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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BEE 21503
PROGRAMME	:	BEV / BEJ
EXAMINATION DATE	:	DECEMBER 2018/JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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Q1 (a) (i) The first partial derivatives could be defined in terms of limit. Therefore, write the first partial derivatives formula for $f_x(x,y)$ and $f_y(x,y)$.

(2 marks)

(ii) State the difference between Chain Rule and Implicit Partial Differentiation in terms of its formula.

(3 marks)

(b) If z is implicity defined as a function of x and y by $x^2+y^2+z^2=1$, show that

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z-\frac{1}{z}.$$

(5 marks)

(c) An electric circuit, i in a microwave transmitter has resistances r and R. The current through r can be found from

$$i=\frac{IR}{r+R};$$

where I is the total current for the two branches. Assuming that I is constant at 85.4 mA, determine $\frac{\partial i}{\partial r}$ and evaluate it for $R=0.150\ \Omega$ and $r=0.032\ \Omega$.

(5 marks)

(d) Given $w=\frac{z^2}{\sqrt{xy+x}}$, $x=\sin\left(\frac{\theta}{2}\right)$, $y=\cos\left(\frac{\theta}{3}\right)$ and $z=\theta$. If $\theta=\pi$, prove that

$$\frac{dz}{d\theta}=2\pi\left(\frac{\pi\sqrt{3}}{24(1.5)^{3/2}}+\frac{1}{(1.5)^{1/2}}\right).$$

(10 marks)

Q2 (a) There are several interpretations of double integrals.

(i) State **THREE (3)** important interpretations of double integrals.

(2 marks)

(ii) Differentiate the **THREE (3)** interpretations in terms of its formula.

(3 marks)

(b) Consider the surface of paraboloid, $z = 1 - x^2 - y^2$.

(i) Sketch its graph.

(2 marks)

(ii) The surface area of paraboloid is given by $\iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$.

Convert the surface area to its associated polar coordinates.

(3 marks)

(iii) Based on the answer in **Q2 (b) (ii)**, find the surface area of paraboloid, $z = 1 - x^2 - y^2$ that lies above xy- plane.

(5 marks)

(c) A solid is bounded on top by an upper hemisphere given by $z = \sqrt{16 - x^2 - y^2}$ and below by a plane $z = 0$ and side by cylinder, $x^2 + y^2 = 4$. Its density function is given by $\delta(x, y, z) = z$. The mass of this solid is given by $m = \int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{0}^{\sqrt{16-x^2-y^2}} z\,dz\,dx\,dy$. Calculate the mass of the solid by using cylindrical coordinates.

(10 marks)

Q3 (a) (i) State **TWO (2)** characteristics of line integrals of scalar field.

(5 marks)

(ii) The work done by a force, **F** by moving a particle along path C is described by

$$\int_c \mathbf{F}.\,dr.$$

Explain in your own word what will happen to the work done if a conservative force, **F** is used to move a particle along different paths.

(5 marks)

(b) The second characteristic of line integrals of scalar field says that the value of line integral is depends on the path (the curve). Different paths will have different value of line integral. Show this characteristic by solving

$$\int_c 2xy + 3y\,dx - 6x\,dy.$$

along path C_1 and C_2 from (-2, 0) to (2, 0). Paths, C_1 and C_2 are shown in **Figure Q3 (b).**

(5 marks)

(c) A force $\mathbf{F}(x, y) = 4xy\,\mathbf{i} + 2x^2\,\mathbf{j} - ze^{2z}\mathbf{k}$ is moving a charge from (1, 4, -2) and (3, -2, -1). Determine the potential function and work done by this force.

(10 marks)

Q4 (a) Define Gauss's theorem and Stokes' theorem.

(5 marks)

(b) Consider the outward flux of vector field,

$$\mathbf{F}(x, y, z) = xy\mathbf{i} + (y^2 + e^{xz^2})\mathbf{j} + sin(xy)\mathbf{k}.$$

across the surfaces of a solid bounded by the paraboloid cylinder , $z = 1 - x^2$ and the planes $z = 0, y = 0$, and $y + z = 2$.

(i) Sketch the graph of the surfaces.

(2 marks)

(ii) Find the divergence of $\mathbf{F}(x, y, z)$.

(3 marks)

(iii) Use Gauss's theorem to evaluate the flux of the vector field $\mathbf{F}(x, y, z)$ across the surfaces.

(5 marks)

(c) Let the vector field is given by $\mathbf{F}(x, y, z) = z^2\boldsymbol{i} + y^2\mathbf{j} + x\mathbf{k}$. Evaluate $\oint_C \mathbf{F}.\mathbf{dr}$ by using Stokes' theorem, if C is the boundary of the triangle with vertices (1, 0, 0), (0, 1, 0) and (0, 0, 1) in the first octant and oriented counterclockwise looking from the top.

(10 marks)

- END OF QUESTIONS -

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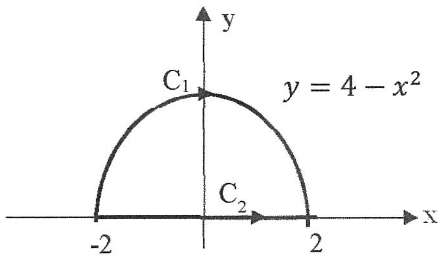

Figure Q3 (b)

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FORMULAS

Polar coordinate

$x = r\cos\theta,\ y = r\sin\theta,\ \theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$

Cylindrical coordinate

$x = r\cos\theta,\ y = r\sin\theta,\ z = z$ and $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)rdzdrd\theta$

Spherical coordinate

$x = \rho\sin\emptyset\cos\theta,\ y = \rho\sin\emptyset\sin\theta,\ z = \rho\cos\emptyset$, then $x^2 + y^2 + z^2 = \rho^2$, for $0 \le \theta \le 2\pi$, $0 \le \emptyset \le \pi$, and $\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\theta,\emptyset)\rho^2\sin\emptyset\, d\rho d\emptyset d\theta$

$m = \iint_R \delta(x,y)\, dA$, where $\delta(x,y)$ is a density of lamina

$$V = \iiint_G dV$$

$$m = \iiint_G \delta(x,y,z)\, dV$$

If f is a differentiable function of x, y and z, then the
Gradient of f, $grad\ \ f(x,\ y,\ z) = \nabla f(x,\ y,\ z) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

If $F(x,\ y,\ z) = Mi + Nj + Pk$ is a vector field in Cartesian coordinate, then the

Divergence of $\mathbf{F}(x,y,z)$, $div\ \ F = \nabla\cdot F = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

Curl of $\mathbf{F}(x,y,z)$, $curl\ \ F = \nabla\times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)i - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)j + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)k$

F is conservative vector field if Curl of **F** = 0.

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FORMULAS

Surface Integral

Let S be a surface with equation $z = g(x,\ y)$ and let R be its projection on the x-y plane.

$$\iint_S f(x,y,z)dS = \iint_R f(x,y,g(x,y))\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA$$

$\iint_S F \cdot n\ dS = \iiint_G \nabla \cdot F\ \ dV$

$\iint_S (\nabla \times F) \cdot n\ dS = \int_C F \cdot dr$