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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BEE 31602 / BWM 30602
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	DECEMBER 2018 / JANUARY 2019
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	ANSWERS **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 Based on statistics issued by Marine Department under Ministry of Tourism and Culture Malaysia, the number of domestic visitors enter that Langkawi via the Kuah's Jetty is higher than international visitors which approximately 2 million since 2012. The number of visitors (in million unit) according to years are: f(2012)=2.11, f(2013)=2.18, f(2016)=2.18, and f(2017)=2.10.

(a) Construct Newton's divided difference for number of visitors enter Langkawi (in million).

(6 marks)

(b) Based on the result in **Q1(i),** approximate the number of visitors that enter Langkawi in the year 2014.

(3 marks)

(c) Approximate the number of visitors in the year 2015 using linear, quadratic, and cubic Lagrange interpolations.

(9 marks)

(d) What is the number of visitors that enter Langkawi in year 2018? Explain your answer.

(2 marks)

Q2 The current i of a circuit at time t is given in the table below:

t, sec	3	5	6	7	8
$i(t)$	0.244	0.405	0.597	0.824	1.093

Given the voltage drop, $V_L(t) = L\dfrac{di}{dt}$ and the inductance, L is 3H.

(a) Give the **FOUR (4)** suitable methods that could be used to find $V_L(6)$.

(4 marks)

(b) Calculate $V_L(6)$ using the **FOUR (4)** suitable methods in 4 decimal places.

(16 marks)

Q3 The π is the ratio of the circumference of a circle to the diameter. Jacob Bernoulli, one of many famous mathematicians of the infamous Bernoulli family proved the following formula:

$$\pi = \int_0^1 \frac{4}{1+x^2}\,dx$$

(a) Compute the value of π using the above formula using trapezoidal and simpson 1/3 rule with $n = 8$ in 4 decimal places.

(10 marks)

(b) The resistance force of a moving car is due to the constant tire resistance and the air resistance coefficient. An experiment is conducted in which the total resistance force can be measured for the velocity of different cars.

Calculate the resistance force by referring the following experimental data using suitable Simpson's rules in 4 decimal places.

Velocity (m/s)	2	5	8	11	14	17	20	23	26	29
Resistance Force (N)	137	133	223	198	345	264	338	450	439	482

(10 marks)

Q4 (a) A simple RL-electrical circuit consists of a constant resistance R (in ohms), a constant inductance L (in henrys) and an electromotive force $E(t)$ (in volts). According to Kirchoff's second law, the current i (in amperes) in the circuit satisfies the equation:

$$L\frac{di}{dt} + Ri = E(t)$$

Given $E(t) = 220$ volts, $L = 4$ henry, $R = 20$ ohms and $i = 0$ when $t = 0$,

(i) Analyze the current at $t = 0.04$ second using the fourth order Runge-Kutta method (RK4) with the step size $h = 0.02$ in 4 decimal places.

(8 marks)

(ii) Estimate the absolute error if the exact solution is $i = 11(1 - e^{-5t})$ in 4 decimal places.

(2 marks)

(b) Given $\frac{d^2y(t)}{dt^2} - 10y(t) = 0$, for $0 \le t \le 0.4$, with the boundary conditions of $y(0) = 0$ and $y(0.4) = 1$, calculate the values of $y(t)$ with step size $h = 0.1$ in 4 decimal places.

(10 marks)

Q5 (a) Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \; t > 0$$

with the boundary conditions $u(0,t) = u(1,t) = 0$ and the initial condition $u(x,0) = \sin 4\pi x$, $\frac{\partial u}{\partial t}(x,0) = 0$ for $0 \le x \le 1$, by using the finite-difference method with $\Delta x = h = 0.2$ and $\Delta t = k = 0.1$ until $t = 0.2$ in 4 decimal places.

(8 marks)

(b) Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, and $t > 0$ with the boundary conditions $u(0,t) = 20t^2$, $u(1,t) = 10t$, and the initial condition $u(x,0) = x(1-x)$ by using the explicit finite-difference method with $\Delta x = h = 0.5$ and $\Delta t = k = 0.1$ until $t = 0.3$ in 4 decimal places.

(12 marks)

-END OF QUESTIONS –

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FORMULAS

Interpolation by Cubic spline:

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1}-x)^3 + \frac{m_{k+1}}{6h_k}(x-x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6}h_k\right)(x_{k+1}-x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6}h_k\right)(x-x_k)$$

where $k = 0,1,2,\ldots,n-1$

$$\left.\begin{aligned} h_k &= x_{k+1}-x_k \\ d_k &= \frac{f_{k+1}-f_k}{h_k} \end{aligned}\right\}, \quad k = 0,1,2,\ldots,n-1$$

$$b_k = 6(d_{k+1}-d_k), \quad k = 0,1,2,\ldots,n-2$$

$$m_0 = 0$$

$$m_n = 0$$

$$h_k m_k + 2(h_k + h_{k+1})m_{k+1} + h_{k+1}m_{k+2} = b_k, \quad k = 0,1,2,\ldots,n-2$$

First Order Numerical differentiation:

3-point central difference: $f'(x) \approx \dfrac{f(x+h)-f(x-h)}{2h}$

5-point difference: $f'(x) \approx \dfrac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

Numerical Integration:

Trapezoidal rule: $\int_a^b f(x)dx \approx \dfrac{h}{2}\left[f_0 + f_n + 2\sum_{i=1}^{n-1} f_i\right]$

Simpson's $\frac{1}{3}$ rule: $\int_a^b f(x)dx \approx \dfrac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i\right]$

Simpson's $\frac{3}{8}$ rule:

$$\int_a^b f(x)dx \approx \frac{3}{8}h[f_0 + f_n + 3(f_1 + f_2 + f_4 + f_5 + \ldots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \ldots + f_{n-6} + f_{n-3})]$$

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Initial value problems:

Fourth-order Runge-Kutta Method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\quad k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems:

Finite difference method:

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h} \qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$