Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2017/2018

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BEE 31602
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	JUNE / JULY 2018
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	ANSWERS ALL QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 Based on statistics issued by Marine Department under Ministry of Tourism and Culture Malaysia, the number of domestic visitors enter Langkawi via Kuah's Jetty is higher than international visitors which approximately 2 million since 2012. The number of visitors (in million unit) according to years are: $f(2012) = 2.11$, $f(2013) = 2.18$, $f(2016) = 2.18$, and $f(2017) = 2.10$.

(a) Construct the third order Newton's divided difference equation to estimate the number of visitors entered Langkawi (in million).

(6 marks)

(b) Approximate the number of visitors that enter Langkawi in the year 2014 using the equation that obtained from **Q1(i).**

(3 marks)

(c) Approximate the number of visitors in the year 2015 using linear, quadratic, and cubic Lagrange interpolations.

(9 marks)

(d) Discuss the number of visitors that will enter Langkawi in year 2018.

(2 marks)

Q2 (a) Find the first derivatives of the function $f(x) = x^3 - 3x^2 - 130x + 150$ at point $x = 3$ using three points central difference with $h = 10^{-k}$ for $k = 1, 2$.

(6 marks)

(b) The following table gives the values of distance traveled by a car at various times from a tollgate at highway.

Time, t $(minute)$	3	5	7	9	11
Distance traveled, x (km)	4.6	8.030	11.966	16.885	19.904

Given that velocity, $v = x'(t)$. By taking $h = 2$ minutes estimate the velocity at time $t = 5$ minutes by using the **FOUR (4)** difference suitable methods in 4 decimal places.

(14 marks)

Q3 (a) If the current flowing in a circuit is related to time by the formula $i(t) = e^{-5t}\cos 5t$, and is applied to a capacitor with capacitance $C = 0.2F$. The voltage drops across the capacitor is given by $V_C = \frac{1}{C}\int i(t)\,dt$,

(i) Approximate V_C, $0 \le t \le 1$ with $h = 0.1$ by using trapezoidal rule and suitable Simpson's rule (9 mark)

(ii) Find the exact integral by using your scientific calculator. (1 mark)

(iii) Find the absolute error for each method (from **Q3(a)(i)**). (2 marks)

(iv) Determine which method approximates better. (1 mark)

(b) The velocity (m/s) of an upward rocket at time t seconds is given by

$$v(t) = u\ln\left(\frac{m_0}{m_0 - qt}\right) - gt,$$

where m_0 (kg) is the initial mass of the rocket at $t = 0\ s$, q (kg/s) is the rate at which fuel is expelled, and u (m/s) is the velocity at which the fuel is being expelled. The initial mass of the rocket is 100,000 kg and the rocket expel fuel at a velocity of 1400 m/s at a consumption rate of 1500 kg/s and $g = 9.8067\,\text{m}^2/\text{s}$.

(i) Find the vertical distance travelled by the rocket from time $t = 10\ s$ to $t = 19\ s$ with $h = 1\ s$ by using suitable Simpson's method. (5 marks)

(ii) If the exact solution is

$$d(t) = \int v(t)\,dt = ut\ln\left(\frac{m_0}{m_0 - qt}\right) + ut + \frac{um_0}{q}\ln(m_0 - qt) - \frac{1}{2}gt^2,$$

find the absolute error.

(2 marks)

Q4 (a) Given $\frac{d^2y(t)}{dt^2} - 10y(t) = 0$, for $0 \le t \le 0.4$, with the boundary conditions of $y(0) = 0$ and $y(0.4) = 1$, calculate the values of $y(t)$ with step size $h = 0.1$ in 4 decimal places.

(10 marks)

(b) A series RL circuit consists of a resistor and an inductor that connected to a AC power supply. According to Kirchoff's second law, the current i (in amperes) in the circuit satisfies the following equation.

$$L\frac{di(t)}{dt} + Ri(t) = E(t)$$

Given $E(t) = 220$ V, $L = 4$ H, $R = 20\ \Omega$, and $i = 0$ A when $t = 0$,

(i) Calculate the current at $t = 0.04$ s using the fourth order Runge-Kutta method (RK4) with the step size $h = 0.02$ in 4 decimal places.

(8 marks)

(ii) Estimate the absolute error if $i(t) = 11(1 - e^{-5t})$ in 4 decimal places.

(2 marks)

Q5 (a) The temperature distribution $u(x,t)$ of one dimensional silver rod is governed by the heat equation as follow.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\quad 0 \le x \le 1, \text{ and } t > 0$$

Given the boundary conditions $u(0,t) = 20t^2$, $u(1,t) = 10t$, for $t > 0$ and the initial condition $u(x,0) = x(1-x)$ for $0 \le x \le 1$, analyze the temperature distribution of the rod with $\Delta x = h = 0.25$ and $\Delta t = k = 0.02$ until $t = 0.04$ s only, by using the explicit finite difference method.

(10 marks)

(b) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 2$, and $t > 0$ with the boundary conditions $u(0,t) = u(2,t) = 0$ and the initial condition $u(x,0) = \sin(\pi x)$, $\frac{\partial u}{\partial t}(x,0) = 0$ for $0 \le x \le 2$, by using the finite-difference method with $\Delta x = h = 0.5$ and $\Delta t = k = 0.1$ until $t = 0.2$ s in 4 decimal places.

(10 marks)

-END OF QUESTIONS –

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FORMULAS

Interpolation

Lagrange Polynomial: $P_n(x)=\sum_{i=0}^{n}L_i(x)f(x_i)$; where $L_i(x)=\prod_{\substack{j=0\\j\neq i}}^{n}\frac{(x-x_j)}{(x-x_j)}$

Newton's Divided Difference

$$f_i^{[0]}=f_i, \qquad i=0,1,\ldots\ldots,n$$

$$f_i^{[j]}=\frac{f_{i+1}^{[j-1]}-f_i^{[j-1]}}{x_{i+j}-x_i}, \quad j=1,2,\ldots\ldots,n$$

$$f(x)\approx P_n(x)=f_0^{[0]}+f_0^{[1]}(x-x_0)+f_0^{[2]}(x-x_0)(x-x_1)\ldots+f_0^{[n]}(x-x_0)(x-x_1)\ldots.(x-x_{n-1})$$

Numerical differentiation:

2-point forward difference

$$f'(x)=\frac{f(x+h)-f(x)}{h}+O(h^2)$$

2-point backward difference

$$f'(x)=\frac{f(x)-f(x-h)}{h}+O(h^2)$$

3-point central difference

$$f'(x)=\frac{f(x+h)-f(x-h)}{2h}+O(h^2)$$

3-point forward difference

$$f'(x)\approx\frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}+O(h^2)$$

Numerical Integration:

Trapezoidal rule: $\int_a^b f(x)dx\approx\frac{h}{2}\left[f_0+f_n+2\sum_{i=1}^{n-1}f_i\right]$

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Simpson's $\frac{1}{3}$ rule: $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i\right]$

Simpson's $\frac{3}{8}$ rule:

$$\int_a^b f(x)dx \approx \frac{3}{8}h\left[f_0 + f_n + 3(f_1 + f_2 + f_4 + f_5 + \ldots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \ldots + f_{n-6} + f_{n-3})\right]$$

Initial value problems:

Fourth-order Runge-Kutta Method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$, $k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$, $k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems:

Finite difference method:

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h} \qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Central time central space (CTCS) finite-difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \Leftrightarrow \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$

$$u_{i,j+1} = \rho^2 u_{i-1,j} + (2 - 2\rho^2)u_{i,j} + \rho^2 u_{i+1,j} - u_{i,j-1} \qquad \rho^2 = \frac{k^2 c^2}{h^2}$$

Forward time central space (FTCS) finite-difference method:

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x}\right)_{i,j} \Leftrightarrow \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$u_{i,j+1} = ru_{i-1,j} + (1 - 2r)u_{i,j} + ru_{i+1,j} \qquad r = \frac{kc^2}{h^2}$$