CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	SIGNALS AND SYSTEMS
COURSE CODE	:	BEB 20203
PROGRAMME CODE	:	BEJ
EXAMINATION DATE	:	DECEMBER 2018 /JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A: ANSWER **ALL** QUESTIONS SECTION B: ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **THIRTEEN (13)** PAGES

SECTION A: ANSWER ALL QUESTIONS

Q1. (a) According to the given signal $p(t)$ that shown in **Figure Q1(a)**, sketch a graph for each of the following signals:

(i) $p(t+1)$

(2 marks)

(ii) $-\frac{1}{2}p(0.5t-1)$

(4 marks)

(b) A system in **Figure Q1(b)** produces an output, $y(t)$ which is represented as:

$$y(t) = [u(t+2) - u(t-2)] + [u(t+1) - u(t-1)].$$

By using graphical method, find the input signal $x(t)$ when $g(t)$ is given by

$$g(t) = -2[u(t+1) - u(t-1)]$$

(4 marks)

Q2 (a) Consider a periodic signal $x(t)$ is defined by

$$x(t) = 3 + 4\sin(2w_o t) + 7\cos(4w_o t + 30°)$$

(i) Find exponential Fourier Series coefficients of signal $x(t)$.

(5 marks)

(ii) Sketch the magnitude and phase spectrum of signal $x(t)$.

(2 marks)

(b) Calculate the average power, P_{av} supplied to a network if the applied voltage, $v(t)$ and resulting current, $i(t)$ are given by

$$v(t) = 50 + 10\cos\left(30\pi t + \frac{\pi}{2}\right) + 15\cos(45\pi t - \frac{\pi}{3}),$$
$$i(t) = 20 + 5\cos(40\pi t + 20°) - 10\cos(100\pi t - 50°).$$

(3 marks)

Q3 (a) A signal $x(t)$ is given by

$$x(t) = 4\,\mathrm{rect}\left(\frac{t}{3}\right).$$

Find the Fourier transform of

(i) $x(t)$

(2 marks)

(ii) $y(t) = \frac{d}{dt}\left(x(t)\right)$

(2 marks)

(b) Given two signals,

$$x_1(t) = u(t - 1.5) - u(t - 2.5),$$

$$x_2(t) = u(t + 2.5) - u(t + 1.5).$$

(i) Sketch $y(t) = x_1(t) + x_2(t)$.

(2 marks)

(ii) Determine $F[y(t)]$.

(4 marks)

Q4. Given the signal

$$x(t) = 2\cos(\omega t)\,u(t).$$

(a) Find the Laplace transform of $x(t)$ using the definition of Laplace transform.

(4 marks)

(b) Sketch the zero-pole plot and region of convergence (if it exists) of the signal $x(t)$.

(2 marks)

(c) Signal $x(t)$ becomes the input signal for a delayed system as illustrated in **Figure Q4(c).** Solve $\mathcal{L}[y(t)]$.

(4 marks)

SECTION B: ANSWER THREE (3) QUESTIONS ONLY

Q5 (a) Test the stability of the following continuous-time systems:

(i) $h_1(t) = e^{2t}u(t+4)$,

(2.5 marks)

(ii) $h_2(t) = e^{-4t}u(t-4)$.

(2.5 marks)

(b) Determine and sketch the convolution of the following two signals:

$$x(t) = u(t) - 2u(t-1) + u(t-2),$$

$$h(t) = \begin{cases} t & ,0 \le t \le 1 \\ 0 & ,elsewhere. \end{cases}$$

(15 marks)

Q6 (a) Consider the periodic signal $x(t)$ as shown in **Figure Q6(a)**.

(i) Show that the amplitude-phase Fourier series of $x(t)$ is

$$x(t) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\frac{n\pi}{3} \cos\left(\frac{2\pi n}{3\times10^{-3}} t\right).$$

(ii) Sketch the amplitude-phase plot for the signal $x(t)$ for $n = 0$ to $n = 5$.

(14 marks)

(b) The signal $x(t)$ in **Q6(a)(i)** is passed through a low pass filter (LPF) given in **Figure Q6(b)**.

(i) Determine the frequency response of the LPF.

(2 marks)

(ii) If the cutoff frequency, f_{cutoff} of the LPF is f_o, where f_o is the frequency of the first harmonic, determine the output of the LPF for the first harmonic.

(4 marks)

Q7 (a) A basic modulator circuit is shown in **Figure Q7(a).** Modulation is a multiplication between message signal $m(t)$, and a carrier signal $c(t)$. The process yields a new signal, $v(t)$.

(i) Analyze the Fourier transform of the output signal $v(t)$ by using modulation properties.

(4 marks)

(ii) Sketch the spectrum of signal $v(t)$.

(1 marks)

(b) A Linear Time Invariant (LTI) system has the impulse response,

$$h(t) = \delta(2t + 3).$$

If the input, $x(t) = e^{-3t}u(t)$, determine the system output, $y(t)$.

(5 marks)

(c) By using the Fourier transform, determine the impulse response, $h(t)$ for the following system:

$$\frac{dy(t)}{dt} + 2y(t) = x(t) + \frac{dx(t)}{dt}$$

(5 marks)

(d) The voltage across a 10-Ω resistor of an RC circuit in **Figure Q7(d)** is given by $v_R(t) = 5\,e^{-3t}u(t)\ V$. Determine the total energy dissipated by this resistor using Parseval's theorem.

(5 marks)

Q8 (a) The unit impulse response of an LTI system is $h(t) = e^{-t}(\sin 2t) \cdot u(t)$. Use the Laplace transform to find its response to

$$x(t) = (\sin(t)) \cdot u(t)$$

(12 marks)

(b) **Figure Q8(b)** shows a transient circuit consists of a DC voltage source, V, a capacitor, C, an inductor, L and a switch. The switch is moved from position 1 to 2 at $t = 0$. Determine, using Laplace Transform,

(i) the current flowing through the capacitor before the transition,

(1 marks)

(ii) current after the switch is moved from position 1 to 2.

(4 marks)

(iii) Sketch the $i(t)$ if $V = 5V, R = 10\Omega,\ C = \frac{1}{10}F$ and $L = \frac{1}{20}H$ for both before and after the transition.

(3 marks)

-END OF QUESTIONS-

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2018/2019 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

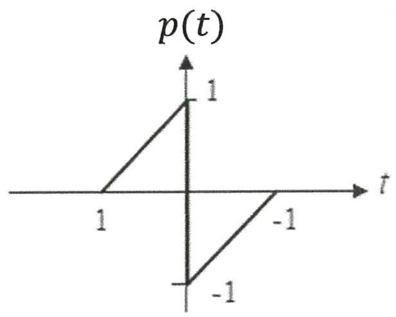

Figure Q1(a)

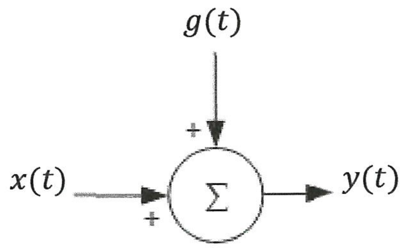

Figure Q1(b)

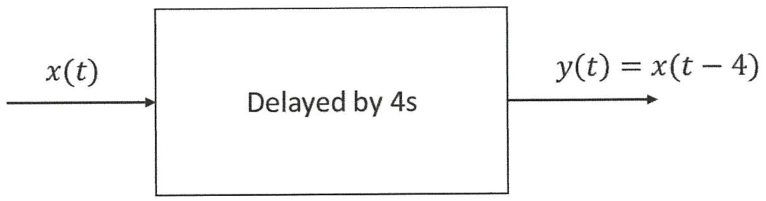

Figure Q4(c)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2018/2019 PROGRAMME CODE: BEJ

COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

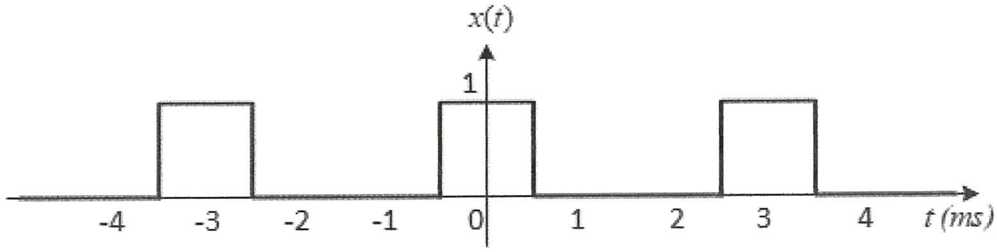

Figure Q6(a)

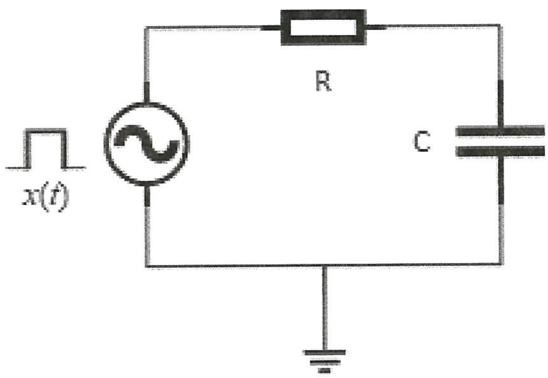

Figure Q6(b)

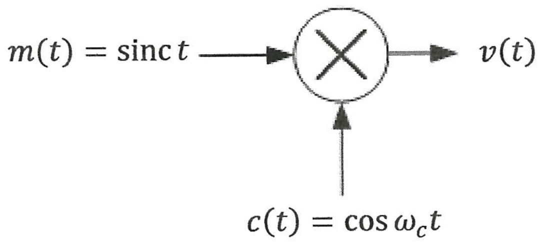

Figure Q7(a)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2018/2019 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

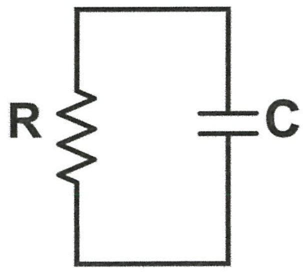

Figure Q7(d)

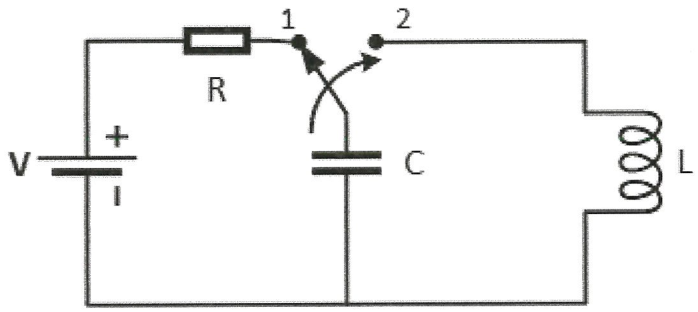

Figure Q8(b)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2018/2019 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 1: INDEFINITE INTEGRALS

$\int \cos at\,dt = \frac{1}{a}\sin at$	$\int \sin at\,dt = -\frac{1}{a}\cos at$
$\int t\cos at\,dt = \frac{1}{a^2}\cos at + \frac{1}{a}t\sin at$	$\int t\sin at\,dt = \frac{1}{a^2}\sin at - \frac{1}{a}t\cos at$
$\int te^{at}dt = \frac{1}{a^2}e^{at}(at-1)$	$\int \frac{1}{(a^2+t^2)}dt = \frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)$

TABLE 2: EULER'S IDENTITY

$e^{\pm j\pi/2} = \pm j$	$A\angle \pm\theta = Ae^{\pm j\theta}$
$e^{\pm jk\pi} = \cos k\pi$	$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$
$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$	$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$

TABLE 3: COMPLEX NUMBER

$s = a + jb = \|s\|\angle \pm\theta = \|s\|e^{\pm j\theta}$	$\|s\| = \sqrt[2]{a^2+b^2}$	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

TABLE 4: TRIGONOMETRIC IDENTITIES

$\sin\theta = \cos\left(\left(\theta - \frac{\pi}{2}\right)\right)$	$\cos\theta = \sin(\theta + \frac{\pi}{2})$
$\sin(\propto \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin^2\alpha + \cos^2\beta = 1$	
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 5: VALUES OF COSINE, SINE AND EXPONENTIAL FUNCTIONS FOR INTEGRAL MULTIPLE OF π.

Function	Value	Function	Value
$\cos 2n\pi$	1	$e^{j2n\pi}$	1
$\sin 2n\pi$	0	$e^{jn\pi}$	$(-1)^n$
$\cos n\pi$	$(-1)^n$	$e^{\frac{jn\pi}{2}}$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = \text{even} \\ j(-1)^{\frac{n-1}{2}} & ,n = \text{odd} \end{cases}$
$\sin n\pi$	0		
$\cos\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = \text{even} \\ 0 & ,n = \text{odd} \end{cases}$	$\sin\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n-1}{2}} & ,n = \text{odd} \\ 0 & ,n = \text{even} \end{cases}$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2018/2019 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 6: FOURIER SERIES

Exponential	$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\frac{2\pi}{T}t}$ $x_n = \frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\, e^{-jn\frac{2\pi}{T}t}\, dt$
Trigonometric	$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\frac{2\pi}{T}t + b_n \sin n\frac{2\pi}{T}t$ $a_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\cos n\frac{2\pi}{T}t\, dt, \quad n = 0, 1, 2, 3 \ldots$ $b_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\sin n\frac{2\pi}{T}t\, dt, \quad n = 1, 2, 3 \ldots$
Amplitude-phase	$x(t) = X_0 + \sum_{n=1}^{\infty} A_n \cos(n\frac{2\pi}{T}t + \theta_n)$ $A_n = 2\lvert X_n\rvert = \sqrt[2]{a_n^2 + b_n^2} \quad , \quad \theta_n = \angle X_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$
Average Power	$P = V_{dc}I_{dc} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_{V_n} - \theta_{I_n})$

TABLE 7: DEFINITION OF FOURIER AND LAPLACE TRANSFORM

FOURIER TRANSFORM	INVERSE FOURIER TRANSFORM
$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$ $\mathcal{F}[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\, dt$	$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega$ $x(t) = \mathcal{F}^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\, df$
LAPLACE TRANSFORM	**INVERSE LAPLACE TRANSFORM**
Bilateral $\mathrm{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$ **Unilateral** $\mathrm{L}[x(t)] = X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$ $s = \sigma + j\omega$	$x(t) = \mathrm{L}^{-1}[X(s)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2018/2019 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 8: FOURIER TRANSFORM PAIRS

Time domain, $x(t)$	Frequency domain, $X(\omega)$	Frequency domain, $X(f)$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	$\frac{\delta(f)}{2}+\frac{1}{j2\pi f}$
$u(t+\tau)-u(t-\tau)$	$\frac{2\sin(\omega\tau)}{\omega}=2\tau\,\mathrm{sinc}(\omega\tau)$	$2\tau\,\mathrm{sinc}\,2f\tau$
$\mathrm{rect}(t)$	$\mathrm{sinc}(\frac{\omega}{2})$	$\mathrm{sinc}(f)$
$\lvert t\rvert$	$-\frac{2}{\omega^2}$	$-\frac{2}{(2\pi f)^2}$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$\frac{1}{\alpha+j2\pi f}$
$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$\frac{1}{\alpha-j2\pi f}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	$\frac{2a}{a^2+4\pi^2f^2}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$\delta(f-f_0)$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$\frac{n!}{(a+j2\pi f)^{n+1}}$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$\frac{\delta(f-f_o)-\delta(f+f_o)}{2j}$
$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$	$\frac{\delta(f-f_o)+\delta(f+f_o)}{2}$
$e^{-at}\sin\omega_0 t\,u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$\frac{2\pi f_0}{(a+j2\pi f)^2+(2\pi f_0)^2}$
$e^{-at}\cos\omega_0 t\,u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$\frac{a+2\pi f}{(a+j2\pi f)^2+(2\pi f_0)^2}$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2018/2019 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 9: FOURIER TRANSFORM PROPERTIES

Property	Time domain, $x(t)$	Frequency domain, $X(\omega)$	Frequency domain, $X(f)$
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(\omega)+a_2X_2(\omega)$	$a_1X_1(f)+a_2X_2(f)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
Time shifting	$x(t-t_0)u(t-t_0)$	$e^{-j\omega t_0}X(\omega)$	$e^{-j2\pi f t_0}X(f)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$	$X(f-f_o)$
Modulation	$\cos(\omega_0 t)\,x(t)$ $\sin(\omega_0 t)\,x(t)$	$\frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$ $\frac{1}{2j}[X(\omega-\omega_0)-X(\omega+\omega_0)]$	$\frac{1}{2}[X(f-f_o)+X(f+f_o)]$ $\frac{1}{2j}[X(f-f_o)-X(f+f_o)]$
Time differentiation	$\frac{d}{dt}(x(t))$ $\frac{d^n}{dt^n}(x(t))$	$j\omega X(\omega)$ $(j\omega)^n X(\omega)$	$j2\pi f X(f)$ $(j2\pi f)^n X(f)$
Time integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{X(\omega)}{j\omega}+\pi X(\omega)\,\delta(\omega)$	$\frac{X(f)}{j2\pi f}+\frac{1}{2}X(0)\delta(f)$
Frequency differentiation	$t^n x(t)$	$(j)^n\frac{d^n}{d\omega^n}X(\omega)$	$\left(\frac{j}{2\pi}\right)^n\frac{d^n}{df^n}X(f)$
Time Reversal	$x(-t)$	$X(-\omega)\ or\ X^*(\omega)$	$X(-f)$
Duality	$X(t)$	$2\pi x(-\omega)$	$X(-f)$
Convolution in t	$x_1(t)*x_2(t)$	$X_1(\omega)\cdot X_2(\omega)$	$X(f)\cdot Y(f)$
Multiplication	$x_1(t)\cdot x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$	$X(f)*Y(f)$
Parseval's Theorem	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2 dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2 d\omega$	$\int_{-\infty}^{\infty}\lvert X(f)\rvert^2 df$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2018/2019 PROGRAMME CODE: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 10: LAPLACE TRANSFORM PAIRS

Time domain, $x(t),\ t>0$	s-domain, $X(s)$	ROC	Time domain, $x(t),\ t>0$	s-domain, $X(s)$	ROC
$\delta(t)$	1	All s	$\cos bt$	$\frac{s}{s^2+b^2}$	$Re(s)>0$
$u(t)$	$\frac{1}{s}$	$Re(s)>0$	$\sin bt$	$\frac{b}{s^2+b^2}$	$Re(s)>0$
t	$\frac{1}{s^2}$	$Re(s)>0$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$Re(s)>-a$
t^n	$\frac{n!}{s^{n+1}}$	$Re(s)>0$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$	$Re(s)>-a$
e^{-at}	$\frac{1}{s+a}$	$Re(s)>-a$	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$	$Re(s)>0$
te^{-at}	$\frac{1}{(s+a)^2}$	$Re(s)>-a$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s)>0$

TABLE 11: LAPLACE TRANSFORM PROPERTIES

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t), x_2(t)$	$X_1(s), X_2(s)$	R_1, R_2
Linearity	$ax_1(t)+bx_2(t)$	$aX_1(s)+bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s-s_0)$	Shifted version of R (i.e., s is in the ROC if $s-s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t)*x_2(t)$	$X_1(s)\cdot X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
		$sX(s)-x(0^+)$ (Unilateral)	R right hand plane
	$\frac{d^n}{dt^n}x(t)$	$s^nX(s)-s^{n-1}x(0^+)-\cdots-sx^{n-2}(0^+)-x^{n-1}(0^+)$	
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{Re(s)>0\}$

Initial- and Final- Value Theorems

If $x(t)=0$ for $t<0$ and $x(t)$ contains no impulses or higher order singularities at $t=0$, then

$$x(0^+)=\lim_{s\to\infty} sX(s)$$

If $x(t)=0$ for $t<0$ and has a finite limit as $t\to\infty$, then

$$\lim_{t\to\infty} x(t)=\lim_{s\to\infty} sX(s)$$