5$. Solve the z-transform of the following using properties:

(i) $y[n] = (2)^n n\, x[n]$

(3 marks)

(ii) $h[n] = x[n] - x[n-1]$

(3 marks)

(c) The z-transform of the convolution $y(n) = x(n) * h(n)$ is:

$$Y(z) = \frac{-3z^{-1}}{1 - 3.5z^{-1} + 1.5z^{-2}}$$

Where $x[n] = u[-n]$ and $h[n] = (0.5)^n u[n]$. The ROC of $Y(z)$ is $\frac{1}{2} < |z| < 3$.

(i) Determine the inverse z-transform using partial fraction.

(7 marks)

(ii) Find the stability of the system.

(3 marks)

Q5 (a) State **TWO (2)** comparison between the Infinite Impulse Response (IIR) filter and Finite Impulse Response (FIR) filter.

(4 marks)

(b) Explain why the digital filter is better than analog filter.

(5 marks)

(c) A digital low pass filter described by $H(z) = \dfrac{z+1}{z^2 - z + 0.2}$ has a cut off frequency,

$f = 0.5$ kHz and operates at a sampling frequency, $S = 15$ kHz. By using the given filter, construct a digital low pass filter with a cut off frequency of 2 kHz.

(11 marks)

-END OF QUESTIONS -

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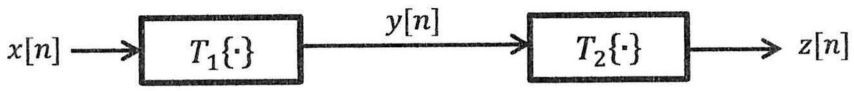

Figure Q1(a)

Encoded signal $x_c[n]$ for $0 \le n \le 3$

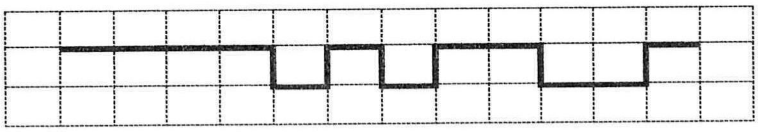

Figure Q2(b)

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Table 1: Properties of the Discrete Fourier Transform (DFT)

Property	Signal	DFT	Remarks
Shift	$x[n - n_o]$	$X_{DFT}[k] \quad e^{-j2\pi k n_o/N}$	No change in magnitude.
Shift	$x[n-0.5N]$	$(-1)^k X_{DFT}[k]$	Half-period shift for even N.
Modulation	$x[n]e^{j2\pi n k_o/N}$	$X_{DFT}[k-k_o]$	
Modulation	$(-1)^n x[n]$	$X_{DFT}[k-0.5N]$	Half-period shift for even N.
Folding	$x[-n]$	$X_{DFT}[-k]$	This is circular folding.
Product	$x[n]y[n]$	$\frac{1}{N} X_{DFT}[k] \otimes Y_{DFT}[k]$	The convolution is periodic.
Convolution	$x[n] \otimes y[n]$	$X_{DFT}[k] Y_{DFT}[k]$	The convolution is periodic.
Correlation	$x[n] \otimes \otimes y[n]$	$X_{DFT}[k] Y^*_{DFT}[k]$	The correlation is periodic.
Central Ordinates	$x[0] = \frac{1}{N}\sum_{k=0}^{N-1} X_{DFT}[k], \quad X_{DFT}[0] = \sum_{n=0}^{N-1} x[n]$		
Central Ordinates	$x\left[\frac{N}{2}\right] = \frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ (N even), $X_{DFT}\left[\frac{N}{2}\right] = \sum_{n=0}^{N-1}(-1)^n x[n]$ (N even)		
Parseval's Relation	$\sum_{n=0}^{N-1}\lvert x[n]\rvert^2 = \frac{1}{N}\sum_{k=0}^{N-1}\lvert X_{DFT}[k]\rvert^2$		

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Table 2: Properties of the z- transform

Property	**Signal**	**z-transform**
Linearity	$a_1x_1[n]+a_2x_2[n]$	$a_1X_1(z) + a_2X_2(z)$
Time reversal	$x[-n]$	$X(z^{-1})$
Time shifting	i) $x(n - k)$ ii) $x(n + k)$	i) $z^{-k}X(z)$ ii) $z^{k}X(z)$
Convolution	$x_1(n)*x_2(n)$	$X_1(z)X_2(z)$
Correlation	$r_{x_1x_2}(l)=\sum_{n=-\infty}^{\infty}x_1(n)x_2(n-l)$	$R_{x1x2}(z) = X_1(z)X_2(z^{-l})$
Scaling	$a^n x(n)$	$X(a^{-1}z)$
Differentiation	$nx[n]$	$z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$
Time differentiation	$x[n]-x[n-1]$	$X(z)(1 - z^{-1})$
Time integration	$\sum_{k=0}^{\infty}X(k)$	$X(z)=\left(\frac{z}{z-1}\right)$
Initial value theorem	$\lim_{n\to 0} x(n)$	$\lim_{\lvert z\rvert\to\infty} X(z)$
Final value theorem	$\lim_{n\to\infty} x(n)$	$\lim_{\lvert z\rvert\to 1}\left(\frac{z-1}{z}\right)X(z)$

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Table 3: z-Transform Pairs

Signal $x(t)$	Sequence $x(n)$	z-Transform $X(z)$
$\delta(t)$	$\delta(n)$	1
$\delta(t-k)$	$\delta(n-k)$	z^{-k}
$u(t)$	$u(n)$	$\frac{1}{1-z^{-1}}=\frac{z}{z-1}$
	$-u(-n-1)$	$\frac{1}{1-z^{-1}}=\frac{z}{z-1}$
$r(t)=tu(t)$	$nu(n)$	$\frac{z^{-1}}{(1-z^{-1})^2}=\frac{z}{(z-1)^2}$
	$a^n u(n)$	$\frac{1}{1-az^{-1}}=\frac{z}{z-a}$
	$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}=\frac{z}{z-a}$
	$na^n u(n)$	$\frac{az}{(z-a)^2}$
	$-na^n u(-n-1)$	$\frac{az}{(z-a)^2}$
e^{-at}	e^{-an}	$\frac{1}{1-e^{-a}z^{-1}}=\frac{z}{z-e^{-a}}$
t^2	$n^2 u(n)$	$z^{-1}\frac{(1+z^{-1})}{(1-z^{-1})^3}=\frac{z(z+1)}{(z-1)^3}$
te^{-at}	ne^{-an}	$\frac{z^{-1}e^{-a}}{(1-e^{-a}z^{-1})^2}=\frac{ze^{-a}}{(z-e^{-a})^2}$
$\sin\omega_o t$	$\sin\omega_o n$	$\frac{z\sin\omega_o}{z^2-2z\cos\omega_o+1}$
$\cos\omega_o t$	$\cos\omega_o n$	$\frac{z(z-\cos\omega_o)}{z^2-2z\cos\omega_o+1}$

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Table 4: Digital- to- digital Transformations

Form	Band Edges	Mapping $s \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-\alpha}{1-\alpha z}$	$\alpha = \frac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$
Lowpass to highpass	Ω_C	$\frac{-(z+\alpha)}{1+\alpha z}$	$\alpha = \frac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\frac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \frac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha K}{K+1}, \; A_2 = \frac{K-1}{K+1}$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\frac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha}{K+1}, \; A_2 = \frac{1-K}{1+K}$

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Table 5 : Direct Analog- to- digital Transformations for Bilinear Design

From	Band Edges	Mapping s →	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-1}{C(z+1)}$	$C=\tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\frac{C(z+1)}{z-1}$	$C=\tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1<\Omega_0<\Omega_2$	$\frac{z^2-2\beta z+1}{C(z^2-1)}$	$C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$
Lowpass to bandstop	$\Omega_1<\Omega_0<\Omega_2$	$\frac{C(z^2-1)}{z^2-2\beta z+1}$	$C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$

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Table 6: Windows for FIR filter design.

Window	**Expression** $w_N[n]$, $-0.5(N-1) \leq n \leq 0.5(N-1)$
Boxcar	1
Cosine	$\cos\left(\frac{n\pi}{N-1}\right)$
Riemann	$\text{sinc}^L\left(\frac{2n}{N-1}\right), L>0$
Bartlett	$1-\frac{2\lvert n\rvert}{N-1}$
Von Hann (Hanning)	$0.5+0.5\cos\left(\frac{2n\pi}{N-1}\right)$
Hamming	$0.54+0.46\cos\left(\frac{2n\pi}{N-1}\right)$
Blackman	$0.42+0.5\cos\left(\frac{2n\pi}{N-1}\right)+0.08\cos\left(\frac{4n\pi}{N-1}\right)$
Kaiser	$\frac{I_0\left(\pi\beta\sqrt{1-4\left(\frac{n}{N-1}\right)^2}\right)}{I_0(\pi\beta)}$

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Table 7 : Characteristics of the windowed spectrum for various windows.

Window	**Peak Ripple** $\delta_p = \delta_s$	**Passband Attenuation** A_{WP}(dB)	**Peak Sidelobe Attenuation** A_{WS} (dB)	**Transition Width** $F_{WS} \approx C/N$
Boxcar	0.0897	1.5618	21.7	$C = 0.92$
Cosine	0.0207	0.3600	33.8	$C = 2.10$
Riemann	0.0120	0.2087	38.5	$C = 2.50$
von Hann (Hanning)	0.0063	0.1103	44.0	$C = 3.21$
Hamming	0.0022	0.0384	53.0	$C = 3.47$
Blackman	1.71×10^{-4}	2.97×10^{-3}	75.3	$C = 5.71$

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Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

Finite Summation Formula

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}, \quad \alpha \neq 1$$

$$\sum_{k=0}^{n} k\alpha^k = \frac{\alpha\left[1-(n+1)\alpha^n + n\alpha^{n+1}\right]}{(1-\alpha)^2}$$

$$\sum_{k=0}^{n} k^2\alpha^k = \frac{\alpha\left[(1+\alpha)-(n+1)^2\alpha^n + \left(2n^2+2n-1\right)\alpha^{n+1} - n^2\alpha^{n+2}\right]}{(1-\alpha)^3}$$

Infinite Summation Formula

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} \alpha^k = \frac{\alpha}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k\alpha^k = \frac{\alpha}{(1-\alpha)^2}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k^2\alpha^k = \frac{\alpha^2+\alpha}{(1-\alpha)^3}, \qquad |\alpha| < 1$$

$$\sum_{k=-\infty}^{\infty} e^{-\alpha|k|} = \frac{1+e^{-\alpha}}{1-e^{-\alpha}}, \qquad \alpha > 0$$