Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2017/2018

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BEE 11403
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	JUNE/JULY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWERS ALL QUESTIONS

THIS QUESTION PAPER CONSISTS OF **TWELVE (12)** PAGES

Q1 (a) Show that the first-order differential equation $xy' - 2x^2 y = 0$ can be obtained by using:

(i) separating-variable method, and

(2 marks)

(ii) power series method without using recurrence relations.

(8 marks)

(b) Solve using power series method for the given second-order differential equation with initial condition $q(0) = 0$.

$$q''(t) + 5q'(t) = 40$$

(10 marks)

Q2 (a) An RLC circuit can be modelled as:

$$3\frac{d^2V}{dt^2} + 3\frac{dV}{dt} + 3V = 0$$

Solve the second order ordinary differential equation for the initial condition $V(0) = 9$ and $V'(0) = 0$.

(5 marks)

(b) Given a system of first order linear differential equation as below:

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} e^{2x} \\ 10 \end{pmatrix}$$

(i) Find the complimentary function, $\boldsymbol{Y_c}$ for the homogeneous system.

(6 marks)

(ii) Find the general solution, $\boldsymbol{Y}$ for the system differential equation.

(9 marks)

Q3 (a) Calculate Laplace transform of the function $v(t) = t + 3e^{-3t}H(t-2)$.

(6 marks)

(b) An RL circuit as shown in **Figure Q3(b)** can be modelled by differential equation:

$$0.1\frac{di(t)}{dt} + 2i(t) = 10t\,H(t-2).$$

Determine the current, $i(t)$ using Laplace transform where the initial current is 0.

(14 marks)

Q4 (a) In a communication system, a cosine signal, $f(x)$ propagates with an amplitude of 2 volts and frequency of $\frac{1}{\pi}$ Hz. In the meantime, the signal experiences 2π shift.

(i) Write the function of the signal $f(x)$.

(2 marks)

(ii) Sketch the graph of the signal $f(x)$ and state whether it is an odd or even or neither odd nor even function.

(3 marks)

(b) An RLC circuit diagram as shown in **Figure Q4(b)** consists of a resistor, capacitor, and an inductor with the respected values of 0.2 Ω, 0.05 F, and 1 H. Given that,

$$E(t) = \frac{1}{2}t^2 \quad , \quad -\pi < t < \pi$$
$$E(t) = E(t+2\pi)$$

By using Fourier Series Expansion, show that the circuit can be governed by:

$$i'' + 0.2\,i' + 20\,i = \sum_{n=1}^{\infty}\frac{2}{n}(-1)^{n+1}\sin nt$$

where $n = 1,2,3,\ldots$

(15 marks)

Q5 (a) Given that a non-periodic signal as follow:

$$v(t) = \begin{cases} e^{5t} & t < 0 \\ e^{-5t} & t > 0 \end{cases}$$

Proof that the Fourier Transform of $v(t)$ is $V(\omega) = \frac{10}{25+\omega^2}$.

(5 marks)

(b) Determine $Y(\omega)$ for the given $y(t)$ if $x(t) = \cos(20t)$ by using Fourier Transform pair and properties,

(i) $y(t) = e^{j1000t}\, x(t)$

(2 marks)

(ii) $y(t) = x(t-8)$

(2 marks)

(iii) $y(t) = x(3t-7)$

(3 marks)

(c) Consider an electrical circuit as visualized in **Figure Q5(c)** with $R = 3\ \Omega$, $C = 0.5$ F, and $L = 1$ H.

(i) Determine the transfer function, $H(\omega)$ of the circuit.

(5 marks)

(ii) Find the time domain transfer function, $h(t)$ of **Q5(c)(i)**.

(3 marks)

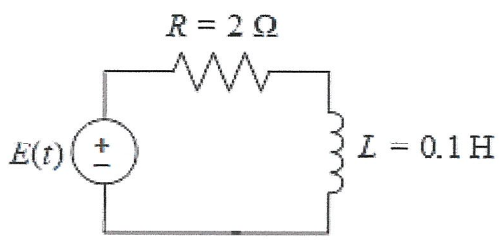

Figure Q3(b)

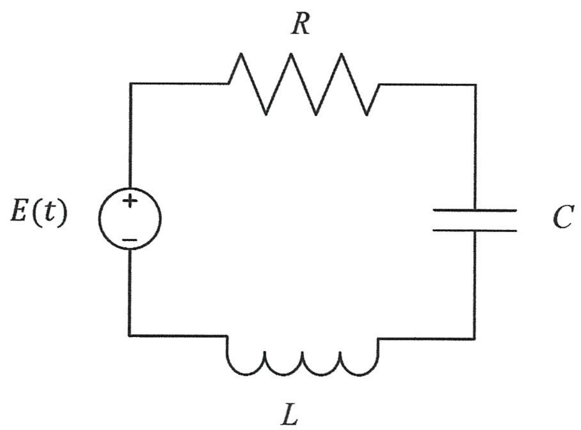

Figure Q4(b)

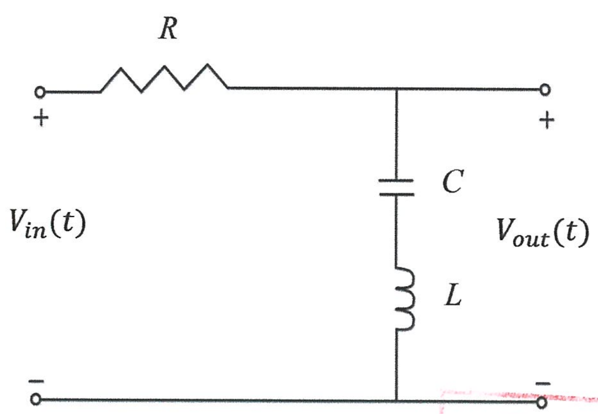

Figure Q5(c)

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FORMULAS

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients for system of first order linear differential equations

For non-homogeneous for system of first order linear differential equations $\mathbf{Y}'(x) = A\,\mathbf{Y}(x) + \mathbf{G}(x)$, the particular solution $\mathbf{Y}_p(x)$ is given by:

$\mathbf{G}(x)$	$\mathbf{Y}_p(x)$	$\mathbf{G}(x)$	$\mathbf{Y}_p(x)$
$\mathbf{u}$	$\mathbf{a}$	$\mathbf{u}e^{\lambda x}$	$\mathbf{a}e^{\lambda x}$
$\mathbf{u}x + \mathbf{v}$	$\mathbf{a}x + \mathbf{b}$	$\mathbf{u}\cos\alpha x$ or $\mathbf{u}\sin\alpha x$	$\mathbf{a}\sin\alpha x + \mathbf{b}\cos\alpha x$

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FORMULAS

Power Series Method

$$\sum_{m=0}^{\infty} c_m x^m = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots$$

Where $c_0,\ c_1,\ c_2$ ... are constants

Representation of Functions in Power Series

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \sum_{m=0}^{\infty} \frac{x^m}{m!}, -\infty < x < \infty$
$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \sum_{m=0}^{\infty} (-1)^m \frac{x^{2m+1}}{(2m+1)!}, -\infty < x < \infty$
$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \sum_{m=0}^{\infty} (-1)^m \frac{x^{2m}}{(2m)!}, -\infty < x < \infty$
$\ln(1+x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \sum_{m=1}^{\infty} (-1)^{m+1} \frac{x^m}{m!}$
$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \sum_{m=0}^{\infty} x^m$
$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \cdots$

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Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
e^{at}	$\frac{1}{s-a}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\sinh at$	$\frac{a}{s^2-a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y(t)$	$Y(s)$
$e^{at}f(t)$	$F(s-a)$	$y'(t)$	$sY(s)-y(0)$
$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

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Table of Fourier Transform

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$\mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t}\, d\omega$$

$$\int_p^q \delta(t-a) f(t) dt = f(a)$$

$f(t)$	$F(\omega)$	$f(t)$	$F(\omega)$
$\delta(t)$	1	$\mathrm{sgn}(t)$	$\frac{2}{i\omega}$
$\delta(t-\omega_0)$	$e^{-i\omega_0\omega}$	$H(t)$	$\pi\delta(\omega)+\frac{1}{i\omega}$
1	$2\pi\delta(\omega)$	$e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{1}{\omega_0+i\omega}$
$e^{i\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$t^n e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{n!}{(\omega_0+i\omega)^{n+1}}$
$\sin(\omega_0 t)$	$i\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	$e^{-at}\sin(\omega_0 t)H(t)$ for $a>0$	$\frac{\omega_0}{(a+i\omega)^2+\omega_0^2}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$	$e^{-at}\cos(\omega_0 t)H(t)$ for $a>0$	$\frac{a+i\omega}{(a+i\omega)^2+\omega_0^2}$
$\int_0^t f(u)g(t-u)\,du$	$F(\omega)\cdot G(\omega)$		
$\sin(\omega_0 t)H(t)$	$\frac{\pi}{2}i[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$		
$\cos(\omega_0 t)H(t)$	$\frac{\pi}{2}[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]+\frac{i\omega}{\omega_0^2-\omega^2}$		

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Properties of Fourier Transform

Property	$f(t)$	$F(\omega)$
Linearity	$\alpha f(t)+\beta g(t)$	$\alpha F(\omega)+\beta F(\omega)$
Time scaling	$f(at)$	$\frac{1}{\lvert a\rvert}F(\frac{\omega}{a})$
Time shifting	$f(t-a)$	$e^{-i\omega a}F(\omega)$
Frequency shifting	$e^{i\omega_0 t}f(t)$	$F(\omega-\omega_0)$
Time differentiate	$f^{(n)}(t)$	$(i\omega)^n F(\omega)$

Fourier Series

Fourier series expansion of periodic function with period $2L/2\pi$	Half Range series
$a_0=\frac{1}{L}\int_{-L}^{L} f(x)\,dx$	$a_0=\frac{2}{L}\int_{0}^{L} f(x)\,dx$
$a_n=\frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$	$a_n=\frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n=\frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$	$b_n=\frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$
$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$	$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$

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IDENTITY OF TRIGONOMETRY

1.	$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$ $\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$ $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$ $\sin A \sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$
2.	Negative angles: $sin(-\theta) = -sin(\theta), cos(-\theta) = cos(\theta), tan(-\theta) = -tan(\theta)$
3.	$sin2\theta = 2sin\theta cos\theta$ $cos2\theta = cos^2\theta - sin^2\theta = 2cos^2 - 1 = 1 - 2sin^2\theta$ $tan2\theta = \frac{2tan\theta}{1 - tan^2\theta}$
4.	$cos\, n\pi = (-1)^n = \begin{cases} -1, & if\ n\ odd \\ 1, & if\ n\ even \end{cases}$ $\sin n\pi = 0\ for\ n = 0,1,2,...$ $cos2n\pi = 1, \quad sin2n\pi = 0 \quad for\ n = 0,1,2,...$ $cos\frac{n\pi}{2} = \begin{cases} 0, & if\ n\ odd \\ (-1)^{n/2}, & if\ n\ even \end{cases}$ $sin\frac{n\pi}{2} = \begin{cases} (-1)^{\frac{n-1}{2}}, & if\ n\ odd \\ 0\ . & if\ n\ even \end{cases}$

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Electrical Formula

1. Voltage drop across resistor, R (Ohm's Law): $v_R = iR$
2. Voltage drop across inductor, L (Faraday's Law): $v_L = L\frac{di}{dt}$
3. Voltage drop across capacitor, C (Coulomb's Law): $v_c = \frac{q}{C}$ or $i = C\frac{dv_c}{dt}$
4. The relation between current, i and charge, q: $i = \frac{dq}{dt}$